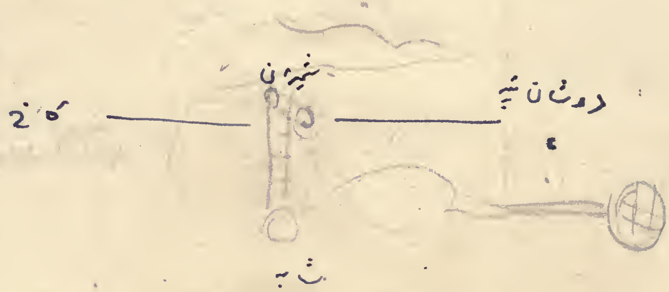

TWENTIETH CENTURY TEXT-BOOKS

LORD KELVIN (SIR WILLIAM THOMSON) (1824–1907)

William Thomson ranks as one of the two or three greatest physicists of the nineteenth century. He was born in Ireland, but spent nearly his entire life in Glasgow, Scotland, where his father was professor of mathematics. At the early age of seventeen he began to write on mathematical subjects, but his attention was soon turned to the study of physics. In 1846 he became professor of natural philosophy at Glasgow, where he remained fifty-three years.

Thomson's early investigations led to the invention of the absolute scale of temperature. Important experiments in heat were carried out by him in collaboration with Joule from 1852–1862.

Later Thomson became almost universally known by his work as electrician of the Atlantic cables, the first of which was laid in 1858. By the invention of the well-known mirror galvanometer which he used in receiving messages, he increased the rate of transmission from two or three to about twenty-five words per minute. This instrument was replaced later by his "siphon recorder," which is still in use.

Thomson was instrumental in establishing the practical system of electrical units. He invented the absolute and quadrant electrometers for measuring potentials, a sounding device for use on moving ships, and many other practical measuring instruments. His favorite subject, however, was the nature of the ether, which, as he said, claimed his attention daily for over forty years. His writings are included in his *Papers on Electrostatics and Magnetism, Mathematical and Physical Papers, Popular Lectures and Addresses,* and Thomson and Tait's *An Elementary Treatise on Natural Philosophy.*

TWENTIETH CENTURY TEXT-BOOKS

A HIGH SCHOOL COURSE IN PHYSICS

BY

FREDERICK R. GORTON, B.S., M.A., Ph.D.
ASSOCIATE PROFESSOR OF PHYSICS, MICHIGAN STATE NORMAL COLLEGE

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CP.

PREFACE

Physics is the summary of a part of human experience. Its development has resulted from the fact that its pursuit has successfully met human needs. Hence it is believed that the presentation of the subject in the secondary school should be the expansion of the everyday life of the pupil into the broader experience and observation of those whose lives have been devoted to the study. Human activity and progress, therefore, should be the teacher's guiding principle, and the bearing of each phenomenon and law on the interests of mankind should be clearly disclosed and emphasized. The author of this book has accordingly endeavored to give great prominence to facts of common observation in the derivation of physical laws, and, further, has attempted to point out plainly the service that has been afforded mankind by a knowledge of nature's laws. No textbook, however, can take the place of the skilled teacher in showing clearly the relation of physical phenomena to human activities, or in the selection of illustrative examples within the range of observation of his pupils.

Large portions of the subject-matter of Physics deal with knowledge already possessed by a pupil of high-school age, and nothing is of more appealing interest to him than the feeling that this information is to be made of some value. By recalling phenomena well within the acquaintance of the pupil and supplementing them with demonstrative experiments, the way is easily paved to the deduction and interpretation of general principles. In the presentation of such experiments, the author has described as simple and inexpensive apparatus as he has found to be consistent with satisfactory results.

No effort has been spared to give the teacher and pupil every possible assistance. The sections have been plainly set off and given suggestive headings; references to related material have been inserted where needed; and the numerous sets of exercises have been carefully graded. In order to bring the problems near the discussions upon which their solutions depend, they have been arranged in more and smaller groups than is usual. The exercises throughout the book have been selected from concrete cases, and the usual problems in pure reductions have been omitted. Illustrative solutions of problems and suggestions have been given wherever difficulties have been found to arise. As an aid to the pupil in reviewing and to the teacher in conducting rapid drill exercises, a summary of the contents of each chapter is presented at its conclusion.

The educational value of the portraits and biographical sketches of many of the great men of science is at once apparent. Emphasis upon the parts that these men have played in the development of Physics has been recognized by eminent educators as an important factor in creating an atmosphere of human interest around the subject. These names should become familiar to every student of Physics.

On account of the rapid advance at the present time in the practical uses made of physical principles, the author believes that the sections involving new applications will be found of general interest and utility.

The book will be found to be free from the more difficult uses of algebraic and geometric principles. The place of first importance has been given to the study of phenomena, and mathematical expressions have been introduced as convenient means of designating exact relations which have been previously interpreted. It is mainly in the subject of Physics that the pupil is brought to realize the value of his mathematical studies in the world of concrete quantities.

The author believes that the class-room work in the subject should be accompanied by a sufficient number of individual laboratory exercises to fix clearly in mind great principles and important phenomena. Further, enough practice with simple drawing instruments should be given to enforce the use of the simplest geometrical relations.

In addition to the many subjects whose treatment is demanded by the achievements of recent times, the author has given careful attention to the various topics recommended by the Committee of Secondary School Teachers to the College Entrance Examination Board.

The author desires to acknowledge here the many helpful suggestions of those who have read and criticised the proof or manuscript, and

wishes especially to mention Professor E. A. Strong of the Michigan State Normal College; Mr. Fred R. Nichols of the Richard T. Crane Manual Training High School, Chicago, Ill.; Mr. Frank B. Spaulding of the Boys' High School, Brooklyn; Mr. Albert B. Kimball, Principal of the High School, Fairhaven, Mass.; Professor Karl E. Guthe of the University of Michigan; Mr. George A. Chamberlain, Principal of East Division High School, Milwaukee; and Mr. J. M. Jameson of the Pratt Institute, Brooklyn, N.Y.

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CHAPTER I

INTRODUCTION

1. **Physics.** — In the experiences of everyday life we witness a great variety of changes in the things around us. Objects are moved, melted, evaporated, solidified, bent, made hot or cold, and undergo a change in their condition, place, or shape in a great many other ways. Physics is the science that treats of the properties of different substances and the changes that may take place within or between bodies, and it investigates the conditions under which such changes occur.

In its broadest sense Physics is the science of phenomena. Every action of which we become aware through the senses is a *phenomenon*. We hear the rolling thunder, we see the shining of a live coal, we taste the dissolving sugar, we smell the evaporating oil, and we feel moving air. By considering his own experience the student will be able to recall numerous examples of physical phenomena and to state the sense by means of which he perceives each of them.

The study of physics, however, not only directs our attention to the phenomena to which we are accustomed, but to a multitude of more unusual but not less important ones. It also strives to put these phenomena to experimental tests that will enable us to understand the laws connecting actions with their causes.

2. **Utility of the Study of Physics.** — Increasing acquaintance with nature and natural law has been the means of

elevating man from the life of limited power and usefulness of the savage to his condition of present-day enlightenment. The early discovery of fire was a great step toward civilization. By some crude experimental study it was found later that fire could be produced at will, as by the striking of flint and by rubbing two pieces of dry wood together. Thus the observation of simple natural phenomena enabled man to secure heat for cooking his food and warming his habitation, besides aiding him in forming implements to procure food, improve his shelter, and give him protection from enemies.

This same observation of natural phenomena has produced every existing artificial device for our protection, convenience, and comfort. The engineer who plans a railroad, with its bridges, tunnels, and grades, together with the locomotive and its train, makes use at every step of knowledge acquired through the study of Physics. The surveyor ascertains how to cut through the hills and fill the valleys by the use of instruments which involve physical principles. By the discovery and application of physical laws scientists and inventors have produced the telescope, telephone, steam engine, electric car, and all the other useful appliances which form so important a part of our everyday life.

3. Matter. — There are three general characteristics by which matter is recognized.

(1) Matter always occupies space. On this account it is said to possess the property of *extension*. Many invisible bodies of matter exist. The air in a bottle or tumbler is such a body. We may show, however, by the following experiment that it is as real as any other body:

1. Place a piece of cork upon the surface of water in a vessel, cover it with an inverted tumbler, and force the tumbler deep into the water as in Fig. 1. The air that was in the tumbler still occupies nearly all of that space, and the water is not allowed to rise and fill it.

(2) All matter is *indestructible*, in the sense that it has never been discovered that the smallest portion can be annihilated by any process known to man. Causing a body, as a piece of coal, to disappear by burning does not destroy the material of which it is composed. A portion is carried away in the smoke and gases, and the remainder left behind in the ash produced. In the process of evaporation a drop of water becomes invisible; but the matter still exists in the atmosphere as a transparent vapor.

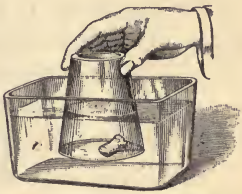


FIG. 1.—Inverted Tumbler Nearly Full of Air.

(3) All matter has *weight*, *i.e.* is attracted by the earth. In a more general sense it may even be said that every body has an attraction for, or pulls upon, every other body. The term *gravitation* is used to express this characteristic of matter.

The fact that air possesses weight as well as extension may be shown by experiment as follows:

2. Remove the brass fixture from an incandescent lamp bulb, and carefully balance the bulb on a sensitive beam balance. Introduce a short nail into the stem of the bulb, and tap lightly with a hammer until the glass is broken and air admitted. If the bulb is now placed upon the balance, a decided increase in weight will be observed. The weight of air admitted has been added to that of the bulb, which originally contained almost no air.

The quantity of matter in a body is called its **mass**. Different kinds of matter, as gold, water, glass, air, mercury, salt, hydrogen, etc., are called **substances**. Substances are recognized by their properties; as, hardness, elasticity, tenacity, fluidity, transparency, etc.

4. Measurement of Quantities. — A little reflection will show the necessity of having systems of measurement for the various quantities that we find in nature, such as length, area, volume, weight, time, etc. The importance of such systems has been recognized by the governments of civilized countries, and the values of the units employed have been fixed by law. Thus the foot, pound, second, etc., are well-established units of quantity, and are in general use throughout Great Britain and the United States.

The English system of measurement, however, is objectionable on account of the inconvenient relations between the units and their multiples and divisions. For example, 1 pound = 16 ounces = 7000 grains; or 1 mile = 320 rods = 5280 feet = 63,360 inches. It is mainly for this reason that many countries have adopted the *metric system* of measurement, in which the relations are always to be expressed by some power of *ten*. This system greatly reduces the effort required in making correct computations.[1] Since in the United States both the English and the metric system are employed, it will be advisable to become proficient in the use of each.

5. Measures of Extension. — Every body occupies space of three dimensions: length, breadth, and thickness; but each of these is simply a *length*, the metric unit of which is the *meter*. The meter is the distance between two transverse lines ruled on a platinum-iridium bar kept in the Archives at Sèvres, near Paris.[2] On account of the

[1] Congress recognized the desirability of introducing the metric system as early as 1866. See *Congressional Globe*, Appendix, Part 5, p. 422, Chap. CCCI: *An act to authorize the use of the metric system of weights and measures.* By this act the *yard* is defined as $\frac{3600}{3937}$ of a meter.

[2] The meter was originally intended to be one ten-millionth of the distance from the equator to the north pole. Accurate copies of the meter and other metric units are kept in the U. S. Bureau of Standards at Washington, D.C.

changes in the length of the bar with variations of temperature, the distance must be taken when the bar is at the temperature of freezing water. The multiples and divisions of the meter are designated by prefixes signifying the relations which they bear to the unit. The multiples have the Greek prefixes, *deka* (ten), *hecto* (hundred), *kilo* (thousand), and *myria* (ten thousand). The divisions have the Latin prefixes, *deci* (tenth), *centi* (hundredth), and *milli* (thousandth). The relations are shown in the following table:

Metric Table of Length

[1] A myriameter		equals	10,000	meters.
A kilometer	(km.)	equals	1,000	meters.
[1] A hectometer		equals	100	meters.
[1] A dekameter		equals	10	meters.
A decimeter	(dm.)	equals	0.1	of a meter.
A centimeter	(cm.)	equals	0.01	of a meter.
A millimeter	(mm.)	equals	0.001	of a meter.

The most important equivalents in the English system are the following:

1 meter	(m.)	equals	39.37 inches, or 1.094 yards.
[2] 1 centimeter		equals	0.3937 of an inch.
1 kilometer		equals	0.6214 of a mile.

The metric equivalents most frequently used are:

1 yard equals 91.44 centimeters, or 0.9144 m.
[2] 1 inch equals 2.540 centimeters.
1 mile equals 1.609 kilometers.

The relative sizes of the inch and the centimeter are shown in Fig. 2.

Because the meter is too large for general use in Physics, the *centimeter* has been chosen as the unit. The *centimeter* and the *gram*, which is the metric unit of mass, and the *second* as a time unit, are together the fundamental units

[1] Seldom used. [2] To be memorized.

of the so-called *centimeter-gram-second* (C. G. S.) system of measurement which is in use throughout the world in scientific work.

6. Surface Measure. — The unit of surface, or area, in the C. G. S. system is the *square centimeter* (cm.^{2}). It is the area of a square whose edge is one centimeter in length. One square inch is thus obviously equal to $(2.540)^2$, or 6.4516 square centimeters. The relative sizes of these two units are shown in Fig. 3.

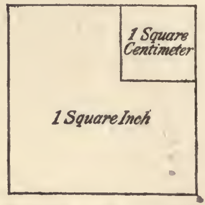


FIG. 3. — Relative Sizes of the Square Inch and Square Centimeter.

7. Cubic Measure. — The unit of volume in the C. G. S. system is the *cubic centimeter* (cm.^{3}). This unit is defined as the volume of a cube whose edge is one centimeter in length. One cubic inch thus equals $(2.540)^3$, or 16.387 cubic centimeters. The relative sizes of these units are shown in Fig. 4.

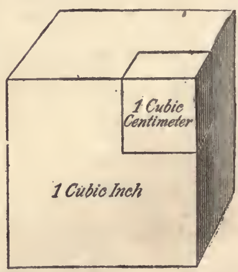


FIG. 4. — Relative Sizes of the Cubic Inch and Cubic Centimeter.

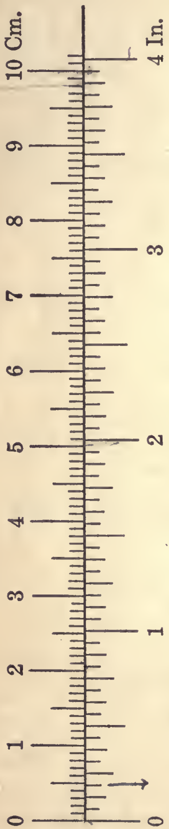


FIG. 2. — Showing the Relative Sizes of the English Inch and the Metric Centimeter.

8. Measures of Capacity. — The unit of capacity in the metric system is the *liter* (pronounced *lee'ter*), which is equal in size to a cubic decimeter, or one thousand cubic centimeters. The liter is somewhat larger than the liquid quart and smaller than the dry quart. More precisely, a liter equals 1.057 liquid quarts and 0.908 of a dry quart. Multiples and divisions of the liter are designated by the prefixes explained in § 5, but are little used in ordinary physical measurements.

EXERCISES

1. The distance from Detroit to Chicago is 280 mi. What is the metric equivalent of this distance?

2. Which is the lower price for silk, $1 per yard or $1.10 per meter? How much is the difference?

3. A tourist while in Paris pays the equivalent of 50 ct. per meter for cloth worth 40 ct. per yard. How much is the loss on a purchase of 20 m.?

4. If the cost of water is 10 ct. per thousand gallons, what is the equivalent cost per cubic meter? (1 gal. = 231 cu. in.)

5. How much dearer in Germany is oil costing the equivalent of 5 ct. per liter than the same in the United States at 15 ct. per gallon? Express the result in cents per gallon.

6. How much more cloth will $1 buy at 20 ct. per meter than at 18 ct. per yard when the width is 30 in.? Express the result in square inches.

7. If a railroad ticket in France costs the equivalent of $19.50 per thousand kilometers, what is the rate per mile?

8. If illuminating gas in Germany is sold at the equivalent of 3.5 ct. per cubic meter, what is the corresponding price per thousand cubic feet?

9. Measures of Mass. — The unit of mass in the metric system is the *kilogram*, and in the C. G. S. system, the *gram*. The gram-mass is the one-thousandth part of the mass of a standard platinum-iridium cylinder preserved in the Archives of France. The entire mass of this cylinder is one *kilogram* (abbreviated to *kilo*, pronounced *kee'lo*). This standard kilogram was intended to be equal to the mass of one thousand cubic centimeters, or one liter, of pure water; in fact, it may be considered so without appreciable error.

This relation between mass and volume in the metric system is of great convenience in physics. Since one cubic centimeter of water has a mass of one gram, if we

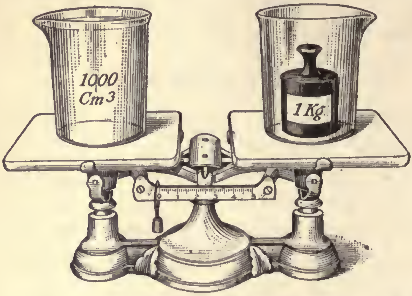


FIG. 5. — Relation of the Unit of Mass to the Unit of Volume.

know the mass in grams of a certain volume of water, the volume also is known, and *vice versa*. (See Fig. 5.)

METRIC TABLE OF MASS

[1] A myriagram		equals	10,000 grams (g.).
A kilogram	(kg.)	equals	1,000 grams.
[1] A hectogram		equals	100 grams.
[1] A dekagram		equals	10 grams.
A decigram	(dg.)	equals	0.1 of a gram.
A centigram	(cg.)	equals	0.01 of a gram.
A milligram	(mg.)	equals	0.001 of a gram.

The English unit of mass used in Physics is the avoirdupois pound containing 7000 grains and defined as being $\frac{1}{2.2046}$ of a kilogram. Its multiple is the ton, or 2000 pounds; its divisions, the ounce and the grain.

The English and metric equivalents most frequently used are as follows:

1 pound	is equal to	453.59 grams.
1 ounce	is equal to	28.35 grams.
[2] 1 kilogram	is equal to	2.20 pounds.
1 gram	is equal to	15.43 grains.

[1] Seldom used. [2] To be memorized.

EXERCISES

1. Express the mass of a cubic inch of water in grams.

2. What is the mass of a cubic decimeter of water in pounds?

3. Sugar at 6 ct. per pound costs how much per kilogram?

4. A cubic centimeter of mercury has a mass of 13.6 g. Find the mass of a cubic inch of mercury in ounces. *Ans.* 7.86 oz.

5. How many pounds are there in a cubic foot of water?

6. How many grams of water will be required to fill a rectangular vessel measuring $20 \times 25 \times 30$ cm.? Reduce to pounds.

7. One cubic centimeter of iron has a mass of 7.5 g. Find the mass of an iron plate 150 cm. square and 2 mm. thick.

8. An empty flask weighs 100 g.; when filled with water, the entire mass is 365 g. What is the capacity of the flask?

9. If the mass of a given volume of gold is 19 times that of an equal volume of water, what is the mass of 25 cm.^{3} of gold? What is the volume of a gold body whose mass is 10 g.?

10. Mass Distinguished from Weight. — *Mass* and *weight* must not be regarded as synonymous terms. If it were not for the fact that any two masses attract each other (§ 3), we should have little use for the word *weight*. This attraction between common bodies is beyond our power to detect by ordinary means, because it is so slight. When, however, one of the attracting bodies is massive, as the earth, and the other is some object, as a stone, the attraction is great enough to be easily perceived as we try to support or lift the smaller body. *This downward pull of the earth upon the stone is called the* **weight** *of the stone.* The weight, or earth-pull, of a body changes when it is taken to a different latitude, or is elevated above, or lowered beneath, the surface of the earth (§ 68). It is therefore obvious that the weight of a body may change while the quantity of matter in it, *i.e.* its mass, remains the same.

11. Processes of Weighing. — Since equal masses are attracted equally by the earth at any given place, weighing offers one of the most convenient and accurate means for comparing masses; thus, to make one mass equal to another, we have only to adjust the quantity of each until they stretch the spring of a dynamometer, Fig. 6, equally; or, as we say, "weigh alike" when placed on any weighing device. The usual process of determining the mass of a body consists in placing it upon one pan of a beam balance, Fig. 7, and known masses, called "weights," upon the other pan until the two balance. The sum of the known masses used gives the mass of the body. Such known masses, ranging from one milligram up to several hundred grams, constitute a so-called "set of weights."

FIG. 6. — A Dynamometer or Spring Balance.

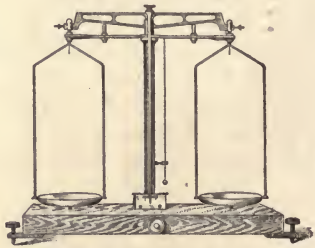


FIG. 7. — A Beam Balance.

12. Density. — Everyday experience teaches us that bodies may have the same size and yet differ greatly in weight. A bar of iron, for example, is much heavier than a bar of wood of the same dimensions, because its mass is much greater. The substances are said to differ in **density**. *The density of a substance is measured by the number of units of mass contained in a unit of volume.* Thus in the C. G. S. system it is expressed as the number of grams per cubic centimeter; in the common, or foot-pound-second (F. P. S.), system, by the number of pounds per cubic foot. For example, the density of lead is 11.36 grams per cubic centimeter (abbreviated 11.36 $g./cm.^3$).

EXERCISES

1. Find the density of a liquid of which a liter has a mass of 850 g.

2. If the volume of a piece of glass whose mass is 10 g. is 3.9 cu. cm., what is the density of the glass?

3. The density of mercury is 13.6 g./cm.^{3} Calculate the mass of mercury that can be contained in a vessel whose capacity is 30 cm.^{3}

4. If mercury is 90 ct. per pound, what will half a liter cost?

5. A vessel will hold 500 g. of mercury. How many grams of water will be required to fill it?

6. The diameter of a steel sphere is 4 cm. If the density of steel is 7.8 g./cm.^{3}, what is the mass of the sphere? Volume of a sphere $= \frac{1}{6}\pi d^3$.

13. States of Matter. — Matter admits of being separated into three classes according to its ability to preserve (1) its shape and volume, (2) its volume only, or (3) neither its shape nor volume. A body that retains both its shape and volume is called a *solid;* one that retains its volume only and shapes itself to the vessel containing it, a *liquid;* while one that occupies completely any vessel in which it is placed, retaining neither form nor size, is called a *gas.* Ice, water, and steam are examples of the same substance in the three states.

14. Time. — All systems of measurement of time, used in Physics, employ the interval called the *mean solar second* as the unit. It is $\frac{1}{86400}$ of a mean solar day, the average length of time intervening between two successive transits of the sun's center across a meridian.

SUMMARY

1. Physics is the science of phenomena (§ 1).

2. Matter is always recognized by its properties of expansion, indestructibility, and weight. Different kinds of matter — gold, water, air, etc., are recognized by their properties; *i.e.* hardness, fluidity, etc. (§ 3).

3. The quantity of matter in a body is called its *mass.*

4. There are two well-known systems of measurement for the quantities of length, area, volume, time, etc., viz. the English system and the metric system (§ 4).

5. The units of the metric system are the centimeter, gram, and second. This system is the more generally used for scientific work. It is called the *centimeter-gram-second* (C. G. S.) system (§ 5).

6. Units of the English system are the foot, pound, and second. This system is therefore called the *foot-pound-second* (F. P. S.) system (§ 12).

7. The terms *mass* and *weight* have not the same meaning. The *weight* of a body refers to the downward pull of the earth upon the body. The mass of a body may remain constant, while the weight varies with the latitude, the altitude above the earth's surface, and the depth to which it may be lowered into the earth (§ 10).

8. Masses are measured and compared by the process of weighing (§ 11).

9. The *density* of a substance is measured by the number of units of mass contained in a unit of volume (§ 12).

10. Matter may be divided into three classes according to its ability to preserve (1) its shape and volume, (2) its volume only, or (3) neither its shape nor volume. Thus bodies are classed as solids, liquids, and gases (§ 13).

CHAPTER II

MOTION, VELOCITY, AND ACCELERATION

1. UNIFORM MOTIONS AND VELOCITIES

15. Motion and Rest Relative Terms. — The position of a body at any instant is defined by its direction and distance from some other body which is usually conceived as being fixed, or *at rest*. **Motion** *is a continuous change in the position of a body.* It is customary to think of the earth as the fixed body when we speak of the motion of a train, a bird, a cloud, etc. Again, a passenger sitting in a moving railway coach is in the condition of rest with respect to the train, while with respect to the earth the same person is in rapid motion. Even the earth, as we know, is not at rest; it not only rotates on its axis, but travels with great speed in its orbit around the sun. Hence a body at rest with respect to the earth is not actually at rest, nor is its motion with respect to the earth the actual motion. However, when no statement is made to the contrary, the earth is regarded as the body to which the motion of an object is referred.

16. Path of a Moving Body. — The line described by a small moving body is called its *path*. When the path described is a straight line, the motion is *rectilinear;* when curved, the motion is *curvilinear*. Let us first consider cases of rectilinear motion.

17. Uniform Rectilinear Motion. — *If a moving body describes equal portions of its path in equal intervals of time, no matter how small the intervals may be, its motion is* **uniform.** In other words, uniform motion is the motion of a body when the distance passed over is proportional to the time occupied. In the case of uniform rectilinear motion the *velocity,* or rate of motion, of the moving body is constant in both magnitude and direction, and is measured by the distance which the body travels per second, per minute, per hour, etc.; for example, 10 centimeters per second (abbreviated 10 cm./sec.), 25 miles per hour, etc.

18. Equation of Uniform Motion. — From § 17 it can easily be seen that *the entire distance passed over by a body having uniform motion can be found by multiplying the velocity by the time.* Thus, if the velocity is 20 cm./sec., the distance passed over in five seconds is 5×20, or 100, centimeters. This relation between the distance d, the velocity v, and the time t is conveniently expressed by the equation

$$\mathbf{d = vt.} \tag{1}$$

19. Representation of a Motion. — In describing completely the rectilinear motion of a body, the following characteristics must be given: (*a*) the *starting point*, (*b*) the *direction*, and (*c*) the *distance traveled.* It will be observed that a straight line, since it has origin, direction, and length, is capable of representing the three characteristics of rectilinear motion. Hence the line AB, Fig. 8, drawn from the point A a distance of 4 centimeters to the right may be used to represent the three qualities of the motion of a body 4 miles in an easterly direction from the place represented by the point A. By letting a centimeter represent 5 miles the same line will represent the characteristics of the rectilinear motion of a body over a distance of 20 miles. Thus any convenient scale may be used, but the same scale should, of course, be used throughout a given problem.

FIG. 8. — Representation of a Rectilinear Motion.

20. Average Velocity. — Absolute uniform motion is of very rare occurrence, except during exceedingly small intervals of time. For instance, a train on leaving a station starts slowly and gains in speed until it acquires the velocity with which it can follow schedule time. It would be practically impossible for the engineer so to regulate the throttle as to maintain an absolutely constant velocity, inasmuch as the resistance due to the track, curves, wind, etc., would vary from time to time. Finally the throttle is closed, the brakes applied, and the train comes gradually to rest. Although the velocity has changed greatly, the train has passed over a certain distance in a definite time; let us say 30 miles in 40 minutes. The train has moved just as far as it would have traveled with a uniform velocity found by dividing the total distance by the time consumed, *i.e.* $\frac{3}{4}$ of a mile per minute. This is called the *average velocity* of the train for the time under consideration. From this explanation it is obvious that equation (1) will hold for motions that are not uniform, provided v is the average velocity for the time t.

21. Velocity at Any Instant. — If we examine the motion of all the objects with which we are familiar, we shall find, as in the case of the train in § 20, that the velocity in almost every instance is either increasing or decreasing. Hence velocity cannot be defined accurately as the distance over which a body moves in a unit of time. We must, therefore, consider the velocity of a body at a given instant, *i.e.* at some stated time. *The velocity of a body at a given instant is the distance it would move in a second if at that instant its motion were to become uniform.*

22. Representation of a Velocity. — A velocity has the characteristics of *magnitude* (*i.e.* speed) and *direction.* Therefore, a straight line of a definite length may conveniently be used to represent the velocity of a body at a given instant. Let the velocity of a body be 10 miles per hour north. A straight line, as AB, Fig. 9, drawn from A to B in an upward direction and having a length of 10 units, will completely represent the given velocity. As in the case of motions (§ 19), any convenient length may be selected to represent a unit of velocity, but the same length should be used throughout any given discussion. Any quantity, as velocity, having direction as well as magnitude, is called a **vector quantity**, and the line representing it, a **vector**.

Fig. 9. — Representation of a Velocity.

EXERCISES

1. A velocity of 60 mi. per hour is how many feet per second?

2. Express a velocity of 10 m. per minute in centimeters per second. Draw the vector that represents the velocity when the direction is eastward.

3. A train travels 100 mi. in two and one half hours. Calculate the average velocity in feet per second.

4. The speed of an electric car averages 20 ft. per second. How far will it travel in three hours?

5. A train whose length is 440 yd. has a velocity of 45 mi. per hour. How long will it take the train to pass completely over a bridge 100 ft. long? *Ans.* 21.51 sec.

6. A wheel 50 cm. in diameter revolves 600 times per minute. Express the speed of a point on its rim in centimeters per second. *Ans.* 1570.8 cm./sec.

7. Assuming that the radius of the earth is 4000 mi. and that it revolves on its axis once in exactly 24 hr., ascertain the speed of a point at the equator. *Ans.* 1047.2 mi./hr.

2. UNIFORMLY ACCELERATED MOTION

23. Acceleration. — We have spoken of the manner in which a train leaves a station. Starting from rest and gradually gaining in speed, it finally attains the desired velocity. Let us suppose that after the train has been moving one second its velocity is 20 cm./sec.; at the end of two seconds from the instant of starting, 40 cm./sec.; at the end of three seconds, 60 cm./sec., etc. While the train continues to move in this manner, the velocity is increasing the same amount during each second, viz., 20 cm./sec. This quantity, *the rate at which the velocity changes with the time, is called the* **acceleration** *of the train.* Since the change in velocity per second is measured in centimeters per second, the acceleration of the train is conveniently expressed thus: 20 cm./sec.^{2}, and read "20 centimeters per second per second." The acceleration of a body is *positive* or *negative* according as the velocity increases or decreases. The acceleration of a falling body, for example, is positive; that of a body thrown upward, negative.

24. Uniformly Accelerated Motion. — *If the rate at which the velocity of a moving body changes with the time be constant,* — i.e. *if the acceleration remain uniform,* — *the motion is called* **uniformly accelerated motion.** The motion of the train considered in § 23 is of this type. There occur in nature many cases in which the condition that defines uniformly accelerated motion is very nearly fulfilled; *e.g.* falling bodies, bodies thrown upward, bodies moving freely along inclined planes, etc.

25. Velocity Acquired and Distance Traversed. — The *velocity acquired* in a given number of seconds by a body having uniformly accelerated motion is found in a manner which the following example clearly illustrates:

A body starts from rest and gains in speed 4 cm./sec. in each second of time.

At the end of one second its velocity is 4 cm. per second.
At the end of two seconds its velocity is 8 cm. per second.
At the end of three seconds its velocity is 12 cm. per second.
At the end of t seconds its velocity is $4\,t$ cm. per second.

It is plain, therefore, that the velocity v at the end of any number of seconds t is found by multiplying the acceleration a by the time; or,

$$\mathbf{v} = \mathbf{at}. \tag{2}$$

Again, since the initial velocity for a given interval of time is 0, and the final velocity is v, and since the gain in velocity is uniform, the mean velocity for the interval is $(0 + v) \div 2$. If the acceleration is 4 cm./sec.^{2} as before,

the average velocity for the first second is $\frac{0+4}{2}$ cm. per second;

the average velocity for the first two seconds is $\frac{0+8}{2}$ cm. per second;

the average velocity for the first t seconds is $\frac{0+4t}{2}$ cm. per second; and if the acceleration is a cm./sec.^{2}, the average velocity for t seconds is $\frac{0+at}{2}$ cm./sec., or $\frac{at}{2}$.

Now since the distance passed over is found by multiplying the average velocity by the time (§ 20), the distance that the body moves in t seconds when the acceleration is 4 cm./sec.^{2} is $\frac{4t}{2} \times t$, or $\frac{1}{2}(4t^2)$ cm.; but if the acceleration is a, we have for the distance, $\frac{at}{2} \times t$, or $\frac{1}{2}at^2$.

Hence $$\mathbf{d} = \tfrac{1}{2}\,\mathbf{at}^2. \tag{3}$$

EXAMPLE. — A car starts to move down an incline that is just steep enough to cause it to gain in velocity 50 cm./sec. during each second of time. Find the velocity and the distance traversed at the end of the fifth second, and the distance that the car moves during the fifth second.

SOLUTION. — The initial velocity is 0. The final velocity is the product of the gain per second and the number of seconds. Hence $v = 50 \times 5$, or 250 cm./sec.

The total distance traversed is the product of the average velocity multiplied by the number of seconds. The average velocity for 5 seconds is $\frac{0+250}{2}$, and the number of seconds is 5. Hence $d = \frac{250}{2} \times 5$, or 625 cm. The same result could be found by substituting the values of a and t in equation (**3**). The analysis of a problem, however, is far more valuable than the mere substitution of numbers in a formula.

The distance traversed during the fifth second is evidently the difference between the distance traversed in 5 seconds and that in 4 seconds. Now in 4 seconds the car moves $\frac{0 + 4 \times 50}{2} \times 4$, or 400 cm. Hence during the fifth second the car moves 625 – 400, or 225 cm.

We thus observe that if any two of the four quantities used in the discussion be given, the others can be calculated by the help of equations (**2**) and (**3**).

The relation of time and distance shown by equation (**3**) may be tested experimentally as follows:

Let a grooved board AB, Fig. 10, about 15 feet long be supported with one end elevated about 18 inches. The groove can easily be formed by nailing a strip of wood about 2 inches wide to the side of

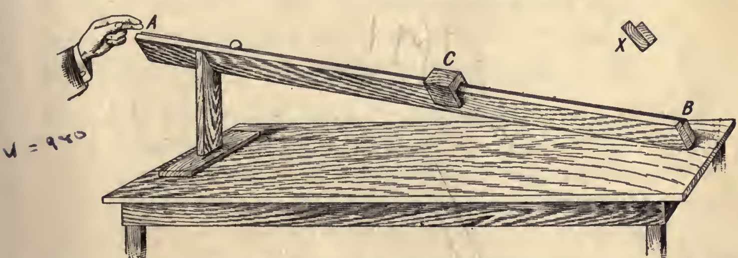


Fig. 10. — Verifying the Relation between the Distance Traversed and the Time in Uniformly Accelerated Motion.

a wider piece, forming a cross section as shown in X. Sufficient supports should be used to keep the groove straight. Arrange a seconds pendulum (§ 84) so as to make and break an electrical contact at the center of its path, and connect a telegraph sounder and a cell of battery in the circuit with the pendulum. The sounder should give a loud click at the end of every second. Release a marble at A precisely at the instant the sounder clicks, and place a block C at such a point on the incline that the click of the marble against C coincides with the click that marks the end of the third second. This point will have to be found by trial, and should be verified by two or three tests. The length AC gives the distance passed over by the marble in three seconds. Let the process be repeated for two seconds and one second. The distances found should be proportional to the squares of the times, as shown by equation **(3)**; *i.e.* as **1 : 4 : 9.**

EXERCISES

1. Solve both equations **(2)** and **(3)** for the acceleration a and the time t.

2. Combine equations **(2)** and **(3)** so as to express the velocity in terms of the acceleration a and the distance d. Express also the distance in terms of velocity and acceleration.

3. Letting a line 1 cm. long express the acceleration a, represent the velocity at the end of each of the first four seconds. Represent also the corresponding distances.

SUGGESTION. — Equation **(2)** gives the length representing the velocity, and equation **(3)**, the distance.

4. A train leaving a station has a constant acceleration of 0.4 m./sec.^{2}. What will be its velocity at the end of the tenth second? At the end of 15 seconds?

5. If the acceleration of an electric car is uniform and 2 ft./sec.^{2}, in how many seconds will it accumulate a velocity of 25 ft. per second?

6. How far will the car in Exer. 5 move during the first 10 seconds? What will be its average velocity during this interval of time?

7. The acceleration of a car is 5 m./sec.^{2}. What velocity will it acquire in going 100 m.? *Ans.* 31.62 m./sec.^{2}.

8. A body has uniformly accelerated motion. What is its acceleration if it passes over 300 cm. in 20 seconds? *Ans.* 1.5 cm./sec.^{2}.

9. A bicycle starts from rest at the top of a hill 150 ft. long and has a uniform acceleration of 1 ft. per second. What will be its velocity at the foot of the hill? *Ans.* 17.32 ft./sec.

10. A car was moving at the rate of 30 mi. per hour when the brakes were applied. What was the rate of retardation if the car came to rest in 10 seconds, the decrease in velocity being uniform?

Ans. 4.4 ft./sec.^{2}.

11. A bicycle rider moving at the rate of 15 mi. per hour applies the brake which brings him to rest in moving 121 ft. Assuming that the velocity decreases uniformly, find the acceleration.

Ans. -2 ft./sec.^{2}.

12. A fly wheel is set in motion with a uniform acceleration of two revolutions per second per second. If the diameter of the wheel is 50 cm., what is the linear acceleration of a point on its rim?

Ans. 314.16 cm./sec.^{2}.

13. What is the velocity of a body having uniformly accelerated motion at the beginning of the tth second? What is the average velocity during the tth second? Show that the distance passed over during the tth second is $\frac{1}{2}a(2t-1)$.

14. Apply the formula developed in Exer. 13 to the conditions given in Exer. 4, and calculate the distance passed over by the train during the fifth and the tenth second. *Ans.* 1.8 m. and 3.8 m.

3. SIMULTANEOUS MOTIONS AND VELOCITIES

26. Composition of Motions. — The actual displacement of a body is often due to two or more causes acting together. For example, a ball rolled along the deck of a vessel has a displacement that is the result of combining the displacement of the boat with that given the ball by the hand. Hence, to find the displacement of the ball with respect to the earth, we must take into account all the separate motions that enter into the case. It will be observed that different cases will arise depending on the relation of the magnitudes and directions of the displacements to be compounded. The individual motions effecting the displacement are called the **components**, and the motion due to the united action of the components, the **resultant.** The process of finding the resultant from the components is called the **composition** of motions.

27. Compounding Motions in a Straight Line. — Let a ball be rolled along the deck of a vessel toward the bow. While the ball moves over a distance of 20 feet along the deck, the boat moves forward 30 feet. Since the component motions are in the same direction, it is plain that the ball actually moves a distance of 50 feet in the direction the vessel is going. Hence the resultant is equal to the sum of the components. Again, imagine that the ball is rolled toward the stern, a distance of 20 feet, while the boat is moving forward 30 feet. In this case we can see that the ball will be carried forward by the vessel 10 feet farther than the distance which it rolls backward, Hence the resultant is 10 feet, and in the direction of the motion of the vessel, because that is the larger component. A rule for compounding motions in the same straight line may be stated as follows:

The resultant of two component motions in the same straight line is equal to their sum when the directions are the same, and to their difference when the directions are opposite. In the latter case the resultant is in the direction of the greater component.

28. Compounding Velocities in a Straight Line. — If the velocities of the boat and the ball considered in § 27 are respectively 15 and 10 feet per second, and if the directions are the same, it is clear that the actual velocity of the ball will be the sum of the velocity of the boat and the velocity given to the ball by the hand, or 25 feet per second; and if the directions are opposite, the actual velocity of the ball will be equal to the difference of the velocities, or 5 feet per second. In the latter case the resultant velocity is in the direction of the boat's motion, since that is the greater of the two components.

29. Compounding Motions at an Angle. — Let a man starting from the point *A*, Fig. 11, row a boat perpendicular at all times to the current of a river 40 rods in width. If there were no current, the boat would land at B. But, while crossing, the current carries the boat down the stream a distance AC, which we may call 30 rods. The boat therefore lands at D, having taken the path AD. Since AD is in this case the diagonal of a rectangle whose sides are 40 and 30 units, representing distances measured in rods, its length, which is 50 units, represents a distance of 50 rods, the resultant motion of the boat. If the angle between the components is not a right angle, a boat starting from E takes a path EH, the diagonal of an oblique parallelogram $EFHG$.

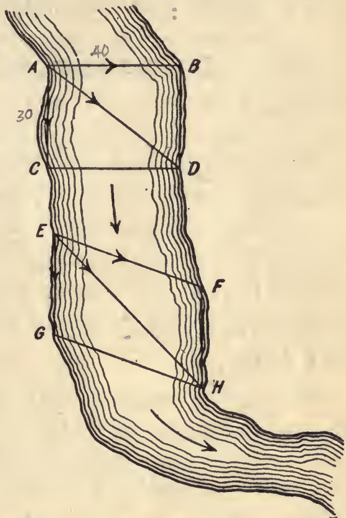


FIG. 11.—Resultant of Two Motions at an Angle.

A thin piece of wood E, Fig. 12, is arranged to slide smoothly along the edge of a drawing board. At m and n wire nails are driven a short distance into the wood and a third into the board at o. Loop one end of a piece of thread around m, pass it over n, and attach the other end to a small weight at A. If the board is now placed in a vertical position and the slide moved from E to E', the weight undergoes a displacement AB. If the loop is transferred to nail o, on the stationary board, a movement of the slide from E to E' gives the weight two simultaneous displacements represented by AB and BD, causing it to follow the diagonal path AD. If the operations are repeated after tilting the board in its plane, it will be seen that the weight follows the diagonal

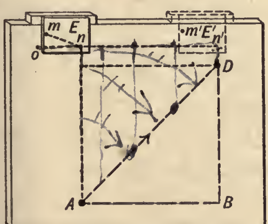


FIG. 12.—Apparatus for Showing the Compounding of Two Motions.

of an oblique parallelogram as the result of the two component displacements.

The facts shown by this experiment may be stated as follows:

The resultant of two component uniform motions not in the same straight line is represented by the diagonal of a parallelogram whose adjacent sides represent the two component motions.

30. Compounding Velocities at an Angle. — Velocities may be compounded in the same manner as motions. For example, if the velocity with which an oarsman rows his boat is represented in magnitude and direction by the line EF in Fig. 11, and the velocity of the stream by the line EG, the actual velocity of the boat is represented by the line EH.

It should be observed that in cases where the angle between the two components is a *right angle*, the resultant is the square root of the sum of the squares of the components. In other cases the parallelogram should be constructed accurately and the diagonal carefully measured.

31. Compounding Several Motions. — When the actual motion of a body is due to the united action of more than two components, the final resultant is found by first determining the resultant of any two of them; and this resultant is then compounded with a third component, and so on until each component has been used.

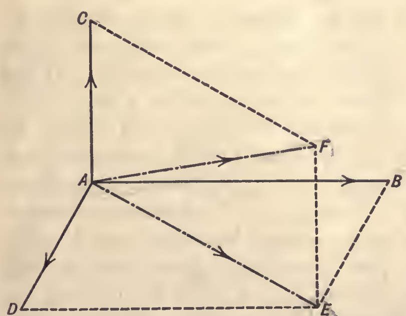


Fig. 13.—Determination of the Resultant of Three Component Motions.

Let AB, AC, and AD, Fig. 13, be three component motions imparted simultaneously to a body at A. The resultant of any two, *e.g.* AB and AD, is found in the manner described in § 29, giving the resultant AE. AE is now treated as a component and compounded with the third component AC. In this construction $ACFE$ is the parallelogram of which AF is the diagonal. AF is the resultant of the three given components.

32. Resolution of Motions and Velocities. — The meaning of this process, which is the reverse of composition, is most readily understood after considering a particular case. For example, let it be required to find the easterly velocity of a vessel sailing with a velocity of 15 miles per hour in a direction east by 30° south.

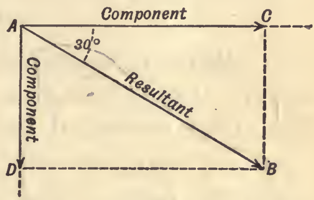


FIG. 14. — Resolution of a Velocity.

Let AB, Fig. 14, represent the given velocity of the vessel, making the angle BAC equal to 30°. If the lines BD and BC are now drawn parallel to AC and AD respectively, the line AC represents the component velocity of the vessel in an easterly direction. AD is the southerly component.

EXERCISES

1. A train approaches Chicago with a velocity of 30 km. per hour, while a brakeman runs along the tops of the cars toward the rear at the rate of 5 km. per hour. How rapidly is the brakeman approaching Chicago?

2. A boy is paddling a canoe along a river in the direction of the current, which has a velocity of 4 mi. per hour; if there were no current, the canoe would move 3.5 mi. per hour. How fast is the canoe moving?

3. Suppose the boy in Exer. 2 should double his effort and paddle upstream. How long would it take him to go 10 mi.?

4. A ship is moving east at the rate of 15 mi. per hour. If a person walks directly across the deck at the rate of 4 mi. per hour, with what velocity will he actually move?

5. A boat is rowed with a velocity of 4 mi. per hour, perpendicular to the current of a stream flowing 5 mi. per hour. Determine the direction of the motion and the velocity of the boat.

6. A ship headed due east under a power that can move it 12 mi.

per hour enters an ocean current whose velocity is 4 mi. per hour south. If a person on deck walks northeast with a velocity of 3 mi. per hour, what is his actual velocity? *Ans.* 14.3 mi./hr.

7. A balloon is driven in a direction east by 30° north. How rapidly is it drifting north if its velocity is 20 mi. per hour? *Ans.* 10 mi./hr.

8. A body moves down an inclined plane 5 m. in length. If the angle between the incline and a horizontal plane is 60°, what are the horizontal and vertical components of its motion?

9. How rapidly is a bird approaching the equator when flying due southeast at the rate of 20 mi. per hour?

SUMMARY

1. *Motion* is a continuous change in the position of a body. Motion and rest are relative terms. When the motion of terrestrial bodies is under consideration, the earth is usually regarded as being at rest (§ 15).

2. The motion of a body is *rectilinear* or *curvilinear* according as the path described by the body is a straight or a curved line (§ 16).

3. When a body moves over equal spaces in equal periods of time, no matter how small the period may be, the motion is said to be *uniform*. When the motion of a body is uniform, the distance passed over is proportional to the time (§ 17).

4. The equation of uniform motion is $d = vt$ (§ 18).

5. The characteristics of the rectilinear motion of a body are its *starting point*, the *direction* of the motion, and its *displacement*. These three qualities may be represented by a straight line (§ 19).

6. The *average velocity* of a body is found by dividing the space passed over by the time consumed (§ 20).

7. The *velocity* of a body *at any instant* is measured by the distance it would move in a second if at that instant its motion were to become uniform (§ 21).

8. The characteristics of a velocity are its *magnitude* (or *speed*) and *direction*. The qualities may be represented by the length and direction of a straight line. A line used in this manner is called a *vector* (§ 22).

9. The rate at which velocity changes with the time is called *acceleration* (§ 23).

10. A body has uniformly accelerated motion when its velocity changes at a uniform rate. The velocity acquired in a given time by a body starting from rest may be found from the equation $v = at$. The distance passed over is given by the equation $d = \frac{1}{2}at^2$ (§ 24).

11. The simultaneous individual motions (or velocities) of a body are called the *components* of its motion (or velocity), and the motion (or velocity) due to the united action of the components is called the *resultant*. The process of finding the resultant from the components is called the *composition* of motions (or velocities) (§§ 26 and 28).

12. The *resultant of two simultaneous motions* (*or velocities*) *along the same straight line* is their sum when the directions are the same, and their difference when they are opposite (§§ 27 and 28).

13. The *resultant of two simultaneous motions* (*or velocities*) *not in the same straight line* is represented in magnitude and direction by the diagonal of a parallelogram whose adjacent sides represent the two components (§§ 29 and 30).

14. The *resultant of more than two simultaneous motions* (*or velocities*) is found by compounding the resultant of any two of them with a third component, then this new one with the fourth, and so on until each component has been used (§ 31).

15. The process of finding the components from the resultant is called *resolution* (§ 32).

CHAPTER III

LAWS OF MOTION — FORCE

1. DISCUSSION OF NEWTON'S LAWS

33. Momentum. — It is a well-known fact that a moving body must always have been put in motion by an effort on the part of some agent. The amount of this effort depends (1) on the *mass* of the body moved and (2) on the rapidity with which it is given velocity, *i.e.* on the *acceleration.* If we observe a locomotive as it starts a train, we readily see that the effort required is greater as the train is longer or more heavily loaded. Furthermore, in order to start the train more quickly, a greater effort is required and a more powerful engine. Again, after the train has acquired its running speed, an effort is required if the motion is to be destroyed and the train brought to rest. This, also, depends on the mass of the train and the rapidity with which its motion is reduced.

The two quantities, *mass* and *velocity*, determine what is called the *quantity of motion* in a body, or its **momentum.** *Momentum is measured by the product of the mass and the velocity of a body*, and is expressed algebraically as *mv*. For example, the momentum of a 10-gram rifle ball moving with a velocity of 25,000 cm./sec. is $10 \times 25{,}000$, or 250,000 C. G. S. units. No name is used for the unit of momentum.

34. Newton's First Law of Motion. — An inanimate body never puts itself in motion. Not only does a body without motion tend to remain in that condition, but on account of that same tendency resists the effort of any agent that tries to start it. On the other hand, a body in motion manifests a tendency to keep moving and resists any effort made to change its motion in any way. These facts may be illustrated by the following experiments:

1. Stand a book upon end on a sheet of paper placed flat upon the table. Grasp the paper and try by a quick pull to give the book a forward motion. On account of the tendency of the book to remain at rest, it will be found to fall backward. Repeat the experiment, but move the paper very slowly at first; and, while the book is in motion, let the paper suddenly stop. On account of the fact that the book tends to remain in motion, it will fall forward.

2. Place a card upon the tip of a finger and lay a small coin upon the card directly above the finger tip. With the other hand give the card a sudden snap in such a manner as to drive the card from beneath the coin. The coin will be left upon the finger. The same experiment may be varied by placing a card upon the top of a bottle and a marble upon the card. The sudden removal of the card leaves the marble resting in the mouth of the bottle.

The first to express these facts of common observation in the language of Physics was Sir Isaac Newton[1] (1642–1727), professor of mathematics at Cambridge, England. The statement of his First Law of Motion is as follows:

Every body of matter continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by force to change that state.[2]

This is known as the **Law of Inertia**, the tendency of matter to act in the manner stated being often ascribed to a property of matter called *inertia*. In this law Newton has given a definition of *force* as that which is able to cause or change motion. Hence *the term* **force** *is the name*

[1] See portrait facing p. 30.

[2] "Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon." — *Newton's Principia*, Motte's Translation.

given to the cause that produces acceleration, retardation, or a change in the direction of the motion of a body.

35. Newton's Second Law of Motion. — According to Newton's First Law a moving body which could be entirely freed from the action of all forces would have uniform motion. A stone thrown from the hand would take a perfectly straight course, and a bullet fired upward would never return to the earth. The curved path described by the stone, however, indicates that a force is acting upon the body while it is moving. This force, as we know, is the force of gravity.

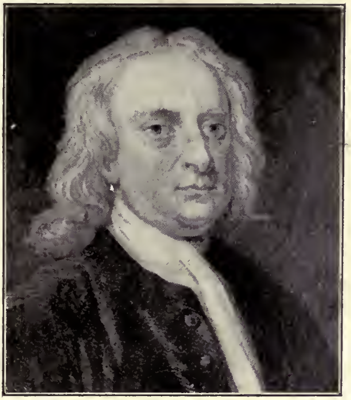
Just as the First Law *defines force*, so the Second Law leads to the *measurement of force*. This law may be stated as follows:

Change of motion, or momentum, is proportional to the acting force and takes place in the direction in which the force acts.[1]

The proportion always existing between force and the rate at which it changes the momentum of a body has led to the adoption of a convenient C. G. S. unit of force called the **dyne** (pronounced *dīne*). *The dyne is that force which, acting uniformly for one second, imparts one C. G. S. unit of momentum;* or, a dyne would give a mass of one gram an acceleration of one centimeter per second per second. This definition implies that the body upon which the force acts is not at all hindered in its motion by external resistances, as friction, etc.; *i.e.* the force has simply to overcome the inertia of the body.

36. Equations of Force. — The relation expressed in the preceding section between force, momentum, and time ad-

[1] "The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which the force is impressed." — *Newton's Principia*, Motte's Translation.



SIR ISAAC NEWTON (1642–1727)

The name of Newton will always be associated with the subject of gravitation on account of the fullness with which he applies and discusses his famous Principle of Universal Gravitation in his book entitled *Principia,* published in 1687. The *Principia,* which ranks as a mathematical classic, treats of the laws governing the motion of bodies under various conditions, and especially of the motion of the planets. This work follows close upon the achievements of Galileo and Kepler in astronomical discovery. Kepler had found by observation that the planets move around the sun in elliptical paths, which Newton showed would be the case if between the sun and each planet there exists a force which decreases as the square of the distance increases.

The *Principia* laid a firm and deep foundation for subsequent discoveries in the field of astronomy; it propounded and showed the application of a new method of mathematical investigation, the Calculus, by which alone it would retain its position at the head of mathematical treatises.

Newton must also be accredited with the announcement and elucidation of the three laws of motion which bear his name and with numerous discoveries in Light. His book entitled *Optics* contains his discoveries and theories in this subject.

Newton was born in Lincolnshire, England, in 1642, graduated from Trinity College, Cambridge, in 1665, and at once began to make the discoveries in mathematics and physics which have immortalized his name. He was professor of mathematics at Cambridge, member of Parliament, and Master of the Mint. He was knighted in 1705, and at his death, in 1727, was buried in Westminster Abbey.

mits of being expressed in a concise algebraic form. Let a mass of m grams be acted upon by a constant force of f dynes for t seconds. If the velocity imparted to the mass by that force is v cm./sec., the momentum produced is mv (§ 33). The momentum imparted per second is found by dividing the quantity mv by the time t. We have therefore, by the definition of the dyne,

$$\mathbf{f}\ (\text{in dynes}) = \frac{\mathbf{m}\ (\text{in grams}) \times \mathbf{v}\ (\text{in cm./sec.})}{\mathbf{t}\ (\text{in seconds})}. \quad (1)$$

From Eq. (2) § 25, we have $v = at$; $a = \frac{v}{t}$

whence $\frac{v}{t}$ is the acceleration a produced by the force f.

Therefore, by substituting in (1),

$$\mathbf{f}\ (\text{in dynes}) = \mathbf{m}\ (\text{in grams}) \times \mathbf{a}\ (\text{in cm./sec.}^2). \quad (2)$$

In the English F.P.S. system the unit of force is the **poundal**. *The poundal is that force which, acting uniformly for one second, imparts one F. P. S. unit of momentum.* Hence, if the mass of the body upon which the force acts is given in pounds, the velocity in feet per second, and the time in seconds, substitution in equation (1) will give us the force in poundals. A force of one poundal is equivalent to 13,825 dynes.

Example. — What constant force acting four seconds will give a body of 15 g. a velocity of 20 cm./sec.?

Solution. — The total momentum produced by the force in four seconds is 15×20, or 300 C. G. S. units. The momentum imparted to the body in one second is $300 \div 4$, or 75 units. Hence the force is 75 dynes.

Another method is to substitute the given quantities directly in equation (1), first being assured that all are given in C. G. S. units. The same may be said of the F. P. S. system in obtaining the force in poundals when the problems deal with English units.

37. Gravitational Units of Force. — A *gram-weight* (or a *gram-force*) is the downward pull exerted by the earth upon a one-gram mass. (See § 10.) If the mass is free to fall, the acceleration due to gravity will be 980 cm./sec.^{2}, or 980 C. G. S. units of momentum per second.

Hence, by Eq. (2), *a gram-weight is equivalent to* 980 *dynes.* Likewise, a *pound-weight* (or a *pound-force*) is the attraction of the earth upon a one-pound mass. The acceleration produced by this force when the body falls freely is 32.16 ft./sec.^{2}, and the momentum is 32.16 F. P. S. units per second. Therefore, one pound-weight is equivalent to 32.16 poundals.

Since the earth's attraction varies with the locality, as explained in §§ 10 and 69, the gram-weight and the pound-weight change accordingly. Hence these units are called *gravitational* units of force. On the other hand, the dyne and the poundal, being independent of gravity, are called *absolute* units.

Commonly, use is made of the pound and the gram (*i.e.* pound-weight and gram-weight) as units of force. Scientific work has, however, demanded a more unvarying unit and has brought the dyne into extensive use in all cases requiring accuracy of expression. The poundal is little used.

38. An Application of the Second Law. — Newton's Second Law implies that any force acting upon a body produces its own effect, whether acting alone or conjointly with other forces. An interesting illustration of this may be observed in the following experiment:

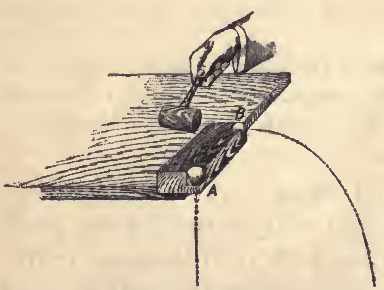


Fig. 15. — Illustration of Newton's Second Law of Motion.

Cut notches *A* and *B*, Fig. 15, in two of the corners of a piece of wood about 2 × 8 inches. By means of a large screw attach the center of the block loosely to the edge of a table as shown, and place a marble in each notch. If the end of the block opposite *A* is struck with a mallet, ball A will be dropped vertically, while ball B is projected horizontally. B is subject to two forces, an impulse which projects it in a horizontal direction and the constant force of gravity acting along a vertical line. The two marbles will be found to strike the floor at the same time.

The experiment shows that the effect of gravity in bringing the balls to the floor is independent of the horizontal component of the motion; *i.e.* a given force (gravity) produces as much "change in momentum" in the vertical direction in one ball as in the other. After ball B leaves the block, its momentum in the horizontal direction suffers no change.

39. Newton's Third Law of Motion. — *To every action there is always an equal and opposite reaction; or, the mutual actions of any two bodies are always equal in magnitude and oppositely directed.*

This law may be illustrated by experiment as follows:

Let two elastic wooden balls A and B, Fig. 16, be suspended by threads in such a manner that they just touch each other when stationary. Draw A aside and let it fall against B. A will be brought to rest by the impact, and B will be moved to the position B'.

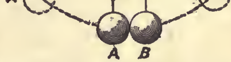


Fig. 16. — Action and Reaction are Equal and Opposite.

In this experiment two results are apparent: first, ball B is acted upon by a force sufficient to carry it to the position B', and, second, ball A loses an equal amount of momentum in that it is brought to rest. In the impact occurs a mutual effect — a force exerted by A toward the right upon B, and an equal and oppositely directed force from B upon A. This process goes on in every case in which force enters. A pressure of the hand against the table is opposed by an equal pressure of the table against the hand. When a person leaps forward from a boat, the boat is pushed in the opposite direction. When a gun is fired, the mutual effect is to give the bullet and the gun *equal momenta;* or, in other words, the mass of the bullet multiplied by its velocity equals the mass of the gun multiplied by its velocity. The velocity of the gun's recoil, or "kick," is small because the mass of the gun is many times greater than that of the bullet.

The question often arises: "Does the earth rise to meet the falling apple?" In the light of the Third Law of Motion, we must admit that the action of the earth which draws the apple down is accompanied by a reaction which is operative for the same length of time upon the earth in an upward direction. Hence the momentum given the apple equals that given the earth. On account of the enormous mass of the earth, however, the distance through which it rises to meet the apple is infinitesimally small.

EXERCISES

1. Why does a person standing in a car tend to fall backward when the car starts, and forward when it stops?

2. Why does a bullet continue to move after leaving a rifle?

FIG. 17.

3. A weight W, Fig. 17, is attached by a cord B to some fixed object. A quick downward pull on a similar cord A will break the cord below W, but a steady pull will break cord B. Explain.

4. A blast of fine sand driven against glass soon cuts away its smooth surface. Why?

5. If a rifle ball is thrown against a board placed on edge, it will knock it down; but when fired from a gun, it will pass through the board and leave it standing. Why?

6. Explain why the head of a hammer or mallet can be driven on by simply striking the end of the handle.

7. Why can an athlete make a longer "running jump" than a "standing" one?

8. Will a stone dropped from a moving train fall in a straight line?

9. Why do moving railway coaches "telescope" in a collision?

10. Explain how heavy fly wheels serve to steady the motion of machinery, as in the case of the sewing machine.

11. A 4-gram rifle ball leaves a gun with a speed of 20,000 cm. per second. Compute its momentum.

12. Which has the larger momentum, a man weighing 150 lb., walking 10 ft. per second, or a boy weighing 60 lb. and running 25 ft. per second?

13. Which has the greater momentum, a man weighing 160 lb. in a railway coach moving 30 mi. per hour, or a 2-ton stone moving 3 ft. per second? Express the difference in F. P. S. units.

14. What force acting for 10 seconds upon a mass of 200 g. will produce a velocity of 5 cm./sec.? Express the change of momentum per second in C. G. S. units.

15. A body whose mass is 20 g. is given an acceleration of 45 cm./sec.^{2}. What is the required force?

16. What acceleration will be given to a mass of 25 g. by a constant force of 500 dynes? Over what distance will the body move in 5 seconds if the force continues to act?

17. If the force given in Exer. 16 ceases to act at the end of the 5th second, how far will the body move during the next 5 seconds?

Suggestion. — Find the velocity imparted in the first 5 seconds and apply the First Law of Motion.

18. An inelastic ball of clay whose mass is 200 g. has a velocity of 25 cm./sec. when it collides with a similar ball at rest whose mass is 50 g. Find the velocity after collision.

Suggestion. — After impact the two masses move on as one mass with the momentum of the first before collision.

19. A projectile weighing 100 lb. is fired with a velocity of 1200 ft. per second from a gun weighing 8 T. Find the velocity with which the gun starts to move backward.

2. CONCURRENT FORCES

40. Representation of Forces. — The three characteristics of a force, its point of application, direction, and magnitude, can, as we have seen, be represented by a straight line. One end of the line shows the point of application, the length of the line shows the magnitude of the force, and the direction in which the line is drawn shows the direction in which the force acts. For example, (1), Fig. 18, represents a force of 10 dynes acting northeast from the point A. The unit of force may be represented by any convenient length, but the same scale should, of course, be used throughout a given problem.

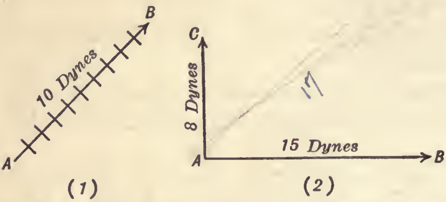


FIG. 18. — The Representation of Forces by Means of Straight Lines.

In a similar manner, (2), Fig. 18, shows that two concurring forces, AB and AC, representing respectively 15 dynes east and 8 dynes north, act upon the point A. The scale adopted in this case is 2 millimeters to the dyne.

41. Composition of Forces. — When a body is acted upon by two forces at the same time, it is easy to imagine a single force that might be substituted for them and would have the same effect. This single force is the **resultant** of the two forces, which are the **components.** *The process of finding the resultant of two or more component forces is called the* **composition** *of forces.* Forces are compounded in the same manner as motions and velocities (§§ 27–31).

42. Forces Acting in a Straight Line. — When two forces act upon a body in the same line and in the same direction, it is clear that the resultant is the sum of the two components. For example, if a weight is to be lifted by two men pulling upward on a rope attached to it, and if one man pulls with a force of 50 pounds while the other pulls with a force of 75 pounds, the two forces result in a single pull of 125 pounds. Hence the resultant is 125 pounds and is directed upward.

When the two forces act in opposite directions along the same line, the resultant is the difference of the two components. Thus, if one man pulls upward on a weight with a force of 75 pounds while the other pulls downward with a force of 50 pounds, the lifting effect is the same as a single force of 75 − 50, or 25 pounds. The action of the resultant is plainly in the direction of the greater of the two components. A special case of opposite forces is that in which the sum of the components acting in one direction is equal to the sum of those acting along the same straight line in the opposite direction. In this case no motion can result from the joint action of all the forces. In other words, the resultant is zero. The body upon which such forces act is said to be in **equilibrium**.

43. Forces Acting at an Angle. — If AB and AC, (2), Fig. 18, represent forces of unequal magnitudes, it is clear that the resultant will divide the angle between them, but will lie nearer the greater force. The actual magnitude and direction of the resultant are found in the same manner as in the case of the resultant of two motions (§ 29).

The resultant of two concurring forces acting at an angle is represented by the diagonal of a parallelogram constructed on the two lines representing the component forces.

This is one of the most important laws of mechanics and is universally known as the **Principle of the Parallelogram of Forces.** The following experiment will illustrate the truth of this principle:

Arrange two dynamometers (see Fig. 6), before the blackboard, as shown in Fig. 19. Let the weight W be great enough to produce a large but measurable tension in each of the oblique cords. Place a rectangular block of wood against each of the cords and trace its direction on the blackboard. Read the dynamometers and record the magnitude of each force on the corresponding line. Adopt a convenient scale and lay off each force along its line of direction, measuring from O. Using the oblique lines as sides, construct the parallelogram. Measure the diagonal OR and write its value in force units upon it. A comparison will show that the force represented by OR is equal to W and might, therefore, be substituted for the two components and would produce the same effect.

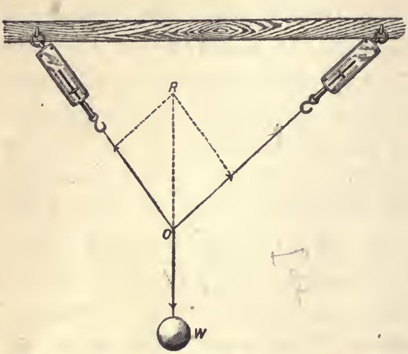


FIG. 19. — Principle of the Parallelogram of Forces Illustrated.

44. Equilibrant and Resultant. — The force W is said to hold the component forces *in equilibrium*, and is therefore called the **equilibrant** (pronounced *ĕ′ qui lī′ brant*). From the definitions given of resultant and equilibrant, we see that they are necessarily equal in magnitude but opposite in direction.

EXERCISES

1. Represent by a diagram the resultant of two forces of 15 dynes and 25 dynes acting (1) in the same direction and (2) in opposite directions from the same point.

2. Find the magnitude and the direction of the resultant of two forces, 3 lb. acting north and 4 lb. acting west, applied at the same point.

3. A ball is acted upon simultaneously by two forces, one of 10 kg. directed upward, the other of 25 kg. directed east along a horizontal line. Find the resultant in both magnitude and direction.

4. The angle between a force of 50 dynes and one of 30 dynes is 60°. Find the resultant and the equilibrant in both magnitude and direction.

5. The angle between two equal forces of 40 lb. each is 120°. Find the resultant.

6. A boat is pulled by two ropes making an angle of 30°. If one force is 10 lb. and the other 20 lb., what is the resultant?

7. A weight is suspended by two cords applied at the same point and each making an angle of 30° with a vertical line. If the tension in each is 25 lb., what is the weight supported?

Ans. 43.3 lb.

3. MOMENTS OF FORCE — PARALLEL FORCES

45. The Moment of a Force. — It can often be observed that when a mechanic wishes to loosen a nut that is difficult to start, he uses a wrench with a long handle. For those that start easily, he uses a short-handled wrench. The results prove that the *effectiveness* of a force in producing rotation against a resistance is greater as the applied *force* is farther from the point about which rotation takes place.

The effectiveness of a force in producing a rotation is called the **moment** *of the force.* The moment of a force depends upon two quantities: (1) the *magnitude of the force* and (2) the *perpendicular distance from the point about which the rotation takes place to the line representing the direction of the force.* The moment is measured by the *product* of these two factors. The following experiment will make the matter clear:

Fasten one end of a light wooden bar to the table top by means of a nail at O, Fig. 20. Let a force, which may be measured by a dynamometer (see Fig. 6), pull upon the bar at A, and another at B, as shown. Measure both forces and the distances AO and BO. The product of the force applied at A multiplied by the distance AO will be found equal to the product of the other force multiplied by the distance BO.

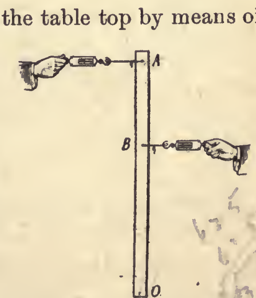


Fig. 20. — The Equality of Moments Illustrated.

It is clear from this experiment that *a force that tends to turn a body* *to the right can be balanced by another of the same moment that tends to produce rotation to the left.*

46. Parallel Forces. — Objects are frequently supported by two or more upward forces acting at different points, thus forming a system of parallel forces. For example, two men may support a heavy beam or carry a loaded bucket on a bar between them. A bridge is supported by the upward pressures of the piers at the ends. The principle of moments given in the preceding section is of service in determining the resultant of such forces as the following experiment illustrates:

Select a bar of wood 4 or 5 ft. in length, of uniform width and uniform thickness. A pine board about 4 in. in width is convenient. Place hooks at several points along one edge, as shown in Fig. 21, but place one hook so that the bar will balance well when hung from that point. Call this point C. Suspend the bar from a dynamometer at C and ascertain the weight of it. This will be the value of the resultant in every case. Now release the bar at C and attach dynamometers at two points, say A and B, and ascertain the forces required to support the bar. Designating these forces by F and F', it will be found that in every case the sum of the two components is equal to the weight W of the bar; or, $F + F' = W$. Furthermore, the moment of the force F about the point B (*i.e.* $F \times AB$) will be found equal to the moment of the weight of the bar *about the same point* (*i.e.* $W \times CB$); or,

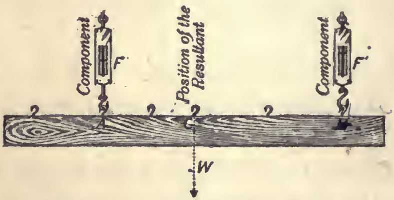


FIG. 21. — Law of Parallel Forces Illustrated.

$$\mathbf{F \times AB = W \times CB.} \tag{3}$$

The moment of the component F' about the point A will be found equal to the moment of W *about the same point*. Hence we may write

$$\mathbf{F' \times AB = W \times AC.} \tag{4}$$

The laws of parallel forces may therefore be stated as follows:

1. *The resultant of two parallel forces acting in the same direction at different points on a body is equal to their sum, and has the same direction as the components.*

2. *The moment of one of the components about the point of application of the other is equal and opposite to the moment of the supported weight about the same point.*

EXAMPLE. — Two men, A and B, carry a bucket weighing 100 lb. on a bar 10 ft. long. If the bucket is 4 ft. from A, how much force is exerted by each?

SOLUTION. — The moment of the force F exerted by A about the opposite end of the bar is $10 \times F$, and the moment of the weight about the same point is $100 \times (10 - 4)$, or 600. Hence $10 \times F = 600$; whence $F = 60$ lb. Let F' be the force exerted by B. Then considering the moments about the other end of the bar, we have $10 \times F' = 100 \times 4$; whence $F' = 40$ lb. Therefore A exerts 60 lb. and B 40 lb.

47. The Couple. — When two equal parallel forces act upon a body along different lines and in opposite directions, as shown in Fig. 22, they have no resultant; that is, no single force will have the same effect as the two components acting jointly. A combination of this kind is called a **couple**. The tendency of a couple is always to rotate the body on which it acts. This tendency is measured by the *moment* of the couple, which is the product of one of the forces multiplied by the perpendicular distance AB between the two forces. This distance is the **arm** of the couple. The equilibrium of the body acted upon can be maintained only by the application of another couple of equal moment acting in the opposite direction.

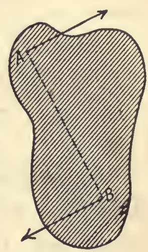


FIG. 22. — The Couple.

A small magnet placed on a floating cork is rotated by the couple formed by the northward-acting force at one end and the equal southward-acting force at the other.

EXERCISES

NOTE. — The student should first draw a diagram representing the conditions of the problem to be solved and then apply the general results deduced from the experiment in § 46.

1. A uniform bar of wood weighing 12 kg. is 120 cm. long; two hooks are placed on opposite sides of the center at distances of 40 cm. and 20 cm. respectively. What forces applied to the hooks will support the bar? *Ans.* 4 kg. and 8 kg.

2. In order to support the bar in Exer. 1, what forces applied at points respectively 15 cm. and 15 cm. from the ends of the bar will be required? *Ans.* 9 kg. and 3 kg.

3. A beam of uniform size is 60 ft. long and weighs 800 lb.; a man at one end supports 200 lb. Find the magnitude and point of application of the other required force.

4. Two parallel forces of 30 g. and 70 g. are applied at the ends of a bar 1 m. long. Find what weight will be supported and its location on the bar, neglecting the weight of the bar itself.

5. A boy and a man are carrying a weight of 150 lb. on a bar 10 ft. in length. If the forces are applied at the ends of the bar, where must the load be placed in order that the boy may have to carry only 50 lb.?

6. Draw a diagram showing a method for attaching three horses to a load so that they must pull equally.

4. RESOLUTION OF FORCES

48. Resolution of Forces. — In many cases it becomes desirable to find the effect of a force in some direction other than that in which it acts. For example, a car on a track running east and west, Fig. 23, is acted upon by a force *AB* directed northeast. The given force has two effects: It produces (1) a tendency to move the car east, and (2) a pressure against the rails toward the north. It is plain that two forces, one directed east and the other north, might have the same effect as the single force

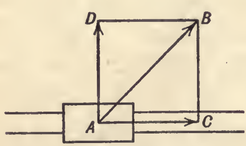


FIG. 23. — Force Resolved into Two Components.

directed northeast. In order to determine these two forces, the given force is represented by the line AB, from whose extremities, A and B, lines are drawn completing the rectangle, as shown in the figure. AC is called the *effective* component, since it acts in the direction in which the car can move; and AD is designated as the *non-effective* component, since it contributes nothing to the production of motion.

A given force may be resolved into two components whose directions are given by making the line of force the diagonal of a parallelogram whose sides are drawn from the point of application of the force in the directions required for the components.

49. The Sailboat and the Aeroplane. — The principle of the resolution of forces explained above is readily applied to the operation of a sailboat. Let the boat be headed north while the wind blows from the east. Now the pressure of the wind AC on the sail SS', (1), Fig. 24 can be resolved into AB, perpendicular to the sail, and a

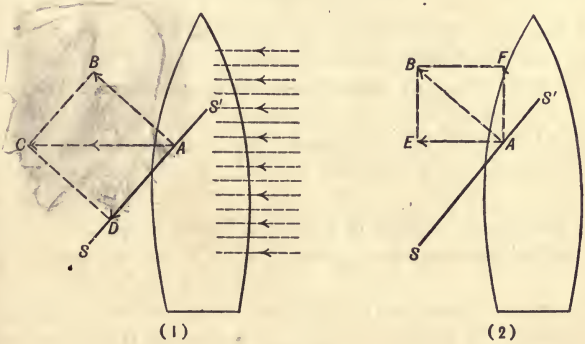


FIG. 24.

second component AD, parallel to the sail, the latter of which is non-effective. Force AB is the effective pressure on the sail. If the vessel were round, it would move in the direction of AB. Now let AB be resolved as shown in (2), Fig. 24 into AE acting parallel to the keel and AF acting perpendicular to it. The former component moves the vessel forward, while the component AE is rendered non-effective by the deep keel of the boat.

In the case of the aeroplane, which is a recent invention, huge planes or sails, AB and CD, shown in the sectional view, Fig. 25, are attached firmly to a light frame, upon which is mounted a powerful gasolene motor (§ 271). The planes are slightly oblique, as shown. The power furnished by the motor turns a propeller whose office it is to drive the aeroplane rapidly forward. When the aeroplane moves forward to the left, it is as though a strong wind were blowing toward the right against the planes, as shown by the dotted lines. As in the case of the sailboat a pressure is produced at right angles to the planes AB and CD. Representing this force by the line EF and resolving it into two components, we find the lifting force EG, and the component EH, which tends to resist the forward motion of the aeroplane. Smaller planes whose positions can be changed by the operator, are used in steering.

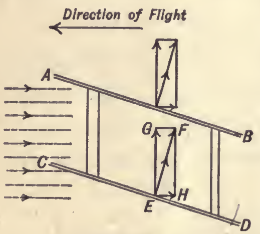


Fig. 25.

EXERCISES

1. If the force represented by the line AB, Fig. 23, is 1000 lb., and the angle BAC, 60°, find the components AC and AD.

2. Resolve a force of 2000 dynes into two components making angles of 30° and 60° with the given force.

3. A weight of 50 kg. is suspended by two cords making angles of 30° and 60° respectively with the vertical. Find the force exerted by each cord. *Ans.* 25 kg. and 43.3 kg.

4. If the mass of the car in Exer. 1 is 20,000 lb., and the resistance offered by the rails may be neglected, what is the acceleration of the car?

Suggestion. — Reduce the effective component to poundals and apply equation (2), § 36. *Ans.* 0.161 ft. per sec. per sec.

5. CURVILINEAR MOTION

50. Uniform Circular Motion. — It is a well-known fact that a ball attached to one end of a cord and whirled about the hand exerts a pulling force against the hand along the cord. This takes place because of the tendency of the ball to move in a straight line according to Newton's First Law of Motion. In order, therefore, to confine the ball to a circular path, a continual force toward the center must be maintained. If this force is removed by the breaking of the cord, the ball will leave its circular path along a tangent. *The force that continually deflects a moving body from a straight line, compelling it to follow a curve, is called* **centripetal force.** If the motion of a body along the circumference of a circle is uniform, the centripetal force is constant.

The name "centrifugal" force is often applied to the reaction of the moving body upon the fixed center. This reaction gives one the erroneous impression that the body would fly away from the center along a radius if the centripetal force should cease acting. If, however, we watch the course taken by water or mud as it leaves a revolving wheel, we readily observe that it moves along the tangent to the wheel at the point where it is set free.

51. Centripetal Acceleration. — Since a force produces a change in momentum in the direction of the force, according to Newton's Second Law (§ 35), a constant centripetal force produces a constant change in the momentum of a body, which has uniform circular motion, toward the center. Since the mass is constant, the acceleration is constant and directed toward the center. If v is the velocity with which a body is moving in a circular path, and r the radius of the circle, the centripetal acceleration a is represented as follows :[1]

$$\mathbf{a} = \frac{\mathbf{v}^2}{\mathbf{r}}. \qquad (5)$$

[1] Let a body m be moving around the circle whose center is o, with a uniform velocity v. Let it move from m to c in the very short time of t seconds. Then the distance $\overline{mc}$ will be equal to vt (Eq. 1, p. 14). If the time is small, the arc mc is practically equal to the chord $\overline{mc}$. On com-

Thus, if the moving body is at the point m, Fig. 26, its tendency is to continue in the direction mb in accordance with the First Law of Motion. But, on account of the cord, it is compelled to keep the same distance from the center o and, consequently, is deflected from b to c. Again, at c, as at every point, the body tends to follow the tangent ce, but is compelled to take an intermediate path along the circumference to f. If the deflecting force is removed at the time the body reaches f, it continues to move in the direction of its motion at that point; that is, along the line fg, which is tangent to the circle at f. It is the component of the motion ma produced by the centripetal force acting along mo whose acceleration is represented in equation (**5**).

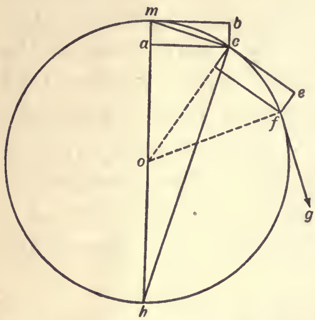


FIG. 26.—A Body m Having Circular Motion Tends to Follow the Tangent to its Path.

52. Equation of Centripetal Force.—A force is equal to the product of the mass of the body upon which it acts and the acceleration that it produces (§ 36). Hence in circular motion the centripetal force is the product of the mass m and the centripetal acceleration $\frac{v^2}{r}$. Therefore the value of the centripetal force may be expressed as follows:

pleting the small rectangle $mbca$, we have, since ca is a perpendicular dropped upon the hypotenuse of the right triangle mch,

$$\overline{mc}^2 = \overline{ma} \times \overline{mh}. \qquad (1)$$

Now the distance the mass m is drawn toward the center by the constant centripetal force in the time t is $\overline{bc}$ and equals $\overline{ma}$. Since the motion toward the center is uniformly accelerated,

$$\overline{mc} = \overline{ma} = \tfrac{1}{2}\,at^2 \text{ (Eq. } \mathbf{3}\text{, p. 18).} \qquad (2)$$

Therefore, by substituting the values $\overline{mc} = vt$ and $\overline{ma} = \frac{1}{2}\,at^2$ in (1), we have $v^2t^2 = \frac{1}{2}\,at^2 \times 2\,r$, where r is the radius of the circle. From this equation $v^2 = ar$, whence $a = \frac{v^2}{r}$.

$$\text{Centripetal force} = \frac{mv^2}{r}. \qquad (6)$$

It should be observed that this equation gives the force in absolute units only; *i.e.* in *dynes*, when C. G. S. units are substituted, and in *poundals*, when F. P. S. units are employed.

53. Illustrations of Circular Motion. — Many examples of circular motion present themselves in everyday life. The bicycle rider must carefully govern his speed as he turns a corner on a slippery pavement lest the force required to change the direction of motion be too great and the wheels slip sidewise. In the modern cream separators the denser portions of the milk are forced to the outside of a rapidly revolving bowl, while the lighter cream remains near the center and is forced out along the axis. Honey is extracted by rapidly whirling the uncapped comb in a machine. Centrifugal driers are used in laundries and factories for removing water from clothing, wool, etc. In the "loop the loop" apparatus a car rides safely along a track within a large vertical circle, its own tendency to follow a tangent keeping it pressed firmly against the rails.

The motion of the bodies of the solar system illustrates the action of centripetal force on the grandest scale. The earth, for example, having an initial motion, tends to move in a straight line. However, the attraction of the sun, like a tense cord, holds it in its orbit. If this force should cease, the earth would at once move away into space along a tangent to its orbit. On the other hand, if it were not for the earth's motion along the curve, it would be drawn with accelerated motion into the sun.

The spheroidal shape of the earth is supposed to be due to the tendency of matter to withdraw from the axis of rotation. This tendency causes bodies to weigh about $\frac{1}{289}$ less at the equator than at the poles.

When the centripetal force is not sufficient to keep the parts of a revolving body in the required circular paths, serious results often follow. This is the case of bursting fly wheels and emery wheels in mills and factories.

EXERCISES

1. Explain why water will not fall from a pail whirled at arm's length in a vertical circle.

2. How is the overturning of a car prevented, as it rapidly turns a curve?

3. Does the rotation of the earth affect the weight of bodies in this latitude?

4. Account for the fact that the moon moves in an orbit around the earth.

5. What keeps the earth in rotation on its axis?

6. Show by equation (**5**) that increasing the rate of rotation of the earth seventeen fold would cause bodies at the equator to "lose" their entire weight.

7. A body whose mass is 50 g. moves in a circle whose radius is 40 cm. with a velocity of 20 cm./sec. What is the required centripetal force?

8. A stone leaves a sling with a velocity of 50 ft. per second. If the mass of the stone is 2 oz. and the radius of the circle 4 ft., what was the pull exerted on the cords of the sling?

Ans. 78.125 poundals.

SUMMARY

1. The *momentum* of a body is measured by the product of its mass and velocity. It is represented by the expression mv (§ 33).

2. The term *force* is the name given to the cause that produces acceleration, retardation, or a change in the direction of the motion of a body (§ 34).

3. Force is measured by the change in momentum produced per second. The C. G. S. unit of force is the dyne. The *dyne* is that force which, acting uniformly for one second, imparts one C. G. S. unit of momentum (§ 35).

4. The equation of force is $f = \frac{mv}{t}$, or $f = ma$ (§ 36).

5. The *absolute* units of force are the *dyne* and *poundal;* the *gravitational* units are the *gram-weight* and *pound-weight*, etc. In everyday use the gravitational units are called simply the "gram" and "pound" (§ 37).

6. A given force produces its own effect, whether acting alone or conjointly with other forces (§ 38).

7. To every action there is always an equal and opposite *reaction;* or, in other words, for every push or pull of one body upon a second body there is always an equal pull or push of the second body upon the first (§ 39).

8. The characteristics of a force are its *point of application*, *direction*, and *magnitude.* Forces are represented by straight lines of suitable length and direction and may be compounded in the same manner as motions and velocities (§ 40).

9. The *resultant of two or more forces acting in the same direction* along a straight line is equal to their sum; but when two forces act in opposite directions in the same line, their resultant is equal to their difference and has the direction of the greater force (§ 42).

10. The *resultant of two forces acting at an angle* is represented by the diagonal of a parallelogram constructed on the lines which represent the component forces. This law is universally known as the *Principle of the Parallelogram of Forces* (§ 43).

11. The *moment of a force about a point* is the effectiveness of the force in producing a rotation. It is measured by the product of the magnitude of the force and the perpendicular distance from the point to the line of direction of the force (§ 45).

12. Any two parallel forces acting upward will support a weight equal to their sum, and the moment of one component about the point of application of the other is equal and opposite to the moment of the supported weight about the same point (§ 46).

13. A system of two equal and opposite parallel forces acting along different lines is called a *couple*. The *moment of a couple* is the product of one of the forces multiplied by the distance between the two forces. A couple can be balanced only by another couple acting in the opposite direction and having an equal moment (§ 47).

14. A force may be *resolved* into two components by making it the diagonal of a parallelogram whose sides are drawn in the directions required for the components (§ 48.)

15. When a body has curvilinear motion, a force is required to deflect the body continually from a straight line. This is called *centripetal force*. The equation of centripetal force is $f = \frac{mv^2}{r}$ (§ 52).

CHAPTER IV

WORK AND ENERGY

1. DEFINITION AND UNITS OF WORK

54. Work. — The use of the expression "to do work" is restricted in the study of mechanics to cases in which a force produces motion in the body upon which it acts. For example, attempting to lift a stone from the ground without succeeding in moving it is not doing work in the scientific sense; but lifting the stone to a higher position implies that work is being done upon it. Similarly, bending a bow is doing work, but holding it in a bent condition is not. Lifting a weight involves the process of doing work, but simply supporting it does not. Work is done when the spring of a clock is wound, or a body is moved along upon a table.

An important case in which work is done is that in which a freely moving body is given acceleration. We have already found (§ 34) that an increase in the velocity of a body requires the action of a force. Furthermore, the tendency of the force is to produce motion in the direction in which the force acts. Hence, work is done by exploding powder when it projects a bullet from a gun, or by gravity when a body is allowed to fall to the earth.

55. Elements Involved in Work. — In each of the examples given in the preceding section, it will be observed that two quantities are involved in the process of doing work. These are (1) the acting *force* and (2) the distance through which the force continues to act, sometimes called the *displacement.* **Work** *is directly proportional to the force* *and the distance through which the force acts, and is measured by their product.* Thus

$$Work = force \times displacement.$$

Or, if f represents the force, d the distance through which the force acts.

$$\mathbf{Work = fd.} \qquad (1)$$

56. Units of Work. — Since work is measured by the product of the force that acts upon a body and the distance the body is moved in the direction of the force, *a unit of work is done when a unit of force acts through a unit of distance.* For every unit of force (§§ 36 and 37) there is a corresponding unit of work. The most important, however, is the C. G .S. unit which is called the **erg**. *The erg is the work done when a force of one dyne acts through a distance of one centimeter.* The erg is also called the *dyne-centimeter.*

The F. P. S. unit of work is the *foot-poundal,* which is the work done when a poundal of force acts through a distance of one foot. In the gravitational system two units are frequently used. The *kilogram-meter* is the work done when a force of one kilogram (§ 37) acts through a distance of one meter. The *foot-pound* is the work done when a force of one pound acts through a distance of one foot.

The following table is given to show the use of equation (1) in the computation of work in the different systems:

Absolute System		
f (in dynes)	$\times\ d$ (in centimeters)	$=$ *Work* (in ergs).
f (in poundals)	$\times\ d$ (in feet)	$=$ *Work* (in foot-poundals).
Gravitational System		
f (in kilograms)	$\times\ d$ (in meters)	$=$ *Work* (in kilogram-meters).
f (in pounds)	$\times\ d$ (in feet)	$=$ *Work* (in foot-pounds).

EXAMPLE. — A mass of 50 g. requires a force of 10 g. to overcome the friction and move the body at a uniform rate along a horizontal table. Find the work done when the mass is moved horizontally 25 cm. Find also the work done when the mass is lifted 25 cm.

SOLUTION. — Since the horizontal force is 10 g. or 9800 dynes (§ 37), and the distance through which the force acts is 25 cm., the work performed is 10×25, or 250 gram-centimeters. Measured in ergs, the work is 9800×25, or 245,000 ergs.

The work performed in lifting the mass is 50×25, or 1250 gram-centimeters. Measured in ergs, the work is $49{,}000 \times 25$, or 1,225,000, ergs.

The numerical relation between the various units of work is shown in the following table:

1 dyne-centimeter	equals 1 erg.
1 kilogram-meter (kg-m.)	equals 98,000,000 ergs.
1 foot-pound	equals 13,550,000 ergs.
1 foot-poundal	equals 421,390 ergs.

2. ACTIVITY, OR RATE OF WORK

57. Activity. — The value of any agent employed in doing work will depend upon the amount of work it is able to perform in a certain time. Some agents work slowly, others rapidly. For example, a man can lift a certain number of bricks to the top of a building in an hour; a horse attached to a suitable hoisting mechanism can lift a greater number in the same time; and an engine can lift the bricks as fast as several horses. Working agents are therefore said to differ in **activity**, or **power**. *Activity is the rate of doing work, and is found by dividing the work performed by the time consumed in the process.*

58. Units of Activity. — The unit of activity or power commonly used is the **horse power** (abbreviated H.P.). *The horse power is the rate of doing work equal to 550 foot-pounds per second.* The activity of an agent that is able to perform 550 foot-pounds of work per second is one horse power. The unit of power in the C. G. S. system is the **watt**,[1] which is equivalent to the work done at the rate of 10^7 ergs per second. One horse power equals 746 watts, or 746×10^7 ergs per second.

EXERCISES

1. Calculate the work done by a force of 25 dynes acting through a distance of 120 cm.

2. Express in ergs and gram-centimeters the work done in lifting a mass of 5 g. through a vertical height of 100 cm.

3. A horse has to exert an average force of 200 lb. in moving a loaded cart a distance of a mile. Find the amount of work done.

4. What amount of work is done when one cubic meter of water is elevated to a height of 10 m.?

5. How much work is done per second by an engine that in one hour lifts 10,000 bricks each weighing 4 lb. to the top of a building 50 ft. in height? Find the necessary horse power.

6. A man shovels 3 T. of coal from a wagon box into a bin 6 ft. above the coal in the wagon. How much work is involved in the process?

7. What must be the power of an engine that hoists 50 T. of ore per hour from a mine 300 ft. deep?

8. A pumping engine is capable of raising 300 cu. ft. of water every minute from a mine 132 ft. in depth. If a cubic foot of water weighs 62.5 lb., what must be the power of the engine?

9. How long will it take a 3-H. P. engine to elevate 5000 bu. of wheat 50 ft.? (A bushel of wheat weighs 60 lb.)

10. A train is moving with a velocity of 30 mi. per hour. If the resistance to the motion is 1500 lb., calculate the power utilized.

11. The motors of an electric car can develop 200 H. P. With what velocity can the car run against a uniform resistance of 2200 lb.?

3. POTENTIAL AND KINETIC ENERGY

59. Energy. — In each of the cases selected in § 54 to illustrate the process of doing work, some agent capable of doing work was assumed to be acting. The stone, for

[1] So called in honor of James Watt (1736–1819), the inventor of the steam engine.

example, was supposed to be lifted by this agent, which may have been an engine, a person, a horse, or any other working medium. In order to be able to perform work, an agent must possess **energy**. *The energy of a body is its capacity for doing work, or its ability to do work.*

If we examine a body upon which work has been done, — any lifted mass, for instance, — we discover that the lifting process has invested the mass with the ability to do work upon some other body. The lifted body may be attached by a cord to the proper mechanism and allowed to fall back to its original position; but during its fall it may turn wheels, lift another body, bend a bow, wind a spring, or do work in some other manner. Thus *in doing work the falling body gives energy or working ability to that upon which the work is done.* Similarly, a hammer by virtue of the velocity given it by the mechanic possesses the capacity for doing work. This is manifested by the fact that it drives the nail in opposition to the resistance offered by the wood.

60. Potential Energy. — The lifted weight in the illustration used in the preceding section possesses energy because of its elevated position. A bent bow has the ability to throw an arrow because of the fact that its form has been changed by some agent. The spring of a watch can keep the wheels moving against resistance on account of the fact that some one has done work upon it in winding it up. *Energy possessed by a body because of its position or form is called* **potential energy**. The potential energy of a body is measured by the work that was done upon it to bring it into the condition by virtue of which it possesses that energy. Hence a body whose mass is 100 pounds which has been lifted a distance of 5 feet has 500 foot-pounds of potential energy, *i.e.* it is able to do 500 foot-pounds of work because of its elevated position.

61. Kinetic Energy. — When a lifted weight is allowed to fall, it does work upon the object that it strikes. At the instant of striking it possesses energy because it is in motion. Moreover, any moving body is able to do work by virtue of its motion. *The energy possessed by a body because of its motion is called* **kinetic energy.** The kinetic energy of a body is measured by the amount of work done upon it to put it in motion.

62. Kinetic Energy Computed. — *The kinetic energy of a moving body is measured by one half its mass multiplied by the square of its velocity.* This may be shown in the following manner: Let a body whose mass is m grams be acted upon by a force of f dynes which will give it an acceleration of a cm./sec.^{2}. From (2), § 36, $f = ma$. Again, since the force produces uniformly accelerated motion in the given mass, at the end of t seconds, as shown by (3), § 25, the body will have been moved through a distance $d = \frac{1}{2}at^2$. Now the velocity v acquired by the mass in t seconds as shown by (2), § 25, is $v = at$ cm./sec.; whence $t^2 = \frac{v^2}{a^2}$. Substituting this value for t^2 in the equation for distance, we obtain $d = \frac{v^2}{2a}$ centimeters.

In order to compute the work done by the force f, we have only to multiply the force by the distance d through which it acts (§ 55). Thus

$$Work = fd = ma \times \frac{v^2}{2a} = \frac{mv^2}{2}\ ergs.$$

Since the work done in producing the motion is the measure of the kinetic energy of the mass m (§ 59),

$$\textbf{Kinetic Energy} = \tfrac{1}{2}\,\mathbf{mv^2}\ \textbf{ergs.} \qquad (2)$$

Let the mass, velocity, and acceleration be given in the units of the F. P. S. system. Then the product *ma* will give the force in poundals (§ 36), and the quantity $\frac{v^2}{2a}$, the distance through which the force acts, in feet. Therefore the product of force and distance will give the work in foot-poundals. Hence, in the F. P. S. system

$$Kinetic\ Energy = \tfrac{1}{2}\, mv^2\ foot\text{-}poundals.$$

It should be remembered that the formula for kinetic energy deduced above gives the result in the absolute system only, *i.e.* in ergs or foot-poundals. These can be readily reduced to kilogram-meters and foot-pounds by the help of the numerical relations given in § 56.

Example. — Calculate the kinetic energy of a 10-gram bullet whose velocity is 40,000 cm./sec.

Solution. — Using equation (**2**), we have for the kinetic energy of the bullet $\frac{1}{2} \times 10 \times 40{,}000 \times 40{,}000$, or 8,000,000,000 ergs. By referring to § 54, we observe that 1 kg-m. equals 98,000,000 ergs. Therefore, the reduction from ergs to kilogram-meters gives for the kinetic energy of the body 8163 kg-m.

EXERCISES

1. Calculate the potential energy given to a mass of 25 g. by lifting it through a vertical height of 10 m. Express the result in kilogram-meters.

2. A ball moving with a velocity of 3500 cm./sec. has a mass of 250 g. Find its kinetic energy in ergs. How much work must a boy do in order to stop it?

3. Compare the kinetic energy of the ball in Exer. 2 with that of a mass of 25,000 g. whose velocity is 350 cm./sec.

4. What is the kinetic energy of a 5-gram bullet just as it is leaving the muzzle of a gun with a velocity of 30,000 cm./sec.?

5. To what height would the bullet in Exer. 4 have to be taken in order to have an equal amount of potential energy?

Suggestion. — First find the force required to lift the bullet in dynes; then apply equation (**1**), § 55.

6. Compute the kinetic energy of a 5-pound mass moving with a velocity of 25 ft. per second. Express the result in foot-pounds.

Suggestion. — First obtain the result in foot-poundals; then reduce to foot-pounds by the help of § 56.

7. A constant force of 200 dynes acts upon a mass of 5 g. Calculate (1) the acceleration, (2) the velocity produced in 3 seconds, and (3) the kinetic energy. What is the distance through which the force acts during the 3 seconds?

8. In order to move a load up a hill 250 ft. long, a horse exerts a constant pull of 125 lb. How much work is done? If the load weighs 900 lb., to what height would an equivalent amount of work lift it?

9. A stone whose mass is 50 kg. is placed on the top of a chimney 30 m. in height. Calculate the amount of work that must be performed in kilogram-meters and foot-pounds.

10. Compute the amount of work done per minute by a pumping engine that forces 100,000 gal. of water into a reservoir 120 ft. high every 10 hr. Assume the density of water to be 62.5 lb. per cubic foot.

11. If a rifle ball whose mass is 8 g. has a velocity of 35,000 cm./sec., how far will it penetrate a block of wood that offers a uniform resistance of 100,000 g.

Suggestion. — Let x be the depth of penetration in centimeters, and place the work done by the ball expressed in ergs equal to the kinetic energy.

12. The elevation of a tank containing 25,000 gal. of water is 75 ft. Find the potential energy of the water.

4. TRANSITIONS OF ENERGY

63. Transference and Transformation of Energy. — No processes in nature are of more common occurrence than *transferences* of energy from one body of matter to another and *transformations* of energy from one form into another. For example, if a body is allowed to fall freely, the *potential* energy that it possesses while elevated is gradually *transformed* into *kinetic* energy which resides in the body until its motion is checked. If, however, the body should fall upon a spring properly placed, the spring would be compressed and thus possess *potential* energy at the expense of the *kinetic* energy of the falling mass. Hence energy is *transferred* from the falling body to the spring. *Whenever one body does work upon another, energy* *is transferred from the body that does the work to the one upon which the work is done.*

Let the elevated mass M, Fig. 27, be suspended by a cord wound around an axle A to which is attached a heavy wheel W. It is plain that the downward pull of M upon the cord will cause the wheel to turn. Thus, as M falls and loses potential energy, it does work upon the wheel in producing motion and thus imparting kinetic energy.

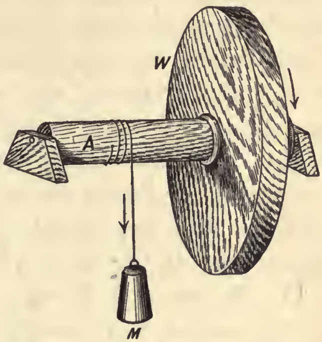


FIG. 27. — Transformation of the Potential Energy of the Raised Mass M into Kinetic Energy in the Wheel W.

When the cord is fully unwound, the action will not cease; but the kinetic energy of the wheel, by winding up the cord around the axle on the opposite side, will enable it to lift M. In this manner the wheel performs work upon M, losing its kinetic energy and imparting potential energy to the mass lifted. If no energy were lost in overcoming friction, the kinetic energy imparted to the wheel in the former case would be completely restored to the mass M in the latter.

Changes in energy occur in a large number of common processes, such as winding a clock or a watch, shooting an arrow from a bow, running a sewing machine, turning a grindstone, running mills by water power, etc.

64. Conservation of Energy. — Although energy is passing continually through transformations and is being transferred from one body to another around us on every hand, no one has ever been able to prove that even the smallest portion can be created or destroyed. The inference is, therefore, that *the same quantity of energy is present in the universe to-day as existed ages ago*; *i.e.* that the quantity of energy present in the universe remains constant. This is known as the **Law of the Conservation of Energy**.

The principal aim of Physics is to trace the various transformations and transferences of energy that accompany natural phenomena. At this point in the study many of these changes will seem obscure because all the forms in which energy may exist have as yet not been considered. For example, we may inquire what becomes of the kinetic energy of a spinning top as it slowly comes to rest. As we pursue the study further, we find that where there is motion in opposition to friction, as in this case, heat is produced. But heat is one of the forms that energy may take. Hence the kinetic energy of the top will appear somewhere in the form of heat.

65. Matter and Energy. — The intimate relation between matter and energy is becoming more and more apparent. Matter is obviously a carrier, or vehicle, of energy. We become acquainted with matter only through natural phenomena. In each phenomenon there is involved some change in energy, and it is in the transformations and transferences of energy that our senses are affected. It is upon these processes that we base our entire knowledge of the material world.

EXERCISES

1. In driving a well a heavy weight is elevated by a horse and then allowed to fall upon the end of a vertical pipe, thus forcing it into the ground. Trace the energy changes taking place in the process.

2. Trace the transferences and transformations of energy in the process of driving a nail; of planing a board; of shooting an arrow; of throwing a stone; of winding a clock; of running a sewing machine; of beating an egg.

3. Account for the energy of the water above a dam in a river. What becomes of this energy?

4. In what form is a supply of energy taken on board an ocean steamer? In what form is energy supplied to a locomotive? to an automobile? to a horse? to a man?

SUMMARY

1. The term *work* is used to express the process of producing motion. Work involves both *force* and *motion in the direction of the force*, and is measured by the product of the force employed multiplied by the distance through which it acts. The equation of work is $Work = f \times d$ (§ 55).

2. The *erg*, or *dyne-centimeter*, is the C. G. S. unit of work and of energy and is the work done by a force of one dyne acting through a distance of one centimeter. The *foot-poundal* is the English absolute unit of work and energy. The *kilogram-meter* and *foot-pound* are the gravitational units in common use (§ 56).

3. The *activity*, or *power*, of an agent is the rate at which it can do work. The activity of an agent is said to be one *horse power* when it can perform work at the rate of 550 foot-pounds (or 746×10^7 ergs) per second (§§ 57 and 58).

4. The *energy* of a body is its capacity for doing work, or its ability to do work (§ 59).

5. *Potential energy* is the energy possessed by a body because of its position or form (§ 60).

6. *Kinetic energy* is the energy possessed by a body by virtue of its motion. The equation of kinetic energy is $K.E. = \frac{1}{2} mv^2$ (§§ 61 and 62).

7. When one body does work upon another, energy is transferred from the body that does the work to the one upon which the work is done (§ 63).

8. Energy cannot be created or destroyed. The quantity present in the universe remains constant. This is known as the *Law of the Conservation of Energy* (§ 64).

CHAPTER V

GRAVITATION

1. LAWS OF GRAVITATION AND WEIGHT

66. Universal Gravitation. — Ancient astronomical observations revealed the fact that the planets move through space in curvilinear paths. Later and more refined observations led to the discovery that the sun is a center about which they revolve in slightly elliptical orbits. Furthermore, it is universally known that several of the planets have satellites which revolve about them, corresponding to the moon which moves in an orbit encircling the earth. Late in the seventeenth century Sir Isaac Newton originated the theory that is now known as the **Law of Universal Gravitation**, in order to account for the motion of heavenly bodies in nearly circular orbits instead of straight lines.

67. Newton's Law of Universal Gravitation. — This law may be stated as follows:

Every body in the universe attracts every other body with a force which is directly proportional to the product of the attracting masses and inversely proportional to the square of the distance between their centers of mass (§ 70).

According to this law a book and a marble, or two bodies of any other kind of matter, attract each other. Between ordinary masses this force remains unnoticed by us in everyday life because it is so minute; in fact, it would require the most refined test to detect it. But since the attraction is proportional to the *product of the masses*, in the case of two heavy bodies — the moon and the earth, for example — the force is enormous. Even between the earth and a marble or a book the force is quite perceptible. When the earth is one of the acting masses, the attraction is called the *force of gravity*, and when expressed in the proper units of measure, this attraction is called the *weight* of the marble, book, etc. Weight, therefore, partakes of the nature of a force and is quite distinct from mass (§ 10). Hence, when we say in ordinary language, for instance, that the intensity of a certain force is 10 pounds, we mean that it is equal to that force with which the earth attracts a mass of 10 pounds. Again, since weight is a force, it may be expressed in any of the units of force (§§ 36 and 37), *i.e.* in dynes, poundals, etc.

68. Weight. — Since for a given locality the mass of the earth, as well as the distance from the center, is constant, *the weight of a body is strictly proportional to its mass.* Again, since the earth is not spherical but flattened slightly at the poles, the same mass at different places will not possess the same weight. On moving north or south from the equator the radius of the earth decreases slightly, which causes the mass to come somewhat nearer the earth's center and thus increases the value of the force of attraction.

69. Law of Weight. — The law of universal gravitation applied to bodies outside the earth's surface is as follows:

The weight of a body above the earth's surface is inversely proportional to the square of its distance from the center of the earth.

If the radius of the earth is assumed to be 4000 miles, the weight of a one-pound mass 4000 miles above the surface, which is 8000 miles from the center, would be only one fourth of a pound. This result is obtained by applying the law of weight as follows:

$$x : 1 \text{ pound} :: 4000^2 : 8000^2;$$

whence $$x = \tfrac{1}{4} \text{ pound}.$$

Since the distance from the earth's center is less at Chicago than at the equator, a mass of 1 pound weighs about $\frac{1}{500}$ of a pound more at the former place than at the latter. For small differences of latitude, however, the difference in weight is so small that it is of little importance. In consequence of the earth's rotation the weight of bodies at the equator is diminished $\frac{1}{289}$ (§ 53) as the result of the centrifugal reaction against the force of gravity. In other latitudes this diminution is less.

It is of interest to consider what the effect would be upon the weight of a given mass if it were to be taken to some point below the surface of the earth. Let it be imagined that the circle in Fig. 28 represents a cross section through the center of the earth, and that P is the location of the body to be weighed. All that part of the earth represented by the shaded portion of the circle above the plane AB will exert a resultant attraction upward, while that represented by the unshaded part has a resultant acting toward the center O. Since these forces oppose each other, the weight of the body will diminish as it approaches the center O. When the body reaches the center, the attractions due to the different portions of the earth will be equal in all directions, and the resultant of all will be zero. Therefore the body will weigh nothing at the center of the earth.

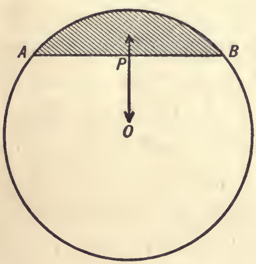


FIG. 28. — A Mass at P is Attracted Upward as well as Downward.

EXERCISES

1. If the mass of the earth were doubled without any change in its shape or size, how would a person's weight be affected?

2. Which is a definite quantity, a gram of matter or a gram of force (*i.e.* a gram-weight)?

3. How much will the potential energy of a mass of 2000 lb. elevated 100 ft. at Chicago differ from that of an equal mass raised 100 ft. at the equator?

4. A certain mass is weighed on a dynamometer (§ 11) at New York. Will the instrument indicate a greater or a less weight when the same mass is weighed at the equator?

5. If two masses are in equilibrium when placed in the pans of a beam balance at the equator, will they still be in equilibrium when tested in the same manner at San Francisco?

6. How far above the earth's surface would a body weigh one half as much as at the surface? *Ans.* 1656.8 mi.

7. What would a 100-pound body weigh at a distance of 200 mi. above the earth's surface?

8. An aëronaut ascends 5 mi. in a balloon. If his weight at the surface is 150 lb., what will it be at that height?

2. EQUILIBRIUM AND STABILITY

70. Center of Gravity. — The weight of a body is the resultant of the weights of the individual particles of which it is composed. Since these innumerable forces are all directed toward the center of the earth, which is 4000 miles away, they form a system of essentially parallel forces whose resultant CA, Fig. 29, is equal to their sum (§ 46). The point of application C is called the **center of gravity** of the body. Since the position of this point depends upon the distribution of matter in the body, it is also called the **center of mass**. In many problems it is convenient to consider the body as though all its mass were located at this point. If a flat piece of cardboard of any shape is balanced on the point of a pin, the center of gravity is located at the point of contact and midway between the two surfaces.

Fig. 29. — Weight is the Resultant of Innumerable Small Forces.

Let a flat piece of cardboard be pierced at any point, as *A*, Fig. 30, and hung loosely on a small nail or pin. The cardboard will turn until the center of gravity falls as low as possible. In this condition a vertical line through *A* will pass through the center of gravity. This line is easily found by hanging a plumb line from the axis in front of the cardboard. If a second point of support, as *B*, be taken and a vertical line determined as before, the center of gravity *C* will lie at the point of intersection of the two lines.

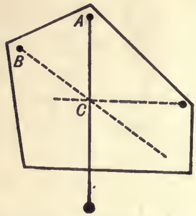


FIG. 30. — Locating the Center of Gravity.

71. Equilibrium of Bodies. — A body is said to be in **equilibrium** when a vertical line through the center of gravity passes through a point of support. A common case of equilibrium is that of a chair or table. In such instances a vertical line through the center of gravity passes through the area of the *base* included within the lines joining the feet. Four typical cases are represented in Fig. 31. Pyramid *A* hangs from its apex, *B* stands upon its base, and *C* rests with its apex at the point of support. Obviously *A* and *B* tend to remain indefinitely in the positions shown, but *C* will overturn with the slightest disturbance. If *A* or *B* should be tilted, the center of gravity would be lifted, necessitating the expenditure of energy upon the body. *A* and *B* are said to be in **stable equilibrium**. On the other hand, a disturbance of *C* lowers the center of gravity and thus lessens the potential energy of the body. When a body is in this condition, it is said to be in **unstable equilibrium**.

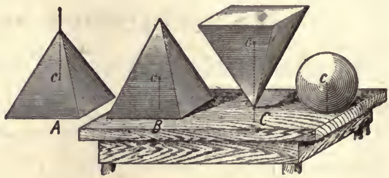


FIG. 31. — Stable, Unstable, and Neutral Equilibrium Illustrated.

A third condition is represented by a sphere of uniform density resting upon a smooth horizontal plane. If the sphere be rolled along the plane, its center of gravity will be neither raised nor lowered. It is said to be in **neutral equilibrium.** A body arranged to turn upon an axis through its center of gravity is also in a condition of neutral equilibrium.

72. Stability of Bodies. — When a body is in stable equilibrium, work must be performed upon it in order to cause it to overturn. This amount of work will depend upon the weight of the body and the distance through which its center of gravity is lifted, and is measured by their product (§ 55). The amount of work required to overturn a body is a measure of its **stability.**

Example. — Find the stability of a box 4 ft. square and 2 ft. high and weighing 500 lb.

Solution. — Referring to Fig. 32, it is plain that the center of gravity C must be moved to the point A while the box is being overturned. The height through which C is raised is BA, equal to $OC - OB$. Now OC is the hypotenuse of the right triangle OBC whose sides are 1 ft. and 2 ft. respectively. Hence OC equals $\sqrt{5}$, or 2.24 ft. Therefore, AB is 1.24 ft., and the work done 1.24×500, or 620 foot-pounds.

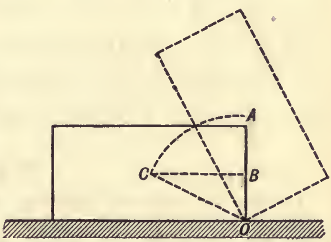


Fig. 32. — Center of Gravity is Lifted Through the Height BA.

It is clear that of two bodies having the same weight, the more stable one is that whose center of gravity has to be lifted through the larger vertical distance when we overturn it. This will depend on the size and shape of the base on which it rests and on the height of the center of gravity above the base. Figure 33 shows a brick in three possible positions. The center of gravity C moves through an arc having the lower right-hand corner of the brick as its center when the body is overturned about this point. It will be seen at once that the greatest

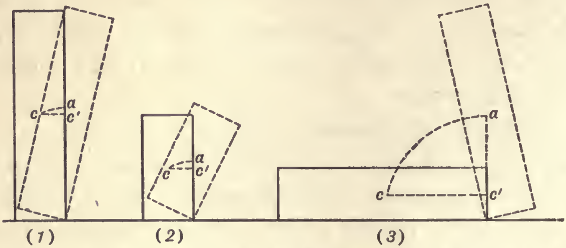


FIG. 33.—The Overturning of a Brick about Different Edges.

stability is possessed by the brick when lying on its largest base; first, because the base is largest, and second, because the center of gravity is in the lowest possible position. In each case $c'a$ is the vertical distance through which the center of gravity would have to be lifted by the overturning agent.

Note the various methods employed to give the proper stability to objects in everyday use, as lamps, clocks, inkstands, chairs, pitchers, vases, etc.

EXERCISES

1. How would you place a cone on a horizontal table in positions representing the three conditions of equilibrium?

2. Arrange two knives in a piece of wood as shown in Fig. 34 and support the point on the finger. Why is the system in stable equilibrium? Where is the center of gravity of the system?

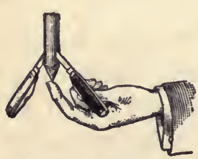


FIG. 34.—A System in Stable Equilibrium.

3. Why is it difficult to walk on stilts?

4. Explain the difficulty experienced in trying to balance an upright rod upon the end of the finger.

5. Why does not the Leaning Tower of Pisa fall? See Fig. 36.

6. Explain the difficulty experienced in trying to balance a meter stick on one end upon a level table.

7. The oil can B shown in Fig. 35 is loaded with lead at the bottom. Explain how this can will right itself while one of the common form A remains overturned.

8. Which of two bodies having equal weights possesses the greater stability, a pyramid or a rectangular box having the same base and height as the pyramid?

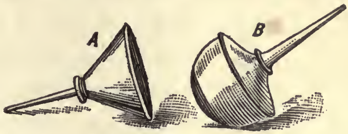


FIG. 35. — Oil Cans.

SUGGESTION. — The center of gravity of a pyramid is located at one third of the distance from the base to the apex.

9. Calculate the stability in foot-pounds of a 4-pound brick placed in three different positions on a horizontal table. Assume the dimensions to be $2 \times 4 \times 8$ in. See example in § 72.

3. THE FALL OF UNSUPPORTED BODIES

73. Falling Bodies. — Before the time of the Italian mathematician and physicist, Galileo Galilei,[1] little was known concerning the way in which bodies fall when unsupported. It is a well-known fact that if we drop a coin and a piece of paper or a feather at the same instant, the coin will reach the floor first. Galileo rightly inferred that the difference was due to the resistance offered by the air. Nevertheless, in order to place on an experimental basis his conclusion that all falling bodies tend to have the same acceleration, he dropped bodies of different kinds from the top of the Leaning Tower of Pisa (Fig. 36) in the presence of many learned men of the time. These experiments demonstrated that *all bodies tend to fall from a given height in practically equal times.* Furthermore, it was readily shown that light materials, as paper, for example, fall in less time when compressed than when spread

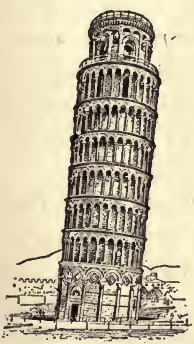


FIG. 36. — Leaning Tower of Pisa, Italy.

[1] See portrait facing p. 70.

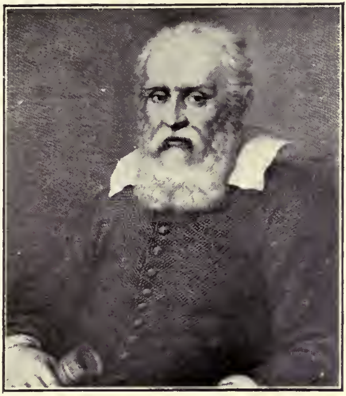
out. Later, after the invention of the air pump, Galileo's inference was verified by allowing a light and a heavy body to fall in a vacuum. For this purpose the "guinea and feather" tube (Fig. 37) is commonly used. On inverting the tube after the air has been exhausted, we find that the feather falls as rapidly as the coin. But when the air is again admitted, the feather flutters slowly along far behind the rapidly falling coin.

FIG. 37. — Bodies Fall Alike in a Vacuum.

74. Uniform Acceleration of Falling Bodies. — Since the attraction existing between a body and the earth is constant, it follows that a body falling freely, *i.e.* without encountering resistance, will have uniformly accelerated motion (§§ 24 and 36). Again, since all bodies fall the same distance in a given time when unimpeded, the acceleration will be the same for all bodies. This acceleration is called the **acceleration due to gravity**, and is designated by the letter g.

75. The Acceleration Due to Gravity. — The acceleration of freely falling bodies varies according to the laws of weight given in § 69. Since the distance to the center of the earth decreases as one travels from the equator toward the poles, the acceleration due to gravity becomes greater. In latitude 38° N. g is 980 cm./sec.^{2}, and in latitude 50° N. 981 cm./sec.^{2}. The value of g also decreases slightly with the elevation above sea-level. (Why?) In the latitude of New York, 40.73° N., a freely falling body gains in velocity at the rate of about 980 centimeters, or 32.16 feet, per second during each second of its motion.

When bodies are thrown upward, the acceleration is negative; *i.e.* the velocity decreases at the rate of 980 centimeters per second



GALILEO GALILEI (1564–1642)

The first successful experimental investigations relating to falling bodies and the pendulum must be attributed to Galileo. For nearly twenty centuries the science of Mechanics had remained undeveloped. Aristotle had announced that the rate at which a body falls depends upon its weight, but Galileo was the first to disprove it by experiment. This he did by dropping light and heavy bodies from the leaning tower of Pisa, Italy, his native town. A one-pound ball and a one-hundred-pound shot, which were allowed to fall at the same time, were observed by a multitude of witnesses to strike the ground together. Hence the rate of fall was shown to be independent of mass.

At another time, while observing the swinging of a huge lamp in the cathedral, Galileo was astonished to find that the oscillations were made in equal periods of time no matter what the amplitude. He proceeded to test the correctness of this principle by timing the vibrations with his own pulse. Later in life he applied the pendulum in the construction of an astronomical clock.

Galileo was the first to construct a thermometer and the first to apply the telescope, which he greatly improved, to astronomical observations. He discovered that the Milky Way consists of innumerable stars; he first observed the satellites of Jupiter, the rings of Saturn, and the moving spots on the sun.

Galileo was made professor of mathematics in the University of Pisa in 1589 and filled a similar position at Padua from 1592 until 1610. He died in the year 1642, the year of Newton's birth.

during each second of its upward motion. Hence, a body thrown upward with a velocity of 2940 centimeters per second will continue to rise 3 seconds, when it will stop and return to earth in the next 3 seconds. However, on account of the hindrance of the air, bodies moving with great velocity deviate considerably from the laws governing unimpeded bodies.

76. Laws of Freely Falling Bodies. — Since the motion of an unimpeded body while falling is uniformly accelerated, the equations of § 25 may be applied by simply substituting for a the acceleration due to gravity g. These equations may be written as follows:

$$\mathbf{v} = \mathbf{gt}, \qquad (1)$$

and

$$\mathbf{d} = \tfrac{1}{2}\,\mathbf{gt}^2. \qquad (2)$$

From equations (1) and (2) other useful formulæ may be deduced. From (1) we find that $t^2 = \frac{v^2}{g^2}$. Substituting this value for t^2 in equation (2), we obtain

$$\mathbf{d} = \frac{\mathbf{v}^2}{2\,\mathbf{g}}. \qquad (3)$$

Solving equation (3) for v, we have

$$\mathbf{v} = \sqrt{2\,\mathbf{gd}}. \qquad (4)$$

77. Distances and Velocities Represented. — The distances passed over and the velocities acquired by a freely falling body are represented graphically in Fig. 38. A vertical line is drawn on which a convenient distance AB is measured off to represent $1 \times \frac{1}{2} g$ (about 16 feet), the distance the body falls during the first second. The distance AC is

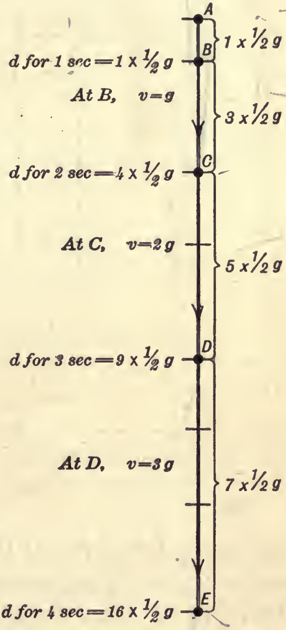


FIG. 38. — Motion of a Falling Body Represented.

made four times the distance AB; AD, nine times AB; AE, sixteen times AB, etc., to represent the distances fallen in one, two, three, and four seconds respectively. The heavy arrows are drawn to represent the velocity at the end of each second. The length of the first is g units (representing about 32 feet per second), the second $2g$, the third $3g$, etc.

78. Bodies Thrown Horizontally. — It is a well-known fact that a body projected in any direction except up or down follows a curved path. An interesting case of this kind is the projection of a body horizontally from some elevated position, as A, Fig. 39. The motion of the body will be the resultant of two component motions, the one vertically downward due to gravity, and the other in a horizontal direction due to the projectile force. Since there is no horizontal force acting on the body after it leaves the point A, the horizontal component of the motion will be uniform, and the body will move (horizontally) over equal distances in equal intervals of time. Let AB', $B'C'$, $C'D'$, etc., represent these horizontal distances for successive seconds. The distances AB, BC, CD, etc., are drawn in the manner described in § 77. Under the combined action of its initial horizontal velocity and the force of gravity the body will pass through the point P_1 at the end of the first second, P_2 at the end of the second, P_3 at the end of the third, etc. Thus the body follows the curved path $AP_1P_2P_3P_4$.

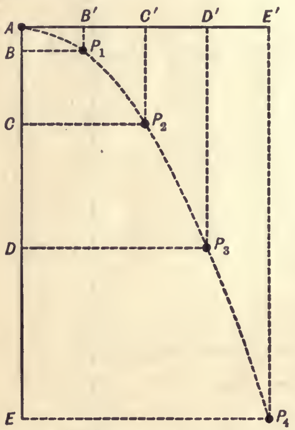


Fig. 39. — Motion of a Body Projected Horizontally from the Point A.

The path of a stone thrown over a tree, for example, is a case in which the initial motion of the body is not horizontal. The motion may be divided into two parts: first, the rise of the stone to the highest point reached; and, second, the fall of the body back to the earth. The latter half of the motion is precisely the case described above. The time during which the stone is rising is practically equal to that of its fall. The time required for the return of the stone to the earth is the same as that of a body falling vertically. Likewise the time occupied by the stone in rising is equal to that required by a body thrown vertically upward to attain the same height. The shape of the path taken by the stone depends on its initial speed and the direction in which it is thrown. The study of the motion of projected bodies, as bullets, cannon balls, shells, etc., forms an important part of military and naval instruction.

EXERCISES

1. A body falls freely from a certain height and reaches the ground in 5 seconds. What velocity is acquired? From what height must it fall?

2. How long does it take a body to fall 100 ft.? 200 ft.?

3. A mass of 50 g. falls for 3 seconds from a state of rest. Calculate its kinetic energy. (See § 62.)

4. A mass of 50 lb. falls from an elevation of 20 ft. Calculate its kinetic energy in foot-pounds at the time it reaches the ground. Compare the kinetic energy with the potential energy of the body before falling.

5. How far must a body fall in order to acquire a velocity of 500 ft. per second?

6. A book falls from a table 3 ft. in height. Find the velocity of the book when it reaches the floor.

7. A stone is dropped from a train whose velocity is 30 mi. per hour. Show by a diagram the path traced by the stone. (See § 78.)

8. Find the kinetic energy of a 10-gram mass after it has fallen from rest a distance of 1960 cm., assuming g to be 980 cm./sec.^{2}.

9. The velocity of a body falling freely from rest was 200 ft. per second. From what height did it fall?

10. Compare the velocity of a body after falling 64.32 ft. with that of a train running 30 mi. per hour.

11. A bullet is fired vertically upward from a gun with a velocity of 25,000 cm./sec. Disregarding the resistance of the air, how many seconds will the bullet continue to rise? How high will it rise?

12. If the bullet in Exer. 11 encountered no resistance due to the air, how many seconds would pass before it returned to earth?

13. The weight of a pile-driver is lifted 10 ft. and allowed to fall. With how much greater velocity will it strike if lifted 20 ft.? With how much greater energy?

14. A stone thrown over a tree reaches the earth in 3 seconds. What is the height of the tree?

15. A boy fires a rifle ball vertically upwards and hears it fall upon the ground in 20 seconds. How high does it rise? What was its initial velocity?

4. THE PENDULUM

79. The Simple Pendulum. — *A heavy particle suspended from a fixed point by a weightless thread of constant length is an ideal* **simple pendulum.** If, however, we suspend a small metal ball *A*, Fig. 40, by a thin flexible thread or wire from a fixed point *O*, it fulfills the ideal conditions almost perfectly. The distance *OA* from the point of suspension to the center of the ball is the *length* of the pendulum. The arc *AC*, or the angle *AOC*, which represents the displacement of the ball from the position of equilibrium, is the *amplitude of vibration.* A *vibration* is one to-and-fro swing, sometimes called a *complete* or *double vibration.* A *single vibration* is the motion of the ball from *C* to *C′*, or one half of a complete vibration. The *period of a single vibration* is the time consumed by the pendulum in moving from *C* to *C′*.

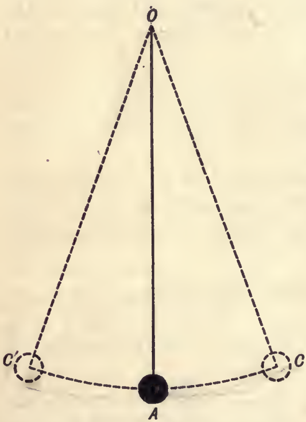


FIG 40. — The Simple Pendulum.

80. Pendular Motion Due to Gravity. — When a pendulum is in the position of rest, the weight of the ball represented by the line *AW*, Fig. 41, is balanced by the equal and oppositely directed tension *AT* in the cord *OA*. Now let the ball be drawn aside to the position *C* and released. The weight of the ball, which always acts vertically downward and is represented by the line *CP*, can be resolved into two components. One of these components is represented by the line *CE* and serves solely to produce tension in the cord *CO*. It is plain that this component has no effect on the motion of the ball. The other component *CB* acts upon the ball along the tangent to the arc of vibration at the point *C*. The effect of this component is to give the ball accelerated motion along the arc. After the ball passes the point *A*, it is clear that the component along the tangent tends to retard the motion and finally succeeds in stopping the ball at the point *C′*, after which it returns the ball again to *A*.

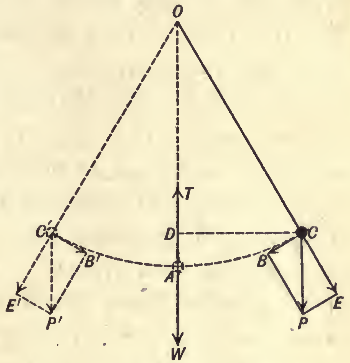


FIG. 41. — Gravity Causes a Pendulum to Vibrate.

If the line *CD* is drawn perpendicular to *OA*, the triangles *CDO* and *CBP* are similar. Why? We may therefore write the proportion

$$\text{CB} : \text{CP} :: \text{CD} : \text{CO} \qquad (5)$$

This proportion may also be written as follows:

$$\text{Force CB} = \text{displacement CD} \times \frac{\text{weight of ball CP}}{\text{length CO}}. \qquad (6)$$

Since the weight of the ball *CP* and the length of the pendulum *CO* remain constant during a vibration, equation (6) shows that the effective force *CB* is proportional to the displacement *CD*. Hence the acceleration of the ball is not the same at all points in the arc CA, but varies directly as the displacement.

81. Transformations of Energy in the Pendulum. — A pendulum is first set in motion by displacing the ball, or pendulum bob. This process requires the performance of work which elevates the ball through the height AD, Fig. 41 (measured vertically), and stores potential energy in it. From equation (1), § 55, it is clear that the amount of potential energy given the ball is measured by the product of its weight and the height AD. As the ball moves toward A, velocity is acquired, and the potential energy is gradually changed into kinetic. At A the energy is all kinetic. After passing the point A the ball rises to C'', while the kinetic energy is transformed back into potential. At C'' the energy is all potential again. Thus recurrent transformations of energy take place, which would occur without loss if it were not for the resistance offered by the air as well as by friction at the point of suspension.

Fig. 42. — Pendulums of Different Lengths and Masses.

82. Laws of the Simple Pendulum. — The first three laws of the simple pendulum may be deduced from the results obtained from the following experiments :

Suspend four balls as shown in Fig. 42. Let A, B, and C be of metal, and D of wood or wax. Make the lengths of A, B, and C as 1 : 4 : 9; *e.g.* 20, 80, and 180 cm. Also let D be made precisely of the same length as C. Now if C and D be set swinging through the same amplitude, it will be readily observed that the period of vibration of D is the same as that of C.

Again, let C and D be set in vibration through different amplitudes. If neither amplitude is large, it will be seen that the period of one is still the same as that of the other.

Finally, let A, B, and C be put in motion successively and the single vibrations of each counted for one minute. If the period of vibration of each pendulum be computed from the number of vibrations per minute, it will be found that the three periods are as 1 : 2 : 3, *i.e.* as $\sqrt{1} : \sqrt{4} : \sqrt{9}$.

The laws governing the vibration of simple pendulums, therefore, may be stated thus:

(1) *The period of vibration is independent of the material, or mass, of the ball.*

(2) *When the amplitude of vibration is small, the period of vibration is independent of the amplitude;* i.e. *the vibrations are made in equal times.* This is called the **Law of Isochronism** (pronounced *i sŏk′ro nism*). If the amplitude exceeds 5° or 6°, the period of vibration will gradually diminish as the arc becomes smaller.

(3) *The period of vibration is directly proportional to the square root of the length of the pendulum.* This is called the **Law of Length.** This law may be represented thus: $t_1 : t_2 :: \sqrt{l_1} : \sqrt{l_2}$, where t_1 and l_1 refer to the period and length of one pendulum, and t_2 and l_2, to the period and length of the other. If, therefore, any three of the terms are given, the fourth may be computed.

Since a pendulum is dependent upon the force of gravity, as shown in § 80, its period of vibration is found to depend upon the value of g. Hence:

(4) *The period of vibration is inversely proportional to the square root of the acceleration due to gravity.*

83. The Pendulum Equation. — The relation between the period and length of a simple pendulum and the acceleration due to gravity is given by the equation

$$\mathbf{t} = \pi\sqrt{\frac{\mathbf{l}}{\mathbf{g}}}, \qquad (7)$$

where t is the period of a single vibration, l the length of the pendulum measured in centimeters (or feet), and g

the acceleration due to gravity measured in centimeters per second per second (or feet per second per second). The value of π is 3.1416, the ratio of the circumference of a circle to the diameter.

This equation is of great assistance (1) in calculating the period of any simple pendulum of known length, (2) in determining the length of a pendulum that vibrates in any given period of time, and (3) in finding the value of g at any place where its magnitude is unknown. (See § 87.)

84. The Seconds Pendulum. — *A pendulum of which the period of a single vibration is one second is a* **seconds pendulum.** As shown by equation (7), the length of a simple pendulum l, when the period t is one second, will depend on the acceleration due to gravity at the place chosen. At all places where the value of g is 980 cm./sec.^{2}, the length l of a simple pendulum that beats seconds is 99.3 cm.; where g is 981 cm./sec.^{2}, l is 99.4 cm.

FIG. 43. — A Compound Pendulum and its Equivalent Simple Pendulum.

85. The Compound Pendulum. — When the conditions defining the ideal simple pendulum (§ 79) are not sufficiently fulfilled, the body is called a **compound** or **physical pendulum.** For experimental purposes a meter bar may be suspended on a smooth wire nail which pierces it at right angles close to one end. The bar may be hung to swing freely between the prongs of a large tuning fork, as shown in Fig. 43. Let a simple pendulum OA be placed by the side of the suspended bar so that their points of suspension lie in the same horizontal plane. Set both pendulums in vibration and adjust the length of the simple one until they vibrate in the same period of time. It will be observed at once that the simple pendulum must be made several inches shorter than the other. Only those points in the compound pendulum very near the point C swing in their natural period; particles below C tend to swing slower, and those above, faster, than the simple pendulum. C is called the *center of oscillation* of the compound pendulum. In this case the point c is located two thirds of the length of the bar from the point of suspension. The simple pendulum is thus seen to be a special case of the compound one in which the entire mass is concentrated near the center of oscillation.

The compound pendulum in Fig. 43 may be set swinging by being struck a sharp blow at the point C, and the axis will not be disturbed. For this reason C is called the *center of percussion*. Thus, for example, when a ball is batted, the bat should be so handled that the ball will strike its center of percussion. This will prevent the jarring of the hands and the breaking of the bat.

86. The Compound Pendulum Reversible. — If the meter bar shown in Fig. 43 be suspended by piercing it at the center of oscillation C and swinging it about this point, the period of vibration will be the same as before. In other words, the point of suspension B and the center of oscillation C are interchangeable. This property of a compound pendulum, which was discovered by the famous Dutch physicist Huyghens (1629–1695), is known as its **reversibility**.

87. Utility of the Pendulum. — The value of the pendulum in the measurement of time is due to the isochronism of its vibrations. Although Galileo was the first to observe this property of the pendulum and the first to make a drawing of a pendulum clock, Huyghens was the first to use a pendulum in controlling the motion of the wheels of a timepiece. This he accomplished in 1656.

The motion of a clock is maintained by lifted weights or by the elasticity of springs. The office of the wheelwork is to move the hands over the dial and to keep the pendulum from being brought to rest by friction. The latter is effected by means of the escapement shown in Fig. 44. The wheel R is turned by mechanism not shown in the figure. When the pendulum swings to the right, motion is communicated to the curved piece MN through the parts A, B, and O; and M is lifted. The wheel is thus released; but, on turning, strikes at N. While the pendulum moves to the left, the slight pressure of the cog against N causes A to deliver a minute force to the pendulum. As N rises, the wheel is again released, but is again detained at M. Thus one cog is allowed to pass for each complete vibration of the pendulum. Every "tick" of the clock is caused by the wheel R being stopped either at M or N. If the wheel is allowed to turn too fast, the clock gains time. This defect is corrected by lowering the bob. If the clock loses time, the bob is raised.

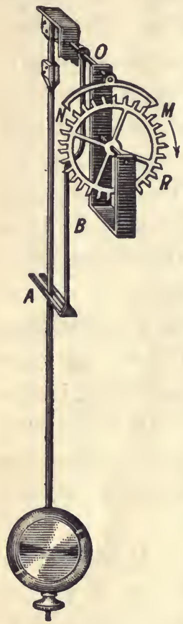


FIG. 44. — Escapement and Pendulum of a Clock.

The pendulum offers the most precise method for measuring the acceleration of gravity. By carefully determining experimentally the period t and the length l of a pendulum, the value of g can be easily calculated by the help of equation (7), § 83.

EXERCISES

1. By the help of equation (7), § 83, find the period of a pendulum 80 cm. long, when g equals 980 cm./sec.^{2}.

2. Calculate the length of a simple pendulum that beats half seconds (*i.e.* t equals $\frac{1}{2}$ sec.) at a place where the acceleration is 981 cm./sec.^{2}.

3. The pendulum of a clock has a period of a quarter second. Find its length if g is 980 cm./sec.^{2}.

4. How long is a simple pendulum that makes 65 single vibrations per minute?

SUGGESTION. — First compute the value of t.

5. What is the value of g where a simple pendulum 99.2 cm. long makes 60 single vibrations per minute?

6. An Arctic explorer finds that the length of the seconds pendulum at a certain place is 99.6 cm. What is the value of g at this place?

7. A simple pendulum is to make 45 single vibrations per minute. If g is 980 cm./sec.^{2}, what must be its length?

8. It is found at a certain place that a simple pendulum 90 cm. long makes 64 single vibrations per minute. Find the value of g at this place.

9. A pendulum whose bob weighs 100 g. is drawn aside until the distance AD, Fig. 41, is 4 cm. How much energy is stored in the bob? How much work was done upon it?

SUMMARY

1. Newton's *Law of Universal Gravitation* states that every body in the universe attracts every other body with a force that is directly proportional to the product of the attracting masses and inversely proportional to the square of the distance between their centers of mass (§ 67).

2. The attraction of the earth for other bodies is called the *force of gravity.* The *weight* of a body is the measure of this force (§ 67).

3. The weight of a body is proportional to its mass (§ 68).

4. The weight of a body above the earth's surface is inversely proportional to the square of its distance from the center of the earth. On account of the spheroidal form of the earth, a body at the equator weighs slightly less than a body of the same mass at some other point on the earth's surface (§ 69).

5. The weight of a body is the resultant of the weights of the individual particles that compose it. The point of application of this resultant is the *center of gravity* of the body (§ 70).

6. A body is in *equilibrium* when a vertical line drawn through its center of gravity passes through a point of support, or within the area included between the extreme points of support. The three kinds of equilibrium are *stable*, *unstable*, and *neutral* (§ 71).

7. The *stability* of a body is measured by the work that must be performed in order to overturn it (§ 72).

8. All freely falling bodies descend from the same height in equal times. Such bodies have uniformly accelerated motion. The *acceleration due to gravity* is about 980 cm./sec.^{2}, or 32.16 ft./sec.^{2} (§§ 73–75).

9. The equations of freely falling bodies are (1) $v = gt$, (2) $d = \frac{1}{2} gt^2$, (3) $d = \frac{v^2}{2g}$, and (4) $v = \sqrt{2gd}$ (§ 76).

10. When bodies are thrown, the horizontal component of the motion is uniform, while the vertical component is uniformly accelerated (§ 78).

11. The swinging of a pendulum is due to the force of gravity (§ 80).

12. The period of vibration of a pendulum is independent of the mass of the bob and the amplitude of vibration when the arc is small, and is directly proportional to the square root of the length and inversely proportional to the square root of the acceleration due to gravity. The equation of the pendulum is $t = \pi \sqrt{\frac{l}{g}}$ (§ 82).

13. A *compound pendulum* may be conceived as being made up of *simple pendulums* of different lengths. The period of vibration depends upon the position of the *center of oscillation.* The center of oscillation and point of suspension are interchangeable. The center of percussion and the center of oscillation coincide (§§ 85 and 86).

CHAPTER VI

MACHINES

1. GENERAL LAW AND PURPOSE OF MACHINES

88. Simple Machines. — The transference of energy from a body capable of doing work to another upon which the work is to be done is often accomplished more advantageously by the use of a simple machine than in any other way. Indeed, it is often impossible for an agent to do the required work without the aid of a machine. For example, a man wishes to load a barrel of lime into a wagon, but finds that he is unable to lift it; with the aid of an inclined plane of suitable length, however, the barrel is easily rolled into the wagon. In other cases use is made of the *pulley*, *lever*, *wheel and axle*, *screw*, and *wedge*, which with the *inclined plane* form the six simple machines.

89. The Principle of Work. — The general law of machines is illustrated by the following experiments:

1. Let a cord be passed over a pulley, as shown in Fig. 45. Let the pull of a dynamometer be used to counteract the weight of the body W. It is obvious in this case that the amount of force F registered on the dynamometer must be equal to the weight W. The experiment will show that this is the case. Furthermore, if force F moves downward 1 foot, W will be elevated through an equal distance.

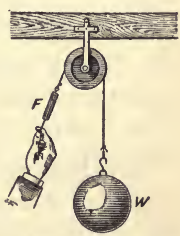


FIG. 45. — The Effort F Equals the Weight W and Moves Through an Equal Distance.

If, now, we designate the distance through which the acting force (or *effort*) F moves by the letter d and the distance the weight W is lifted by d', the work put into the machine by the acting agent is $F \times d$, and the work done by the machine is $W \times d'$. It is plain that the experiment shows that

$$F \times d = W \times d'.$$

2. Let the pulley be now attached to the weight W, Fig. 46, and let one end of the cord be fastened to some stationary object at A. If W is made 1000 grams, for example, it will be found that the upward effort registered by the dynamometer will be 500 grams. For any value of W, F will be one half as great. However, when F moves a distance of 1 meter, for example, W is elevated only one half a meter.

FIG. 46.—The Effort F Equals One Half of the Weight W, but Moves Twice as Far.

In this experiment $W = 2F$, and $d' = \frac{1}{2}d$. Therefore, we may write as before

$$\mathbf{F} \times \mathbf{d} = \mathbf{W} \times \mathbf{d}'. \qquad (1)$$

The relation shown by this equation is one of the most important laws of mechanics and is known as the **Principle of Work.** The principle may be stated as follows:

The work done by an agent upon a machine is equal to the work accomplished by the machine; or, the effort **F** *multiplied by the distance through which it acts equals the resistance* **W** *that is overcome by the machine multiplied by the distance it is moved.*

This law may be applied to any machine, no matter how simple or complicated it may be, provided the friction of the moving parts can be disregarded.

EXAMPLE. — An agent capable of doing work exerts a force of 50 lb. upon a machine. If a weight of 250 lb. is lifted 8 ft. by the machine, through what distance must the applied force act?

SOLUTION. — The work done by the machine is 250 × 8, or 2000 foot-pounds. Hence, the force applied by the agent must act through the distance 2000 ÷ 50, or 40 ft.

It is clear from this example that the gain in force is accomplished at the expense of distance, since the effort must move five times as far as the resistance. Its speed also is five times as great. On the other hand, a machine may be made to increase the distance as well as the speed at the expense of force. Such is the case in many practical applications of the simple machines.

90. Mechanical Advantage of a Machine. — It is frequently desirable to know the multiplication of force that is brought about by the use of a machine; or, in other words, the ratio of the resistance W to the effort F. This ratio is called the **mechanical advantage** of the machine.

From equation (1) we may write

$$\mathbf{W : F :: d : d'}. \qquad (2)$$

Hence, *the mechanical advantage of a machine is the ratio of the distance through which the effort moves to the distance through which the resistance is moved by the machine.*

For example, the mechanical advantage of the pulley as used in the first experiment of the preceding section is 1, in the second experiment 2, and in the example given on page 84 it is 5.

2. THE PRINCIPLE OF THE PULLEY

91. The Pulley. — The pulley consists of a grooved wheel, called a sheave, turning easily in a block that admits of being readily attached to objects. When the block containing the sheave is attached to some stationary object, the pulley is said to be *fixed;* when the block moves with the resistance, the pulley is *movable*. A fixed pulley is shown in (1), Fig. 47, and a single movable pulley in (2).

Let experiments be made with pulleys arranged as shown in Figs. 47 and 48. In each case ascertain by means of a dynamometer or weights the force required at F to balance a weight applied at W. The effect of friction may largely be avoided by taking the mean of the forces applied at F, first when W is slowly raised and then when it is slowly lowered.

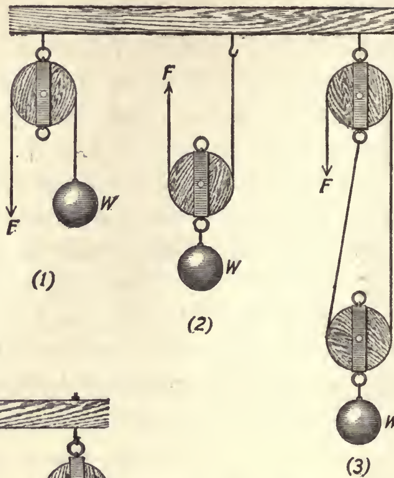


FIG. 47.—(1) A Fixed Pulley. (2) A Single Movable Pulley. (3) A System of One Fixed and One Movable Pulley.

When a single fixed pulley is used, $W = F$. When a single movable pulley is used, the resistance is applied to the movable block, as shown in (2), Fig. 47. A study of the figure will show that W is balanced by two equal parallel forces, each of which is equal to F. Hence $W = 2F$. The mechanical advantage is therefore 2. In (3) also the mechanical advantage is 2, and the only gain secured by

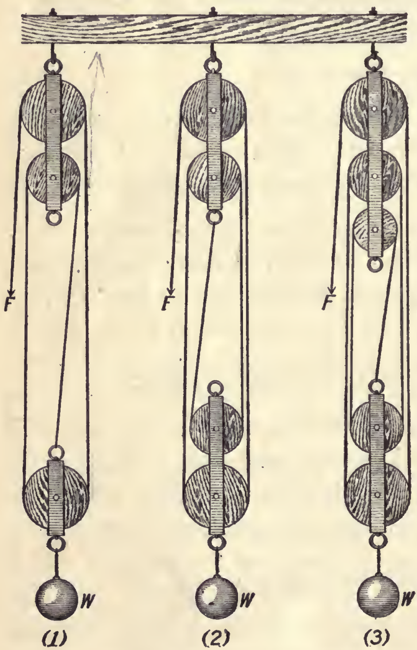


FIG. 48.—(1) One Movable and Two Fixed Pulleys. (2) Two Movable and Two Fixed Pulleys. (3) Two Movable and Three Fixed Pulleys.

the use of the fixed pulley is one of direction, *i.e.* the effort may now act downward instead of upward as in (2).

In (1), Fig. 48, one end of the rope is attached to the movable block, so that W is supported by three upward parallel forces, each of which is equal to F. Hence $W = 3F$. A similar consideration of (2) will show that $W = 4F$, and of (3), that $W = 5F$.

It is obvious from the cases already considered that whenever a continuous cord is used in the pulley system, the mechanical advantage is equal to the number of parallel forces acting against the resistance W. If n is the number of the parts of the rope supporting the movable block, then n is the number of these parallel and equal forces of which W is the sum. Therefore

$$\mathbf{W = nF.} \qquad (3)$$

92. Principle of Work and the Pulley System.—Equation (3) can be derived by applying the general law of work stated in § 89. If there are n portions of the rope supporting the movable pulley, and W is lifted 1 foot, for example, each portion of the cord must be shortened that amount. Consequently the effort F must move n feet. By the principle of work the product of the effort and the distance through which it acts equals the resistance multiplied by the distance through which it is moved; or,

$$W = nF.$$

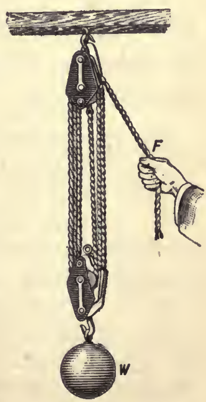


FIG. 49.—A Tackle.

EXERCISES

1. Diagram a set of pulleys by means of which an effort of 100 lb. can support a load of 500 lb.

2. What is the mechanical advantage of the system of pulleys shown in Fig. 49? Find the effort required to balance a weight of 1200 lb.

3. Each of two pulley blocks contains two sheaves. Show by a diagram how to arrange these into a system that will enable an effort of 75 Kg. to move a resistance of 300 Kg.

4. Show by a diagram the best arrangement of two blocks, one containing two sheaves, the other containing one. Ascertain the mechanical advantage.

5. In each case shown in Fig. 48 let the effort be applied to the movable block and the resistance W to the end of the rope in place of F. If the effort F moves 1 ft., how far will W be moved? State the advantage secured in each instance. NOTE. — This plan is frequently employed in the operation of passenger elevators in tall buildings.

6. In a pulley system consisting of a continuous cord attached at one end to a movable block containing one sheave, the rope passes through a fixed block having two sheaves. Find the effort required to support a block of marble weighing a ton.

3. THE PRINCIPLE OF THE LEVER

93. The Lever. — Of all the simple machines the lever is the most common. It is of frequent occurrence in the structure of the skeleton of man and animals, as well as in many mechanical appliances. In its simplest form the lever is a rigid bar, as AB, Fig. 50, arranged to turn about a fixed point O called the *fulcrum*. The effort is applied at the point A, and the resistance at B.

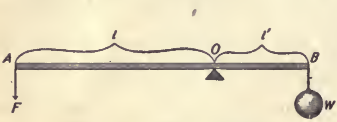


FIG. 50. — The Lever.

Balance a meter stick upon a long wire nail, piercing it a little above the center, as shown in Fig. 51. By means of a small cord or thread, suspend a weight of 150 grams at B, 15 centimeters from the fulcrum O. Now place a weight of 50 grams upon the opposite side of the fulcrum and move it along the bar until it just balances

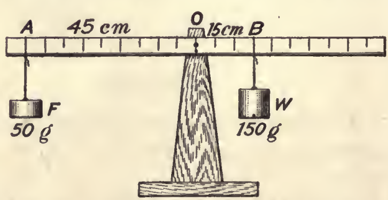


FIG. 51. — The Moment of the Effect F equals the Moment of the Resistance W.

the weight at B. Call this point A. It will now be found that the distance AO is 45 centimeters. From this it is plain that the force F multiplied by the arm AO is equal to the weight W multiplied by the arm BO. The experiment may be extended by using other forces and distances, but in every case it will be found that the product $F \times AO$ equals the product $W \times BO$.

The product of the applied force F multiplied by its distance AO from the fulcrum is called the *moment* of that force (§ 45). Likewise, the product of the resistance W multiplied by its distance BO from the fulcrum is the moment of the resisting force. Thus the fact shown by the experiment may be stated as follows:

The moment of the effort is equal to the moment of the resistance.

The distances AO and BO are called the *arms* of the lever. Representing these distances by l and l' respectively, the law of the lever is represented by the equation

$$\mathbf{F} \times \mathbf{l} = \mathbf{W} \times \mathbf{l'}. \tag{4}$$

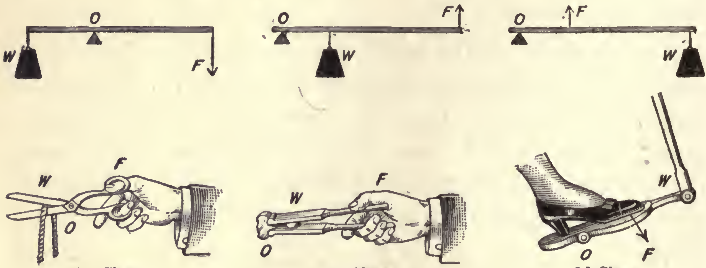
From this equation it is clear that the mechanical advantage $\frac{W}{F}$ (§ 90) is represented by the ratio $\frac{l}{l'}$. Hence, *a lever will support a weight or other resistance $\frac{l}{l'}$ times as great as the effort.*

94. Classes of Levers. — Levers are usually divided into three classes (Fig. 52) depending upon the relative location of the fulcrum, the effort, and the resistance.

(1) In levers of the *first class* the fulcrum is between the effort F and the resistance W; as in the crowbar, beam balance, scissors, steelyard, pliers, wire cutters, etc.

(2) Levers of the *second class* are distinguished by the fact that the resistance W is between the fulcrum O and the effort F; as in the nutcracker, wheelbarrow, etc.

(3) Levers of the *third class* are distinguished by the fact that the effort F is between the fulcrum O and the resistance W; as in the fire tongs, sheep shears, sewing-machine treadle, etc.



1st Class. 2d Class. 3d Class.

FIG. 52.—Levers of the Three Classes.

If the forces acting on the lever are not parallel, it is spoken of as a *bent* lever. A hammer used in pulling a nail (Fig. 53) is a lever of this kind. Bent levers are frequently found in complicated machines; as in farming implements, metal-working machinery, clocks, etc.

FIG. 53.—The Hammer used as a Bent Lever.

In levers of all classes the arms are measured from the fulcrum O on lines perpendicular to the lines which represent the direction of the forces. The law stated in the preceding section holds for all cases.

95. Extension of the Principle of Moments.—The relation of the moments of the forces acting on the arms of a lever as expressed in § 93 may be extended to cases in which any number of forces act upon the bar, provided they produce equilibrium. The case may be illustrated by experiment as follows:

Let weights be placed on the balanced meter stick used in § 93 precisely as shown in Fig. 54. The forces will be found to balance.

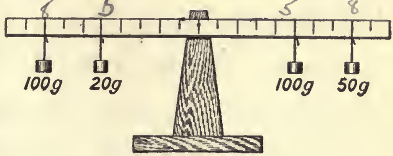


FIG. 54.

Since there is no rotation of the lever, it is obvious that the forces tending to produce a clockwise ↻ rotation are just balanced by those which tend to rotate the bar in the counter-clockwise ↺ direction. On computing the moments of the forces, we find on the right-hand side 50×40 and 100×25. On the left we find 20×25 and 100×40. Now if the moments acting clockwise be added, their sum will be found equal to the sum of those acting counter-clockwise; or,

$$50 \times 40 + 100 \times 25 = 20 \times 25 + 100 \times 40.$$

This equation illustrates a general law which may be stated thus:

An equilibrium of forces results when the sum of the moments which tend to make the lever rotate in one direction equals the sum of the moments which tend to make it rotate in the opposite direction.

The mechanical advantage of the lever can also be found by applying the principle of work set forth in § 89. Let the lever shown in Fig. 55 turn slightly about the fulcrum O until it has the position cd. Drawing the perpendiculars ac and bd, we have from similar triangles

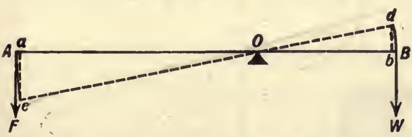


FIG. 55. — The Work Done by the Effort F Equals That Done upon the Weight W.

$$ac : bd :: Oc : Od.$$

But, since Oc equals OA and Od equals OB,

$$ac : bd :: AO : BO.$$

Now the work done by the effort F is the product $F \times ac$, and the work done against the resistance W is $W \times bd$. Since the work done by the agent is equal to that accomplished by the lever,

$$\mathbf{F \times ac = W \times bd}, \text{ or } \mathbf{W : F :: ac : bd :: AO : BO}. \qquad (5)$$

Therefore, the mechanical advantage of a lever is the ratio of the arm AO to which the effort is applied to the arm BO on which the resistance acts.

EXERCISES

1. Show by diagrams the relative position of effort, resistance, and fulcrum in the following instruments of the lever type: oars, sugar tongs, lemon squeezer, rudder of a boat, pitchfork, spade, can opener, pump handle.

2. Two boys weighing respectively 60 and 45 lb. balance on opposite ends of a board. If the fulcrum is 6 ft. from the larger boy, how far is it from the smaller one?

3. The arms of a lever of the first class are 3 ft. and 7 ft. What is the greatest weight that a force of 60 lb. can support?

4. A lever of the second class is required to support a weight of 500 kg.; the effort to be applied is only 50 kg. If the bar is 20 ft. long, where should the weight be attached?

5. Two masses weighing respectively 10 and 15 kg. balance when placed at opposite ends of a bar 2 m. long. Where is the fulcrum?

6. The short arm of a lever is 30 cm. long, the long arm 270 cm. If the end of the long arm moves 1 cm., how far will the end of the short arm move?

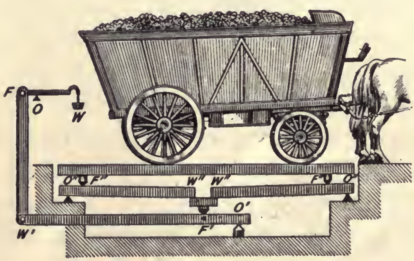


FIG. 56. — Wagon Scales.

7. The forearm is raised by a shortening of the biceps muscle. Considering the forearm as a lever whose fulcrum is at the elbow, to what class does it belong?

What is gained, force or speed? When the arm is being extended in striking a blow, to what lever class does it belong?

8. The scales used in weighing heavy loads, Fig. 56, are a series of levers arranged as shown. Explain how a small weight W can balance a load of coal weighing a ton or more.

9. Explain how a parcel is weighed by means of the steelyards shown in Fig. 57.

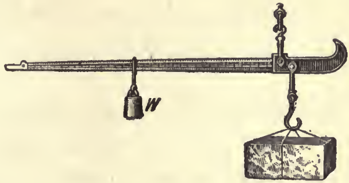


FIG. 57. — Steelyards.

4. THE PRINCIPLE OF THE WHEEL AND AXLE

96. The Wheel and Axle. — A simple form of the wheel and axle is shown in Fig. 58. The applied force, or effort, acts tangentially to the rim of the wheel B through a cord, thus producing rotation. As the wheel revolves, the cord to which is attached the weight W is wound up around the axle A, and the weight is thus lifted.

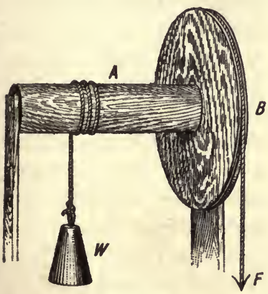


FIG. 58. — The Wheel and Axle.

If the force F turns the wheel through one revolution, the distance through which it acts is equal to the circumference of the wheel, or $2\pi R$, where R is the radius of the wheel. The work done by the agent is therefore $F \times 2\pi R$ (§ 55). One revolution of the wheel lifts the weight W a distance equal to the circumference of the axle, *i.e.* to $2\pi r$, where r is the radius of the axle. The work done therefore by the machine upon the weight is $W \times 2\pi r$. Applying the principle of work as stated in § 89,

$$F \times 2\pi R = W \times 2\pi r;$$

whence $\mathbf{F \times R = W \times r}$, or $\mathbf{W : F :: R : r}$. **(6)**

The mechanical advantage of a wheel and axle is therefore equal to the radius of the wheel divided by the radius of the axle. Hence, *a wheel and axle may be used to overcome a resistance* $\frac{R}{r}$ *times as great as the effort applied to the circumference of the wheel.*

It should be observed that the circumferences of the wheel and the axle bear the same ratio as their respective radii, and therefore may be used to replace R and r in equation (**6**).

In the compound windlass, Fig. 59, two machines of the wheel and axle type are combined. The effort F acting along the circumference of the circle described by the crank produces a force that is transmitted by means of a small cog wheel, or pinion, to the rim of the second wheel to which the axle is attached. As the crank is turned, a rope is wound around the axle, thus lifting a weight or overcoming some other resistance. The mechanical advantage of a compound windlass can be calculated by resolving the machine into simple wheels and axles. The mechanical advantage of the compound machine is the product of the mechanical advantages of its constituent parts.

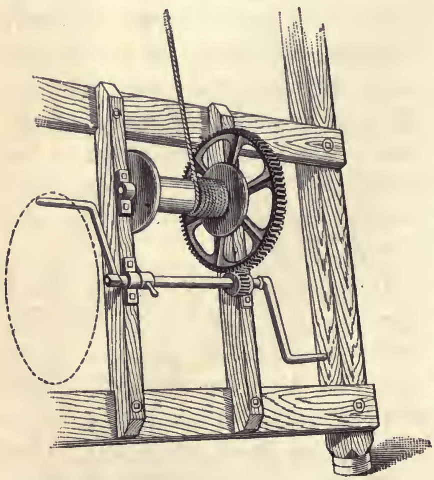


FIG. 59.—A Compound Windlass, or Winch.

In many instances in which the wheel and the axle are compounded the motion is transmitted by belts, chains, or cables, and occasionally by the friction of the circumferences.

The wheel and axle and the pulley may be looked upon as special cases of the lever. Fig. 60 (1) shows that the effort F acting tangentially to the rim of the wheel really acts upon a lever arm equal to R, the radius of the wheel. Similarly, the resistance W acts upon a lever arm equal to the radius of the axle. Applying the principle of moments,

$$F \times R = W \times r.$$

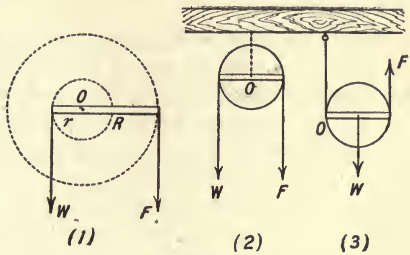


FIG. 60.

The case of a single fixed pulley shown in (2) may be regarded as a lever of the first class whose fulcrum is at O, or as a wheel and axle of which the radii are equal. Either view leads to the relation $F = W$. A single movable pulley shown in (3) is similar to a lever of the second class, the fulcrum being at O, and the resistance at the center as shown. Again, applying the principle of moments, we find that $W = 2F$ as in § 91.

EXERCISES

1. Show that the ordinary kitchen meat chopper and coffee grinder are examples of the wheel and axle. Give other commonplace examples.

FIG. 61.—The Capstan.

2. The radius of a wheel is 3 ft., and that of its axle 12 in. What effort would be required to overcome a resistance of 600 lb.?

3. How much work would be done upon the resistance in moving it a distance of 10 ft.? How much work would have to be done upon the wheel?

4. The arm of a capstan, Fig. 61, measured from the center, is 2 m.; the radius of the barrel is 25 cm. What effort would be required to produce a tension of 500 kg. in the rope attached to the barrel?

5. What is the mechanical advantage of the machines in Exercises 2 and 4?

6. If the effort were applied to the axle, and the resistance to the wheel, what would be gained by using a wheel and axle? Illustrate by using the wheel and axle described in Exer. 2.

7. The pedal of a bicycle describes a circle whose radius is 7 in. If the radius of the attached sprocket wheel is 3 in., find the pull on the chain when the foot pressure is 45 lb.

8. If the front sprocket of a bicycle contains 21 teeth, and the rear one 7, how far will one turn of the pedal move a 28-inch wheel along the ground? Find the number of turns of the pedal per mile.

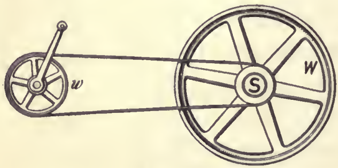


FIG. 62.

9. The length of the crank arm shown in Fig. 62 is 10 in., and the radius of w is 6 in.; wheel w is belted to S, whose radius is 3 in.; and S is attached to W, whose radius is 20 in. One turn of the crank produces how many revolutions of the wheel W? A point on the rim of W moves how much faster than the crank?

10. The large wheel of a sewing machine is 12 in. in diameter, and the small one to which it is belted is 3 in. One up-and-down movement of the treadle produces how many stitches?

5. THE INCLINED PLANE, SCREW, AND WEDGE

97. The Inclined Plane. — Let AB, Fig. 63, represent an inclined plane whose surface is smooth and unbending. Let the height BC be h, and the length AB be l. If the effort F acting parallel to AB causes the ball whose weight is W to move the distance AB, the work done by the agent is $F \times l$. The weight W is lifted a distance equal to BC, and the work

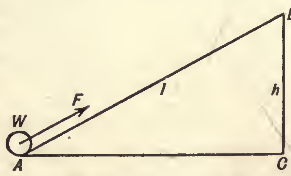


FIG. 63. — The Inclined Plane.

done is $W \times h$. According to the general principle of work,

$$\mathbf{F \times l = W \times h}. \tag{7}$$

The mechanical advantage $\frac{W}{F}$ is therefore equal to $\frac{l}{h}$, *i.e.* the ratio of the length of the plane to its height.

98. The Screw. — The screw is sometimes considered to be a modification of the inclined plane. If a right triangle, Fig. 64, be cut from paper and wound around a cylindrical rod, as shown, the hypotenuse of the triangle forms a spiral similar to the threads of a screw. The distance between two consecutive turns of the thread measured parallel to the axis of the rod is the *pitch* of the screw.

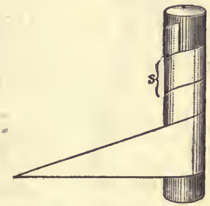


FIG. 64. — A Screw is a Spiral Incline.

The equation giving the mechanical advantage of the screw is readily derived by applying the principle of work (§ 89) as follows:

Let the screw shown in Fig. 65 be turned once around by applying the effort F to the end of arm A, tangent to the circle which it describes. During one revolution the effort acts through the distance $2\pi r$, where r is the length of the arm. The work done is $F \times 2\pi r$. While the screw is making one revolution, the weight W is obviously lifted through a distance equal to the pitch of the screw, which may be represented by the letter s. The work done by the screw is therefore $W \times s$. Hence

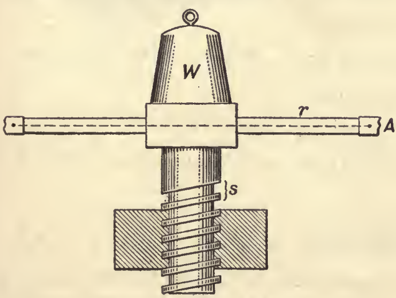


FIG. 65. — The Screw.

$$F \times 2\pi \times r = W \times s. \quad (8)$$

The mechanical advantage of the screw is therefore the ratio $2\pi r/s$.

The screw as a mechanical power owes its importance to the fact that its mechanical advantage can be enormously increased simply by making r large and s small, as in the jackscrew, Fig. 66, used to lift buildings from their foundations. It is extremely useful in the form of bolts, wood screws, vises, clamps, presses, water taps, and in many other cases where a great multiplication of force is desired.

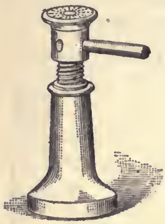


FIG. 66.—The Jackscrew.

99. The Wedge. — The wedge may be considered as two inclined planes placed base to base. It is used for splitting logs (Fig. 67), raising heavy weights small distances, removing the covers from boxes, etc. The importance of the wedge is due to the fact that the energy imparted to it is delivered by the blows of a hammer or heavy mallet. Although the amount of friction to be overcome in driving a wedge is necessarily very large, by its use a man can overcome enormous resistances. Since friction cannot be disregarded, no definite relation between effort and resistance can be given.

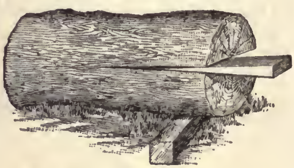


FIG. 67.—One Use of the Wedge.

EXERCISES

1. A ball weighing 10 lb. rests upon an inclined plane. If the height of the plane is 6 in. and the length is 30 in., what effort acting parallel to the plane will be required to hold the ball in equilibrium?

2. On an icy slope of 45°, what force is required to haul a sled and load weighing a ton, neglecting friction?

3. The radius of the wheel of a letter press is 12 in.; the pitch of its screw, $\frac{1}{4}$ in. Neglecting friction, what pressure is produced by an effort of 50 lb.?

4. Neglecting friction, what constant force must a team of horses exert in hauling a load of coal weighing 3000 lb. up an incline of 30°?

5. What is gained by making an inclined plane of a given height longer? What is lost? Illustrate by means of examples.

6. In a machine the effort of 50 lb. descends 20 ft., while a weight is raised 10 in. What is the weight?

7. What is the mechanical advantage of a screw press of which the pitch of the screw is 5 mm. and the diameter of the circle described by the effort 50 cm.?

8. A smooth railroad track rises 50 ft. to the mile. A car weighing 20 T. would require how much force to keep it from moving down the slope?

6. EFFICIENCY OF A MACHINE

100. Friction and Efficiency. — On account of the fact that there is always a resistance due to friction wherever one part of a machine moves over another, some work must be done in moving the parts of the machine itself. The useful work done by a machine is therefore less than the work done upon it; *i.e.* $W \times d'$ is always less than $F \times d$. *The ratio of the useful work done by a machine to the work done upon it is the* **efficiency** *of the machine.*

Example. — In the working of a pulley system an effort of 50 lb. acts through a distance of 20 ft. and lifts a weight of 180 lb. 5 ft. What is the efficiency of the system?

Solution. — The work done on the machine is 50×20, or 1000 foot-pounds. The work done by the machine is 180×5, or 900 foot-pounds. The efficiency is therefore $900 \div 1000$, or 0.9.

Efficiency is usually expressed as a percentage of the total work applied to a machine; thus in the example above the efficiency is 90%.

101. Sliding and Rolling Friction. — Since friction always tends to decrease the efficiency of machinery, advantage is taken of every method that will reduce it to the smallest possible amount.

Let a block of wood or metal and a car of the same weight be drawn up a slight incline and the force measured by a dynamometer. It will be found that the car is more easily moved than the block.

Again, place a piece of sheet rubber on the incline under the car. A greater force will be required to produce motion than before.

The first experiment shows clearly a great difference between the sliding friction of the block and the rolling friction of wheels on a hard surface. The second experiment demonstrates the value of a hard, unyielding road when heavy loads are to be drawn. If, however, the wheels are provided with wide tires, they sink less deeply into the roadbed and meet with less resistance. The unyielding surfaces of car wheels and the track enable a locomotive to pull enormous loads on account of the small amount of rolling friction.

It is plain that in the axle bearings of ordinary vehicles the moving part must *slide* over the stationary part. The friction thus brought about is greatly reduced by means of lubricating oil, but is avoided in the construction of bicycles, automobiles, etc., by substituting ball bearings, as shown in Fig. 68. In this way the moving part S is separated from the stationary part A by balls which roll as the wheel turns. Thus rolling friction takes the place of sliding friction.

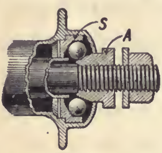


FIG. 68. — Ball Bearings of a Bicycle Axle.

EXERCISES

1. In what way are we dependent upon friction in the process of walking? Why do we encounter difficulty in walking on smooth ice?

2. Why is it difficult for a locomotive to start a train on wet rails? How is the difficulty overcome?

3. Why do smooth nails hold two pieces of wood together? State other ways in which we take advantage of friction.

4. Calculate the efficiency of a wheel and axle when an effort of 20 lb. acting through 30 ft. lifts a weight of 80 lb. 7 ft.

5. On account of the loss of energy due to friction in a pulley system, an effort of 70 kg. acting through 30 m. moves a resistance of 340 kg. through 5.5 m. What is the efficiency of the machine?

6. What is the efficiency of a screw if an effort of 5 kg. applied at the end of an arm 1 m. long produces a pressure of 4000 kg., the pitch of the screw being 4 mm.?

7. The efficiency of an inclined plane is 50%. If the length of the plane is 20 ft. and its height 4 ft., what effort acting parallel to the plane will be required to move a body weighing 500 lb.?

SUMMARY

1. The simple machines are the pulley, lever, wheel and axle, incline plane, screw, and wedge. Complicated machinery is made up of simple machines (§ 88).

2. The work done by an agent upon a machine is equal to the work accomplished by it. This is known as the *Principle of Work.* Hence a machine may gain force at the expense of distance (or speed), or it may gain distance (or speed) at the expense of force (§ 89).

3. The *mechanical advantage* of a machine is the ratio of the resistance overcome by it to the effort applied to it; *i.e.* $W : F$ (§ 90).

4. When a continuous cord is used in a system of pulleys, the equation is $W = nF$, in which n is the number of parts of the rope supporting the movable block (§ 91).

5. When a lever is used, the moment of the effort equals the moment of the resistance, or $F \times l = W \times l'$ (§ 93).

6. Levers are classified according to the relative position of fulcrum, effort, and resistance. Lever arms are always measured from the fulcrum on lines perpendicular to the acting forces (§ 94).

7. When several forces act at different points on a lever, the sum of the moments tending to produce a clockwise rotation equals the sum of the moments tending to produce rotation in the opposite direction (§ 95).

8. The equation of the wheel and axle is $F \times R = W \times r$ (§ 96).

9. The equation of the inclined plane is $F \times l = W \times h$ (§ 97).

10. The equation of the screw is $F \times 2\pi r = W \times s$ (§ 98).

11. Friction tends to reduce the work accomplished by a machine. The *efficiency* of a machine is the ratio of the useful work done by a machine to the work done upon it (§ 100).

12. In general the friction of sliding parts of machinery is greater than that of rolling parts. Friction may be reduced by lubricants, or by substituting rolling friction for sliding friction, as in the case of ball and roller bearings (§ 101).

CHAPTER VII

MECHANICS OF LIQUIDS

1. FORCES DUE TO THE WEIGHT OF A LIQUID

102. Pressure of Liquids against Surfaces. — When a hollow rubber ball or a piece of light wood is forced under water, the body resists the action of the force submerging it and manifests a strong tendency to return to the surface. The fact that some bodies float on water and other liquids shows also that there exists a force acting against the lower surfaces sufficient to counteract their weight.

Let a lamp chimney against the end of which a card has been placed be forced partly under water, as shown in Fig. 69. The card will be pressed firmly against the end of the chimney by a force acting in an upward direction, and a large quantity of shot or sand may be poured into the vessel before the card is set free. The person performing the experiment will also perceive that a strong upward force acts against the hand. By lowering the chimney to a greater depth, the upward force against the hand will be increased and a larger quantity of shot or sand will be required to free the card.

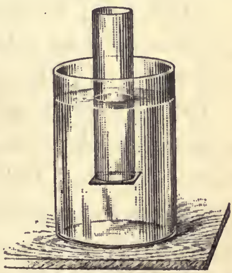


Fig. 69. — A Liquid Exerts an Upward Pressure against the Vertical Tube.

103. Relation between Force and Depth. — The manner in which the force exerted by a liquid against a surface varies with the depth may be experimentally tested by means of a gauge constructed as shown in Fig. 70.

A "thistle tube" having a stem about 80 centimeters long is bent at an angle of 90 degrees about 35 centimeters from one end.

A piece of thin sheet rubber is tied tightly over the large end A. If a drop of ink is placed in the tube at B, its movements along the tube will indicate changes in the force against the rubber surface A. Furthermore, the distance the drop moves when A is submerged in a liquid will be very nearly proportional to the applied force.

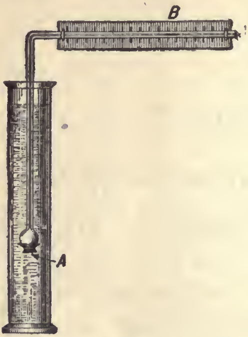


FIG. 70. — The Upward Force against A Varies with the Depth.

Let the gauge be clamped in the position shown in the figure, and let a graduated linear scale be attached to the horizontal tube containing the drop of ink. Now let the position of the drop be read upon the scale and a tall vessel of water brought up until the surface A is 3 centimeters beneath the free surface of the liquid. Let the new position of the drop be read and its displacement computed. If, now, the positions of the drop be read after submerging the surface A successively to the depths of 6, 9, and 12 centimeters, the movements of the drop will be found to be proportional to the depths.

Therefore, as shown by this experiment, *the upward force exerted by a liquid of uniform density against a given surface is directly proportional to the depth of that surface.*

104. Direction of Forces at a Given Depth. — Common experience shows that liquids press against surfaces that are vertical or oblique as well as horizontal. In every case the force exerted is perpendicular to the surface against which it acts. Thus water will be forced through a hole in the side of a pail near the bottom as well as through one in the horizontal bottom itself. A comparison of the forces in all directions at a given depth may be made by modifying slightly the construction of the gauge shown in Fig. 71 as follows:

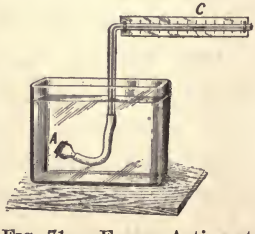


FIG. 71. — Forces Acting at a Point are the Same in all Directions.

Cut the glass tube about 1 centimeter from the bulb, and insert a piece of rubber tubing about 10 centimeters long. Now, if the bulb is lowered in a large vessel of water, the position of the drop will not change when the rubber surface is turned in different directions, *provided the depth of its center is kept constant.*

This experiment, together with the one described in § 103, leads to the following conclusions:

(1) *The force exerted by a liquid at a given depth is the same in all directions, and*

(2) *The force exerted by a liquid in any direction is directly proportional to the depth.*

105. Pressures Due to the Weight of a Liquid. — From a study of Fig. 72 it may readily be observed that the forces exerted by liquids against surfaces are due to the *weight* of the liquid, *i.e.* to gravity. If a vessel were filled with smooth blocks of wood of equal size and density, block *2* would suffer a downward force equal to the weight of block *1*, and would in turn exert an equal and opposite reaction upward against it. Likewise, *3* must support the weight of *1* and *2*, *4* must support *1*, *2*, and *3*, etc. In each instance the upward reaction is equal to the downward action. Thus it is clear that at a given surface the upward and downward forces are equal, and also that the force against any horizontal surface is proportional to the depth of that surface below the upper surface of the blocks in the vessel.

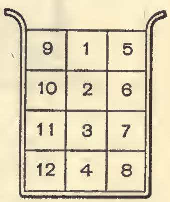


FIG. 72. — Forces Due to Gravity.

Now liquids, unlike solids, do not tend to keep their form, but must be supported on the sides. Hence, if we imagine the vessel in Fig. 72 to contain a liquid, section *4*, for example, will press sidewise against *8* and *12*, whose reactions back against *4* preserve equilibrium. If the liquid were without weight, section *4*, for instance, would suffer no crushing force due to the weight of the portions above it, and consequently would exert no lateral force against the portions surrounding it.

106. Total Pressure on a Given Area. — If the experiment described in § 102 be repeated, and water substituted for the shot or sand (Fig. 73), it will be found that the card is set free from the end of the chimney at the instant the water on the inside reaches the height of that on the outside.

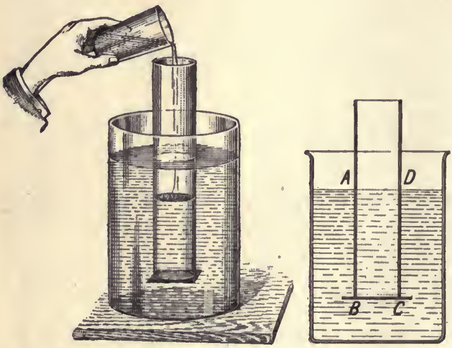


Fig. 73. — Force against BC is Equal to the Weight of Column $ABCD$.

If the chimney is cylindrical, the downward force within the chimney is obviously equal to the weight of the column of water $ABCD$. If the area of the end of the chimney is a square centimeters, and the depth h centimeters, the volume of the column of water in the chimney is ah cubic centimeters. Since the density of water is 1 gram per cubic centimeter (§ 9), the weight of the column $ABCD$ is ah grams. Hence the force exerted on the card is ah grams. If another liquid is used whose density is d grams per cubic centimeter, the weight of the column, and hence the force, is ahd grams.

The following rule may therefore be given:

The force exerted by a liquid on any horizontal surface is equal to the weight of a column of the liquid whose base is the area pressed upon, and whose height is the depth of this area below the surface of the liquid, or **Force = ahd.**

Since the force exerted by a liquid against a surface at a given depth is the same in all directions (§ 104), the following rule for computing the force against surfaces that are not horizontal is often employed:

The force exerted by a liquid against any immersed surface is equal to the weight of a column of the liquid whose base is the area pressed upon, and whose height is the distance of the center of mass of this area below the surface of the liquid.

In the English system the area should be expressed in square feet, the depth in feet, and the density of the liquid in pounds per cubic foot. The density of water may be taken as 62.5 pounds per cubic foot.

EXAMPLE. — A tank 4 ft. deep and 8 ft. square is filled with water. Find the force exerted against the bottom and one side.

SOLUTION. — The area of the bottom of the tank is 8×8, or 64 sq. ft. Hence the volume of the column whose weight equals the force exerted on the bottom is 4×64, or 256 cu. ft. The force against the bottom of the tank is therefore 256×62.5 lb., or 16,000 lb.

The area of one side of the tank is 4×8, or 32 sq. ft. The depth of the center of mass of the side is 2 ft. Hence the volume of the column of water whose weight equals the force exerted against the side is 2×32, or 64 cu. ft. The force exerted by the water upon this side is therefore 64×62.5 lb., or 4000 lb.

The term *pressure* should be confined to the meaning of force per unit area. The pressure at a given point is expressed in terms of the force exerted over a unit area at that depth; for example, 1000 grams per square centimeter, 15 pounds per square inch, etc.

EXERCISES

1. Find the entire force exerted against the bottom of a rectangular vessel 5×8 cm. and filled with water to a depth of 15 cm.

2. Find the pressure per square foot at the bottom of a pond 10 ft. in depth.

3. A cylindrical glass jar 5 cm. in diameter is filled to a depth of 15 cm. with mercury. Find the force against the bottom and the pressure per unit area. The density of mercury is 13.6 g. per cubic centimeter.

4. A tank is 4 ft. wide, 8 ft. long, and 3 ft. deep. Compute the force exerted against one end and the bottom when the tank is full of water.

5. At a depth of 10 m. of sea water, what is the pressure in grams per square centimeter? (The density of sea water is 1.026 g. per cubic centimeter.)

6. At a depth of 25 ft. of sea water, what is the pressure per square inch?

SUGGESTION. — Find first the force exerted on a surface 1 ft. square at the given depth.

7. A cubic inch of mercury weighs 0.49 lb. Compute the force exerted against the bottom and one side of a glass tank 4 in. wide, 6 in. long, and 5 in. deep when full of mercury.

8. Find the force exerted against the bottom of a cubical vessel whose volume is 1 liter when the vessel is filled with mercury.

9. What depth of water will produce a pressure of 1 lb. per square inch?

10. What is the pressure per square centimeter at the bottom of a column of mercury 76 cm. in height? (For the density of mercury see Exer. 3.)

11. To what height would a mercurial column be supported by a pressure of 1000 g. per square centimeter?

12. A gauge connected with the water mains of a city showed a pressure of 65 lb. per square inch. What was the height of the water in the standpipe above the level of the gauge?

SUGGESTION. — Find the depth of water required to produce a pressure of 65 lb. on a surface of 1 sq. in.

13. A diver is working at a depth of 45 ft. How much is the pressure per square inch upon the surface of his body?

14. A rectangular block of wood is placed under water so that its upper face, which is 8×10 cm., is 20 cm. below the surface. If the thickness of the block is 4 cm., what is the force exerted by the liquid against each of its faces?

15. How much is the force against a dam 20 ft. long and 10 ft. high when the water rises to its top?

16. A hole in the bottom of a ship which draws 30 ft. of water is temporarily covered with a piece of canvas. How much is the pressure against the canvas from the outside?

17. The water level is at the top of a dam 30 ft. high. Compute the pressure per square foot at the bottom of the dam. How much is the pressure halfway down?

18. If the dam in Exer. 17 is 100 ft. long, how much is the total force against its surface?

107. Pressure in Vessels of Different Shapes. — It may be correctly inferred from § 106 that the force exerted by a liquid against a given area does not depend on the shape of the vessel containing the liquid used, inasmuch as the computation of this force involves only the area and depth of the surface pressed upon and the density of the liquid. This fact can be demonstrated experimentally as follows:

Let a glass funnel be selected the mouth of which is of the same area as the end of the lamp chimney used in § 106. Place a card across the mouth of the funnel and submerge it, as shown in Fig. 74. The card will be pressed against the funnel with the same force as it was when the chimney was used, *i.e.* with a force equal to the weight of a column of water *ABCD*. If water is now poured into the stem of the funnel at *E*, the card will become free precisely when the level of the water in the funnel has reached the height of the water in the vessel outside. In this condition the water in the funnel exerts the same force downward against the card as the water on the outside exerts in an upward direction. Hence the downward force is equal to the weight of the column of water *ABCD*. In other words, *the force downward against the card is exactly the same as it was when the cylindrical chimney was used.*

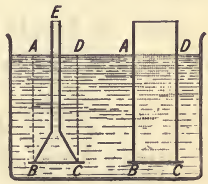


FIG. 74. — Equal Downward Forces in Vessels of Different Shapes.

108. The Hydrostatic Paradox. — An apparent contradiction arises when we apply the laws of liquid pressure to vessels of the forms shown in Fig. 75. Let the vessels have bases of equal size and

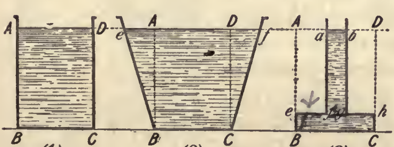


FIG. 75. — The Hydrostatic Paradox.

d x h x area

be filled to the same depth with water. In each case the force exerted by the water against the bottom is equal to the weight of the liquid column $ABCD$. Because it is apparently an impossibility for different masses of a liquid to produce the same pressure, this conclusion is often called the *hydrostatic paradox.*

An application of the laws of pressure in liquids to vessel (3), Fig. 75, will show how the total pressure on the bottom BC can be far greater than the weight of liquid contained in the vessel. Although the total pressure on the surface BC is equal to the weight of a column $ABCD$ of the liquid (§ 107), there are upward forces against the surfaces ef and gh equal to the weight of the liquid that would be required to fill the spaces $Aefa$ and $bghD$. Hence the resultant of all the forces is the difference between the downward force on BC and the upward forces on ef and gh. This is obviously the weight of the liquid in the vessel.

109. A Liquid in Communicating Vessels. — Let tubes of various shapes and sizes open into a hollow connecting arm, as shown in Fig. 76. Any liquid poured into one of the tubes will come to rest at the same level in all. Although different quantities of the liquid are present in the several tubes, yet for the same depth the parts are in equilibrium. The explanation is as follows:

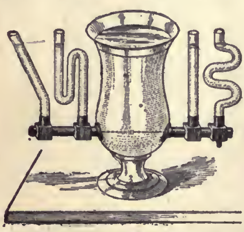


Fig. 76. — Liquid Level in Communicating Vessels.

Let two vessels containing water be in communication, as shown in Fig. 77. Let A be the area of a cross-section of the connecting tube. The force tending to move the water to the right is Ahd, where h is the depth of the center of the area considered, and d the density of the water. The force tending to move the liquid to the left is $Ah'd$. These two forces will be in equilibrium only when h and h' are equal, *i.e.* when the upper surfaces in the two vessels lie in the same horizontal plane.

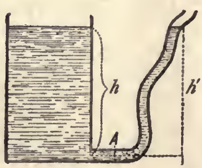


Fig. 77. — A Liquid in Equilibrium.

EXERCISES

1. A cone-shaped vase has a base of 100 cm.^{2} and is filled with water to a depth of 45 cm. Find the force and pressure per square centimeter acting on the bottom.

2. The water in a reservoir supplying a city is 150 ft. above an opening made in a pipe being laid along a street. Find the pressure in pounds per square inch required to prevent the water from running out. *Ans.* 65.1 lb.

3. A glass tube 1 m. long is filled with mercury (density 13.6 g. per cubic centimeter). Find the pressure against the closed end of the tube in grams per square centimeter when the tube is (1) vertical and (2) inclined at an angle of 45°.

Ans. 1360 g. per square centimeter.
961.7 g. per square centimeter.

4. A column of water is lifted 25 ft. in a pipe. Calculate the pressure per square inch that it exerts against the bottom of the pipe.

2. FORCE TRANSMITTED BY A LIQUID

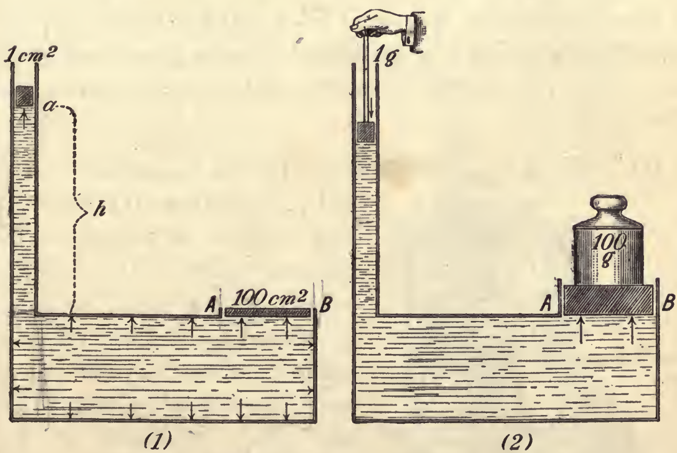


Fig. 78. — The Multiplication of Force by a Liquid.

110. Transmission of Pressure — Pascal's Law. — Let a vessel of the form shown in (1), Fig. 78, be filled with water to the point *a*. A pressure will be exerted on every square centimeter of area depending on the depth of that area. The force exerted upward against the shaded area AB, assumed to be 100 square centimeters, is 100 h grams, if h is the depth of the water in the tube. This force is entirely independent of the area of the portion of the vessel at a. Let this area be 1 square centimeter. Now, if 1 cubic centimeter of water is poured into the vessel, the depth of the liquid is increased 1 centimeter, and the depth of the surface AB becomes $h + 1$ centimeters. The force now exerted against AB is 100 $(h + 1)$ grams, *i.e.* each square centimeter of AB receives an additional force of 1 gram. Hence, *the force exerted on a unit area at* a *is transmitted to every unit area within the vessel.*

This fact was first published in 1663 by Pascal, a French mathematician. The law may be expressed as follows:

Force applied to any area of a confined liquid is transmitted undiminished by the liquid to every equal area of the interior of the containing vessel and to every part of the liquid.

111. The Hydraulic Press. — In the discussion of Pascal's Law given in the preceding section, it makes no difference whether the pressure added to the small area a is produced by a gram of water poured into the tube or exerted by a small piston fitting the tube, as shown in (2), Fig. 78. The shaded area AB to which the force is transmitted by the liquid may be made the area of a large piston,

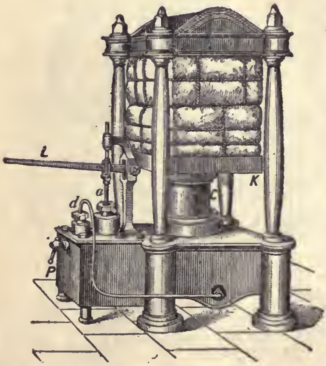


FIG. 79. — An Hydraulic Press.

as shown. In this case a force of 1 gram on the small piston will be transmitted to each square centimeter of the large one. Hence 1 gram on the small piston will balance 100 grams placed on the large one. Furthermore, any effort F applied to the small piston will balance a resistance 100 times as great as itself. Hence a mechanical advantage (§ 90) of 100 is secured. The hydraulic press, a machine employed in factories for exerting great force, is one of the most important applications of Pascal's Law. (See Fig. 79.)

The hydraulic press is analyzed in Fig. 80. When the lever L is raised, the small piston P is lifted, and water from the cistern T enters the cylinder B through the valve v. As the small piston is forced down, v closes, and the water in the cylinder B is driven past the valve v' into the chamber below the large piston P'. By Pascal's Law, the force exerted against the large piston P' is as many times that applied to P as the area of P' is times that of P. In other words, the mechanical advantage is P'/P. By making P very small and P' large, any desired mechanical advantage may be secured. Thus hand presses are sometimes used that are capable of exerting a force of several hundred tons.

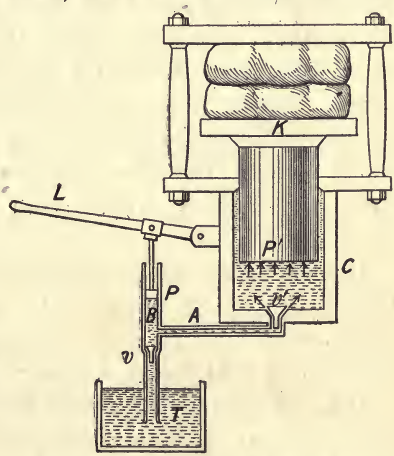


FIG. 80. — Sectional Diagram of the Hydraulic Press.

112. Principle of Work and the Hydraulic Press. — It may be observed from (2), Fig. 78, that when the effort F moves the small piston a distance of 1 centimeter, the large piston, having to make room for the 1 cubic centimeter forced below it, must rise $\frac{1}{100}$ of a centimeter.

Although the larger force is 100 times as great as the smaller, it acts through only $\frac{1}{100}$ as great a distance. Hence the work done by the larger piston as it rises is no greater than the work done on the smaller.

113. Artesian or Flowing Wells. — The tendency of water to flow from a point of higher level to one of lower level has a wide application in artesian, or flowing, wells. In many localities there are formed so-called artesian basins of great extent in which a stratum of porous material, as sand or other substance through which water can pass with comparative ease, lies between strata of clay or rock which are impervious to water. At distant points these layers have been crowded to the surface by geologic processes where the porous layer has been laid bare, thus rendering the entrance of water possible. Hence when borings are made into the earth through the various layers and into the porous stratum, water often rises to the surface when it is lower than the region where the water enters the porous layer.

Deep artesian wells exist at St. Louis, Mo.; Columbus, Ohio; Pittsburg, Pa.; and Galveston, Texas. Noted wells are found at Passy, France (1923 feet); Berlin, Germany (4194 feet); Leipzig, Germany (5735 feet).

114. Supplying Cities with Water. — Comparatively few cities are so favorably situated that they derive their supply of water from mountain springs. The output of such springs is collected in large artificial reservoirs or lakes from which it is piped to the towns where it is distributed in the usual manner. In such cases the elevation of the reservoir is such as to produce an adequate pressure for all ordinary purposes. The pressure may be estimated at 43.5 pounds per square inch for every 100 feet of elevation.

In towns, however, whose location is less favored by nature, the water from springs or wells must be elevated by means of pumps (§ 156) to suitable reservoirs or tanks constructed upon high ground from which it is distributed through pipes to the consumers. In some large cities no reservoir is used, the pumping engines being so adjusted as to supply the water at a given pressure as fast as it is consumed.

115. Water Motors. — In cities where water is delivered through pipes under sufficient pressure, its power may be utilized in running sewing machines, polishers, lathes, etc., by employing a rotary *water motor*. A common type is shown diagrammatically in Fig. 81. Water issues with great velocity from the jet *J* against cup-shaped fans attached to the axle of the motor. These are inclosed in a metal case *C* from which the water flows into the sewer or other drain. The impact of the water against the fans suffices to turn the shaft to which are connected either directly or by belts the machines that it is desired to operate.

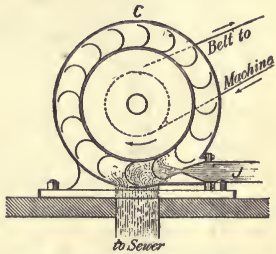


Fig. 81. — Sectional View of a Water Motor.

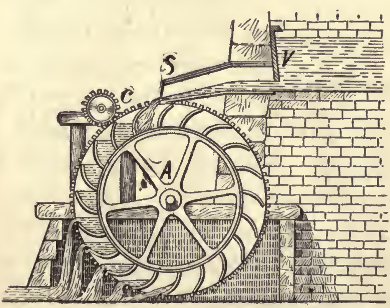


Fig. 82. — An Overshot Water Wheel.

116. Water Wheels. — An elevated body of water, like a lifted weight, is a source of potential energy. It is the falling of this water to a lower level that supplies the power for operating innumerable mills, factories, electric power plants, etc., found throughout this and other countries. In order to enable water to do the required work, the so-called *overshot water wheel* has long been employed. A wheel of this kind is shown in Fig. 82. It is plain that the impact and weight of the moving water from above the wheel conspire to turn the wheel, to which is geared or belted the machinery to be operated. Another old but common form of water wheel is the *undershot* wheel. In this case the wheel is turned by the impact of the current of water against fans at the bottom.

The most efficient form of water wheel, however, is the *turbine*, which is utilized in all modern plants employing water power. Water is conducted from the reservoir above a dam through a closed cylindrical tube, or flume, *F*, Fig. 83, to a penstock (shown by the dotted lines) which surrounds the stationary iron case *T* containing the rotating wheel, or turbine. This case rests upon the floor of the penstock and is submerged in water to a depth equal to the "head," or height, of the water supply. The turbine is attached to the shaft *S* and is set in rotation when the water is admitted. The small shaft *P* serves to control the size of the openings in the case through which the water gains entrance to the turbine. Figure 84 is a sectional view through the turbine *R* and the case *S*. Water enters as shown by the arrows and strikes the blades of the turbine at the most effective angle for producing rotation. When the water

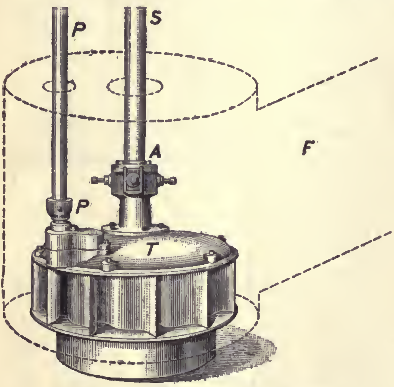


Fig. 83. — A Water Turbine.

has expended its energy, it falls from the bottom of the case into the tailrace below the penstock. The efficiency (§ 100) of turbines is frequently as high as 85 or 90 per cent.

Fig. 84. — Section of a Water Turbine.

117. Hydraulic Elevators. — Another use made of the energy of water under pressure is in the operation of elevators, or lifts. One form is shown in Fig. 85. When water is admitted to the cylinder C through the controlling valve V, the piston is forced to the right, thus producing a pull upon the cable (shown by the dotted lines) and causing the car A to ascend. Checking the flow of water by means of the rope r stops the car, while turning valve V to the position shown in the figure allows the water to flow from the cylinder and causes the car to descend by its own weight. A study of the figure will show that the car moves four times as fast and four times as far as the piston.

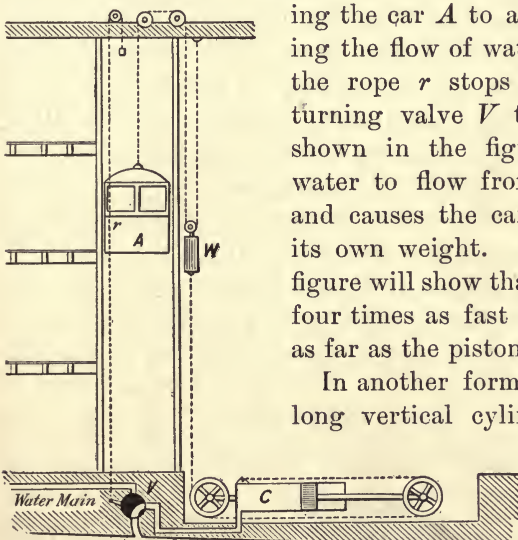


Fig. 85. — An Hydraulic Elevator.

In another form the piston is a long vertical cylinder extending from the bottom of the car into a deep hollow cylinder set in the ground. The admission of water under pressure acting against the lower end of the long piston lifts the car. The method of controlling such elevators is the same as that shown in Fig. 85.

118. The Hydraulic Ram. — The hydraulic ram is a useful device for automatically elevating water in small quantities when an abundant supply of spring water is available which has a fall of only a few feet. Water flows from the source through the pipe *P*, Fig. 86, and out through the opening at *v*. As the water increases in speed, the valve at *v* is lifted, which causes a sudden interruption of the flow in *P*. Since the momentum of the water cannot be destroyed instantaneously, a portion of the water is driven forcibly past valve v' into the air chamber *C*. A slight rebound of the water in the large pipe relieves for a moment the pressure against valve *v*, which falls by its own weight, thus opening again the orifice at that point. A repetition of the process causes more water to enter *C* until finally the pressure is sufficient to lift a portion of the water to a height of many feet. The compressed air in *C* acts as an elastic cushion and serves also to keep a steady flow of water in the small pipe. The quantity of water delivered by an hydraulic ram is dependent on the height of the source, the elevation to which it is to be lifted, the friction of the pipes, the length of pipe *P*, and the size of the ram itself.

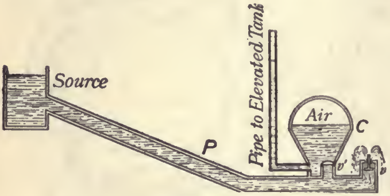


Fig. 86. — The Hydraulic Ram.

EXERCISES

1. The area of the small piston of an hydraulic press is 2 cm.^{2} and that of the large one 80 cm.^{2} How much force will 50 Kg. applied to the former produce upon the latter?

2. The small piston of an hydraulic press is operated by a lever of the second class 4 ft. in length, and the piston rod is attached 12 in.

from the fulcrum. If the diameters of the pistons are 1 in. and 8 in. respectively, how great an effort will produce a force of 2 T.?

3. If the effort applied to the small piston in Exer. 2 moves through 1 ft., how much will the large piston be raised?

4. A piston moves in a cylinder that is in communication with a water system whose pressure is 65 lb. per square inch. If a force of 1 T. is to be developed by the piston, what is the least diameter that it can have? *Ans.* 6.26 in.

5. Pressure against a piston 20 cm. in diameter is produced by a column of water 30 m. high. Calculate the force against the piston and the work performed when the piston moves 4 m.

6. Give suitable dimensions to the pistons and lever of an hydraulic press in order that an effort of 1 lb. may produce a force of 3000 lb.

ARCHIMEDES' PRINCIPLE

119. Buoyancy of Liquids. — It is a matter of common observation that bodies apparently become lighter when placed under water. If the hand, for example, be submerged in a vessel of water, it becomes evident at once that it is supported almost without muscular effort. Let a one- or two-pound stone be weighed in air and then weighed again while immersed in water. A decrease of several ounces will be observed. When blocks of wood or many other bodies are placed in water, the buoyancy of the water is sufficient to cause them to float.

120. The Principle of Archimedes.[1] — On account of the importance of the laws relating to the apparent decrease

[1] ARCHIMEDES (287–212 B.C.). The name of Archimedes will be remembered by students of Physics in connection with the buoyant action of liquids on immersed solids. The story is related that Hiero, King of Syracuse, had ordered a crown of pure gold which, when delivered, although it was of the proper weight, was suspected to contain a quantity of silver. Archimedes was asked to investigate. A method of procedure occurred to him while in the public bath as he noticed that his body experienced a greater buoyancy the more completely it was submerged. Recognizing in this effect the key to the solution of the problem, he leaped from the bath and hurried homeward, exclaiming "Eureka!

in weight that bodies undergo when submerged in a liquid three experiments will be described:

1. Measure the dimensions of a metal cylinder or rectangular metal block, and compute its volume in cubic centimeters. From the volume compute the weight of an equal volume of water. Suspend the metal body from one arm of a balance, and ascertain its weight in air. Weigh the body also when submerged in water, and compute the loss of weight. Compare the loss of weight with the weight of an equal volume of water found at first.

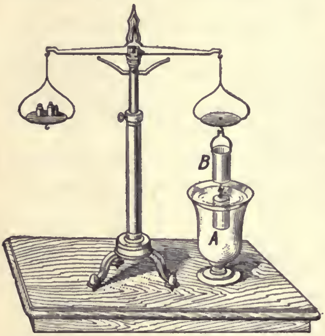


Fig. 87. — Verifying Archimedes' Principle.

2. From one arm of a balance, Fig. 87, suspend a metal cylinder A and the bucket B whose capacity is precisely equal to the volume of the cylinder. Counterbalance these by means of sand or weights placed in the opposite scale pan. Submerge A in water, and equilibrium will be destroyed. Fill the bucket with water, and equilibrium will be restored to the system.

3. Place a small glass beaker (or tumbler) on one pan of a balance, and suspend a stone from beneath the same pan. Counterpoise by

Eureka!" which means "I have found it!" Experiments showed that equal masses of gold and silver weigh unequal amounts when submerged in water. Therefore, when the crown was weighed against an equal mass of pure gold, both being submerged, the fraud was at once detected.

Archimedes is regarded as the founder of the science of Mechanics. From his time nearly 2000 years elapsed before any great advance was made. His investigation of levers is noteworthy. He is said to have made this remark, "Give me a fulcrum on which to rest my lever and I will move the earth." Among his countrymen, he was probably best known on account of the numerous instruments of war which he invented.

As a mathematician, Archimedes was first to determine the value of π, and the first to compute the area of a circle. He is supposed to have been killed by a Roman soldier while engaged in the investigation of a problem in geometry.

using sand, shot, or weights. Next fill a vessel provided with a spout with water, and allow all the excess to flow out at the spout. Place an empty tumbler under the spout, and lower the counterpoised stone into the water. Completely submerge the stone, and catch all the displaced water, the volume of which will be equal to that of the stone. Pour the displaced water into the beaker on the scale pan, and equilibrium will be restored.

From the results obtained by making any one of the experiments just described **Archimedes' Principle** may be deduced:

A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid that it displaces.

121. Explanation of Archimedes' Principle. — The reasons for Archimedes' law become clear when the laws of liquid pressure stated in § 104 are applied to a submerged body. Let a rectangular block *abcd* be immersed in a liquid, as shown in Fig. 88. The force against the upper surface of the block is the weight of the column of the liquid *efad* and acts *downward*. The force exerted by the liquid against the lower surface of the block *cb* is the weight of a column of the liquid *efbc* and acts *upward*. The resultant of these two forces is obviously in an *upward* direction and equal to the weight of a volume *abcd* of the liquid. The lateral forces exerted by the liquid against any two opposite faces of the block are equal and opposite, and therefore produce no tendency to move the block. Hence the immersed body is buoyed up by a force equal to the weight of a volume of the liquid equal to its own volume, *i.e.* to its displacement. It should be observed that the depth to which the body is submerged does not affect the final result.

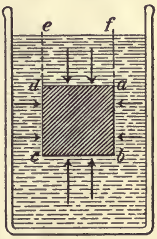


FIG. 88. — The Upward Force Exceeds the Downward Force.

If the weight of the body submerged in the liquid just equals the weight of the displaced liquid, the body will be in equilibrium and remain where it is placed provided the density of the medium be uniform. If the body weighs more than the amount of liquid displaced, it will sink; if less, it will rise to the surface and float.

122. Floating Bodies. — **1.** Obtain a square stick of pine about 30 centimeters long and 1 centimeter square. Measure its dimensions carefully, and beginning at one end, lay off cubic centimeters after taking due account of the cross-sectional area. Drill a deep hole in the end of the bar and embed a nail heavy enough to cause the bar to float in an upright position, as shown in (1) Fig. 89. Melt paraffin into the pores of the wood over a flame in order to make it water-proof. Float the bar in water, and read off the number of cubic centimeters of the portion beneath the liquid surface, *i.e.* the displacement. Compare the weight of the water displaced with the weight of the bar itself.

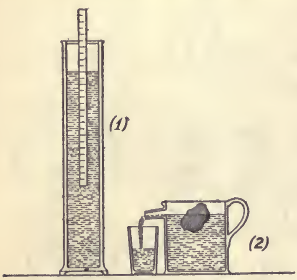


FIG. 89. — Flotation Illustrated.

2. Counterpoise a glass beaker (or tumbler) on a balance. Prepare a vessel, as shown in (2) Fig. 89, to catch displaced water. Carefully float a piece of well-paraffined wood weighing about 100 grams and catch the displaced water. Wipe the block of wood dry, and place it on one scale pan of the balance, and pour the displaced water into the beaker placed on the other. The two should balance.

If a block of wood is entirely immersed in water, the displaced water weighs more than the wood, and therefore the buoyant force is greater than the weight of the block. Hence the block will be forced toward the surface. As a portion of the block rises above the surface of the water, the displacement decreases until the weight of the block and that of the displaced water are equal. This fact will be found to be verified by either of the experiments just described. The law may be expressed as follows:

A floating body sinks to such a depth in the liquid that the weight of the liquid displaced equals the weight of the body.

123. Measuring Volumes by Archimedes' Principle. — Archimedes' principle affords an easy and accurate method for ascertaining the volumes of solid objects of irregular shapes. A body immersed in water evidently displaces a volume of water equal to its own volume, and, according to Archimedes' principle, loses in weight an amount equal to the weight of the displaced volume of water. (See Fig. 90.) Since 1 cubic centimeter of water weighs 1 gram, the body immersed contains as many cubic centimeters as the number expressing its loss of weight in grams. For example, a piece of metal that weighs 45.75 grams in air and 40.23 grams when immersed in water displaces the difference, or 5.52 grams of water. The volume of water displaced is therefore 5.52 cubic centimeters. Hence the volume of the immersed body is 5.52 cubic centimeters. When the loss of weight is measured in pounds, the volume expressed in cubic feet is found by dividing that loss by 62.5. Why?

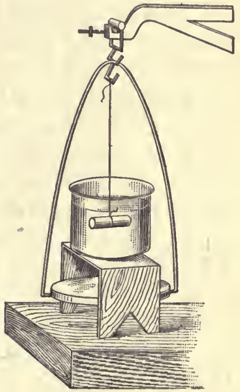


FIG. 90. — Weighing a Body in Water to Find its Volume.

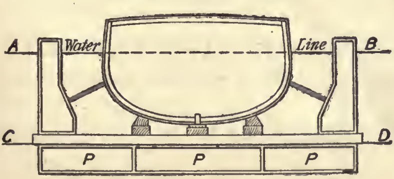


FIG. 91. — The Floating Dry Dock.

124. The Floating Dry Dock. — Among the useful modern inventions depending upon the law of flotation (§ 122) is the floating dry dock, which is shown diagrammatically in Fig. 91. When the several water-tight compartments P, P, P are allowed to fill with water, the dock sinks until the water level is at

AB. A vessel to be repaired is then floated into the dock, and the water pumped out of the various compartments. As the chambers are emptied the dock rises sufficiently to bring the water level to the line *CD*. The boat is thus lifted clear of the water.

EXERCISES

1. A stone weighing 400 g. under water weighs 480 g. in air. What mass and volume of water does it displace? What is the volume of the stone?

2. What is the volume of a metal cylinder that weighs 30 g. in air and 19 g. when immersed in water?

3. A solid weighs 20 lb. in air and 12 lb. when suspended under water. What is the weight of an equal volume of water? What is the volume of the body in cubic inches?

4. A body weighing 50 g. in air weighs 35 g. when immersed in water and 38 g. when immersed in oil. Find the mass and volume of the oil displaced.

5. A block of iron weighing 12 g. and a piece of wood weighing 4 g. are fastened together and weighed in water; their weight when immersed is 7.5 g. If the iron alone weighs 10.2 g. when immersed in water, what is the volume of the wood?

Suggestion. — From the combined volumes subtract that of the iron.

6. A block of wood is floated in a vessel full of oil. If 200 g. is the weight of the oil displaced, what is the weight of the wood?

7. A boat that weighs 450 lb. displaces how many cubic feet of water?

8. A ferry-boat weighing 700 tons takes on board a train weighing 550 tons. Express the total displacement in cubic feet.

9. Why does throwing the hands out of water cause the head of a swimmer to be submerged?

10. An egg will sink in water and float in brine. A solution of salt may be made of such a strength that an egg will remain at any depth. Explain.

11. What is the volume of a man weighing 150 lb. if he floats with $\frac{1}{20}$ of his body above water?

4. DENSITY OF SOLIDS AND LIQUIDS

125. Density of a Solid. — (*a*) *When the solid is more dense than water.* The density of a body is defined as its *mass per unit volume* and is found by dividing the number of units of mass by the number of units of volume (§ 12). In the C. G. S. system density is expressed in grams per cubic centimeter. For example, the density of mercury is 13.59 grams per cubic centimeter.

In ascertaining the density of a solid that sinks in water and does not dissolve, the mass is first found by weighing the body in air, and the volume is then measured by the method described in the preceding section. By definition,

$$\textbf{Density}\ (\textit{in grams per cm.}^3) = \frac{\textbf{mass}\ (\textit{in grams})}{\textbf{volume}\ (\textit{in cm.}^3)}. \qquad (1)$$

But, if the number of units of volume is found, as in practice, by ascertaining the apparent loss of weight sustained when the solid is submerged in water, we have for the *numerical value of the density* in the metric system,

$$\textbf{Density} = \frac{\textbf{mass}\ (\textit{in grams})}{\textbf{weight lost in water}\ (\textit{in grams})}. \qquad (2)$$

(*b*) *When the solid is less dense than water.* When a solid is not dense enough to sink in water, it may be attached to a sinker that is sufficiently heavy to submerge it. Let a sinker S and a light solid A whose mass is M grams be suspended from the arm of a balance, as shown in Fig. 92, so that the sinker alone is submerged. Let the weight of the two bodies thus arranged be W_1 grams. Now let both solids be immersed, and the combined weight be W_2 grams. The difference $W_1 - W_2$

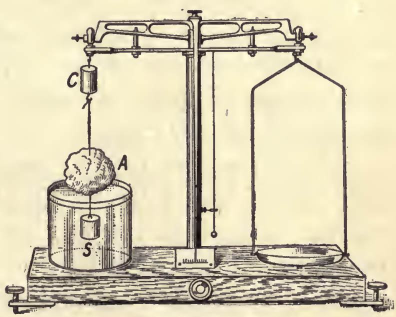


FIG. 92. — Ascertaining the Volume of a Solid Less Dense than Water.

is evidently caused by the buoyant force of the water on the light solid A, and, according to Archimedes' principle, is equal to the weight of the water displaced by this body. This difference is *numerically* equal to the volume of the body expressed in cubic centimeters. Hence an equation expressing the numerical value of the density of the solid may be written as follows:

$$\textbf{Density} = \frac{\mathbf{M}\ (\textit{in grams})}{\mathbf{W}_1 - \mathbf{W}_2\ (\textit{in grams})}. \qquad (3)$$

126. Specific Gravity.— Occasional use is made of the term *specific gravity* (abbreviated *sp. gr.*) to express the heaviness or lightness of a body as compared with the weight of some standard substance. The **specific gravity** *of any solid or liquid is the ratio of the weight of the body to the weight of an equal volume of pure water at* 4° *C.* In the case of gases the standard with which they are compared is either air or hydrogen.

Since one cubic centimeter of pure water at 4° C. weighs one gram, it follows that the density of a body in grams per cubic centimeter is numerically equal to its specific gravity. For example, the density of lead is 11.36 grams per cubic centimeter; hence lead is 11.36 times as heavy as an equal volume of water. Therefore, the specific gravity of lead is 11.36.

Again, since the specific gravity of lead is 11.36, one cubic foot of lead will weigh 62.5 pounds × 11.36, or 710 pounds. Hence in the English system of measurement the density of lead is 710 pounds per cubic foot.

It is to be carefully observed that specific gravity and density are not the same thing; they are numerically equal only when density is given in C. G. S. units. In the English system, density is entirely different from specific gravity, as the example plainly shows.

The equation of specific gravity is

$$\textbf{Specific gravity} = \frac{\textbf{weight of a body}}{\textbf{weight of an equal vol. of water}}. \quad (4)$$

127. Density of Liquids. — There are several methods for finding the density of a liquid. Let a solid that is denser than water be weighed in air, then in water, and finally in a liquid whose density is to be found. The loss of weight in water then represents *numerically* the volume of the submerged solid. The loss of weight in the liquid of unknown density represents the mass of an equal volume of that liquid. Therefore the density of the liquid is found by dividing the latter loss by the former. For example, a solid weighs 80 grams in air, 55 grams in water, and 60 grams in another liquid. The mass of water displaced is 25 grams and that of the other liquid 20 grams. Hence the volume of the liquid of unknown density displaced by the solid is 25 cubic centimeters, and its density $20 \div 25$, or 0.8 gram per cubic centimeter.

128. Hydrometers. — The law of floating bodies (§ 122) affords a convenient method for finding the density of a liquid. Let the bar of wood used in Experiment 1, § 122, be placed in a jar of water and the volume of the submerged portion ascertained. Next place the bar in a liquid whose density is to be found, and again find the volume of the part beneath the liquid. If the displacement of water is v_1 cubic centimeters, the weight of the displaced water is v_1 grams. If d is the density of the other liquid and v_2 its displacement, the weight of the liquid displaced is $v_2 d$ grams. According to the law of floating bodies the weight of the liquid displaced in each instance is equal to the weight of the floating bar. Hence the two displacements are of equal weight. We may therefore write

$$\mathbf{v_2 d = v_1}; \text{ whence } \mathbf{d = \frac{v_1}{v_2}}. \quad (5)$$

A floating body that is made in a convenient form for measuring the densities of liquids is called an *hydrometer*. If the weight of the hydrometer remains constant as in the case of the wooden bar used in the experiment just described, the instrument is called an *hydrometer of constant weight.* In some instances the densities are indicated upon the bar and may be read off at once by observing the point to which the instrument sinks. Many forms of hydrometers are in daily use, the size, shape, and graduations being adapted to the particular commercial purpose that they serve. A common form is shown in Fig. 93. A cylindrical glass tube terminates below in a small bulb filled with mercury, which causes the instrument to float in an upright position. The upper portion of the tube is made small, and the graduations are upon a paper scale sealed within.

FIG. 93.—An Hydrometer of Constant Weight.

Nicholson's Hydrometer.—The Nicholson hydrometer shown in Fig. 94 represents the type known as hydrometers of *constant volume*, *i.e.* of constant displacement. This instrument is used in finding the density of a solid body. Let w_1 grams be the weights required in the upper pan A, to sink the hydrometer to a certain mark placed on the slender stem a. After placing the solid of unknown density in the upper pan, the weight required to sink the instrument to the same mark is w_2 grams. Then $w_1 - w_2$ is the mass of the solid. Let w_3 be the weights required in the upper pan after the solid has been transferred to the lower pan B. Then $w_3 - w_2$ is the mass of the water displaced and numerically equal to the volume of the solid.

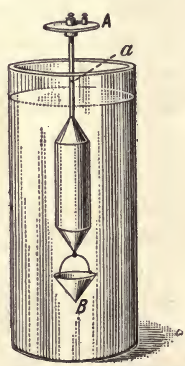


FIG. 94.—Nicholson's Constant Volume Hydrometer.

Therefore, the numerical value of the density of the solid expressed in metric units is:

$$\text{Density} = \frac{w_1 - w_2}{w_3 - w_2}. \tag{6}$$

EXERCISES

1. A piece of lead weighs 56.75 g. in air and 51.73 g. when suspended in water. Find the volume and density of the lead.

2. A cylinder of aluminium weighs 28.35 g. in air and 17.85 g. when immersed in water. Calculate the volume and density of the metal. Compute the sp. gr. of aluminium.

3. A piece of glass weighing 45 g. in air weighs 22.5 g. in water and 23.75 g. in oil. Calculate the densities of the glass and the oil.

4. What would be the weight of the glass in Exer. 3 when immersed in a liquid whose density is 0.922 g. per cubic centimeter?

5. The density of marble is 2.7 g. per cubic centimeter. What is the weight of a rectangular block 1 m. long, 40 cm. wide, and 15 cm. thick? Compute the sp. gr. of marble. What is its mass per cu. ft.?

6. Silver is 10.4 times as heavy as an equal volume of water. What will 20 g. of silver weigh when immersed in water?

7. Ice is 0.9 as heavy as an equal volume of water. If a piece of ice weighing 500 g. floats on water, what is the volume of the submerged portion? What is the volume of the ice?

SUGGESTION. — Apply the law of floating bodies and ascertain the weight of water displaced.

8. A bar of wood weighing 100 g. floats on water with 0.82 of its volume submerged; when placed in oil, 0.80 of its volume is submerged. Calculate the sp. gr. and density of the oil.

9. A piece of paraffin weighs 69 g. in air and when attached to a sinker and suspended in water, 85.8 g. If the weight of the sinker in water is 95.7 g., what is the volume and density of the paraffin?

10. The weight required to sink a Nicholson hydrometer to the mark on the stem is 45 g.; when a piece of marble is placed in the upper pan, the weight required is 15.3 g. and with the marble in the submerged pan 26.3 g. Find the density of the marble.

11. Find the density of paraffin from the following data:

Weight required to sink Nicholson's hydrometer to mark	56.4 g.
Weight to sink hydrometer with paraffin in upper pan	45.6 g.
Weight required with paraffin in submerged pan	57.6 g.

12. The density of mercury is 13.59 g. per cubic centimeter. If a cubic foot of water weighs 62.5 lb., what is the weight of a cubic inch of mercury?

5. MOLECULAR FORCES IN LIQUIDS

129. Cohesion and Adhesion. — Many of the phenomena of nature lead to the conclusion that bodies are made up of extremely minute particles to which is given the name *molecules.* The molecules of bodies possess more or less freedom of motion among themselves depending on the nature of the body; this freedom is greatest in gases and least in solids. It is due to the very perfect freedom of motion among the molecules of a liquid (*i.e.* to the property of *fluidity*) that they are able to transmit pressures in all directions, thus giving rise to the principle stated in (1) § 104 and to Pascal's Law discussed in § 110.

When near together molecules attract one another, thus producing the resistance found when we attempt to break a wire, to remove paint from glass, to tear paper, etc. The name *cohesion* is given to this attraction when it is between molecules of the same kind, and the term *adhesion* applies to the attraction when the molecules are of different kinds.

Cohesion serves to bind individual molecules into bodies, or masses, and adhesion to hold together bodies of different kinds. Two clean surfaces of lead will cohere when pressed firmly together, and the dentist hammers gold leaf into a solid lump to form the filling for a tooth. The blacksmith brings together white-hot pieces of iron and by blows brings their molecules into such close proximity that cohesion takes place. Clean graphite powder is pressed into a solid mass to form the lead of a pencil; the force of cohesion holds the particles together. The attraction between wood and glue, stone and cement, paint and iron, etc., affords cases illustrating the force of adhesion.

130. Surface Films of Liquids. — Although steel is nearly eight times as dense as water, a small sewing needle may be caused to "float" on water by placing it carefully upon the surface. If the surface of the liquid is closely examined, it will be observed that the needle rests in a slight depression, as shown in Fig. 95. In fact, the appearance is as if a thin membrane were stretched across the surface of the water.

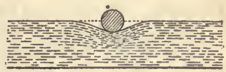


FIG. 95. — An Ordinary Steel Needle "Floating" on Water.

The three following experiments show an important property of the surface films of liquids:

1. Let a soap bubble be blown on the bowl of a clay pipe or the mouth of a small glass funnel. If the stem is left open a moment, it may be observed that the bubble diminishes in size. A candle flame held near the opening in the stem will be deflected by the current of air that is forced out by the contracting bubble.

2. Let a loop of thread be tied to one side of a wire frame 4 or 5 centimeters in diameter so that it will hang near the center, as shown in (1), Fig. 96. If now the frame is dipped into a soap solution, a liquid film will form across it with the loop closed, as shown. If, however, the film within the loop of thread be broken by means of a hot wire, the loop instantly opens out into a circle, as shown in (2).

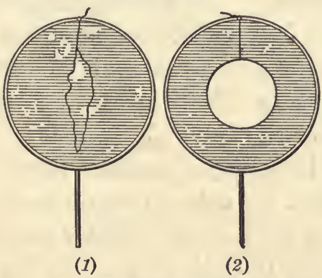


FIG. 96. — An Effect of Surface Tension in a Liquid Film.

3. Let a mixture be made of alcohol and water of such strength that a drop of olive oil will remain in it at any depth. The oil may be introduced beneath the surface by means of a small glass tube. It will be found that the globule of oil at once assumes a spherical form. In order to avoid an apparent flattening of the drop of oil, a body with flat sides should be used.

These experiments show that the surface film of a liquid tends to contract and become as small as possible. In Experiment 1 the contraction produces a pressure that drives the air from the tube. In Experiment 2 the film assumes the least possible area. This is the case when the area within the loop is as large as it can become, *i.e.* when it is circular. In Experiment 3 the film of oil completely incloses the globule and causes the oil to assume the geometrical form requiring the least superficial area for a given volume. The spherical form fulfills this condition. For a similar reason a soap bubble takes the shape of a sphere.

131. Surface Tension. — It will be seen from a study of Fig. 97 that the conditions under which the surface molecules of a liquid exist is very different from that of the molecules lower down. Molecule A at the center of the small circle is attracted equally in all directions by the neighboring molecules. This is the force of cohesion and acts across a very small distance represented here by the radius of the circle. Very near the surface the forces acting downward on molecule B are greater than those that act upward, while for molecule C there is no upward attraction whatever. Hence the surface layers of molecules are greatly condensed by this excess of force acting always toward the body of the liquid. In this manner is formed a tough, tense surface film whose constitution is different from that in the interior mass. *The measure of the tendency of the surface layers of a liquid to contract is called* **surface tension.**

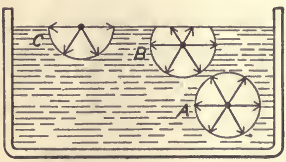


FIG. 97. — Molecular Forces near the Surface of a Liquid.

132. Capillary Phenomena. — The effects of surface films were first investigated in small glass tubes of hair-like dimensions. Hence the name "capillarity" arises from the Latin word *capillus*, meaning "hair." Figure 98 represents a series of glass tubes varying from 0.2 millimeter to 2 millimeters in diameter. If the tubes are moistened and then set upright in a shallow vessel of water, the liquid will rise in the tubes, — highest in the smallest tube and least in the largest. It may also be observed that the surface of the liquid turns upward wherever it comes in contact with the glass. Hence the surface within the tube is concave, and a so-called *meniscus* is formed.

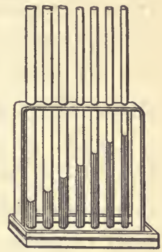


FIG. 98. — Elevation of a Liquid in Capillary Tubes.

If, now, glass tubes of small bore are placed in mercury (Fig. 99), the liquid, which in this case does not wet the glass, will be depressed within the tubes. The depression is greatest in the smallest tube and least in the largest. The edges of the film in contact with the glass may be seen to turn downward so that the surfaces within the tubes are convex, and the meniscus thus formed is inverted.

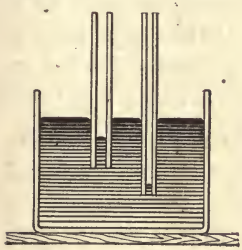


FIG. 99. — Depression of Mercury in Capillary Tubes.

The following laws of capillary action may be stated:

(1) *Liquids are elevated in tubes which they wet, but are depressed in tubes which they do not wet.*

(2) *The elevation or depression is inversely proportional to the diameter of the tubes.*

133. Capillary Action Explained. — If a plate of glass, shown in cross section in (1), Fig. 100, be placed upright in a

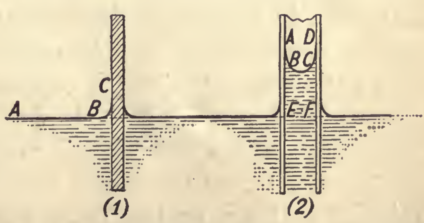


FIG. 100. — Water Lifted by the Contraction of the Surface Film.

shallow vessel of water after having been moistened, there will be formed a continuous film ABC lying partly upon the glass and partly upon the water. On account of the tendency of this film to contract, as shown in § 131, the corner at B will be rounded and a small portion of the water lifted against the glass.

Again, if a moistened tube of glass, (2), Fig. 100, be dipped into water, a film $ABCD$ is formed adhering to the glass and extending across the water in the tube. Owing to the tendency of this film to contract, a force is produced that is sufficient to lift the column of water $BEFC$ in opposition to the force of gravity. When the tube is dipped into mercury, the liquid forms no film adhering to the glass above the surface level. On the other hand, the surface film of mercury (see Fig. 99) continues downward along the glass walls of the tube and then upward into the tube at the lower end. The force which is developed by the contraction of this film tends to pull the liquid down within the tube. The depression thus produced is such that the downward force of the contracting film is balanced by the upward pressure of the liquid within the tube.

134. Capillary Action in Soils. — The principles of capillary action find an important application in the distribution of moisture in the soil. In compact soil water is brought to the surface in much the same manner as it rises in a piece of loaf sugar which is allowed to come in contact with it. As rapidly as evaporation goes on at the surface, the loss is supplied by capillarity from below. In dry weather it is desirable to prevent this surface loss, which is done by "mulching" and loosening the soil by cultivation. In this latter process the grains of soil are broken apart and the interstices thus made too large for effective capillary action to take place.

The moisture then rises to a level a few inches beneath the surface, where it is made use of by growing plants.

EXERCISES

1. Why is a drop of dew spherical? Examine a small globule of mercury placed on glass. How do you account for its form?

2. By heating a piece of glass until it softens the sharp corners become rounded and smooth. Explain.

3. In the manufacture of shot molten lead is poured through a small orifice at the top of a tower. In falling the stream breaks up into drops which solidify before reaching the earth. What gives the spherical form to these masses of lead?

4. Why does oil flow upward in the wick of a lamp? Will mercury do the same?

5. Grease may be removed from a piece of cloth by covering it with blotting paper and passing a hot flatiron over it. Explain.

6. Place two toothpicks upon water about a centimeter apart. Touch the liquid surface between them with a glass rod moistened with alcohol. From the manner in which the pieces of wood move about, observe which liquid film has the greater tension.

7. Explain how an insect can run on the surface of water without sinking.

8. Explain the action of a towel; of blotting paper; of sponges.

9. Why can we not write with ink upon unglazed paper?

10. Why are the footprints made in newly cultivated soil moist while the loose earth is dry?

11. Explain why it is necessary to pack the earth around plants and small trees when they are first set out. Later on we loosen the soil about them. Why?

SUMMARY

1. The force exerted by a liquid of uniform density against a given surface is directly proportional to the depth of the surface (§ 103).

2. At a given depth the force exerted by a liquid is the same in all directions and acts always in a direction perpendicular to the surface against which it presses (§ 104).

3. The force exerted by a liquid on any horizontal surface is equal to the product of the area and depth of the surface and the density of the liquid. The equation is *total pressure* = *ahd* (§ 106).

4. When the surface pressed against is not horizontal, the product of the area and the density must be multiplied by the depth of the center of mass of the given surface (§ 106).

5. The force exerted by a liquid against the bottom of a vessel of given depth is independent of the form of the vessel (§ 107).

6. A liquid at rest in communicating vessels remains at the same level in all its parts (§ 109).

7. A force applied to any area of a confined liquid is transmitted undiminished by the liquid to every equal area of the interior of the containing vessel and to every part of the vessel. This is known as *Pascal's Law* (§ 110).

8. The mechanical advantage of the hydraulic press is the ratio of the area of the large piston to that of the small one (§ 111).

9. A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid that it displaces. This is known as *Archimedes' Principle* (§ 120).

10. A floating body displaces a mass of the liquid in which it floats, whose weight equals its own (§ 122).

11. The volume of a body insoluble in water may be found, according to *Archimedes' Principle*, by ascertaining the weight of water that it will displace (§ 123).

12. $Density = \frac{mass}{volume}$ (§ 125).

13. Bodies are assumed to be composed of extremely small particles called molecules. When near together, molecules attract each other. *Cohesion* is the attraction between molecules of the same kind; *adhesion* is the attraction between molecules of different kinds (§ 129).

14. The surface of a liquid tends to contract and become as small as possible; hence the spherical form assumed by soap bubbles, dew drops, globules of mercury, etc (§ 130).

15. Liquids are elevated in tubes which they wet, but are depressed in those which they do not wet. The elevation or the depression is inversely proportional to the diameter of the tube (§ 132).

16. Moisture rises readily in compact soils, but ceases to rise in those of loose texture. Hence the loss of soil water by evaporation is effectually prevented by "mulching" or by cultivation (§ 134).

CHAPTER VIII

MECHANICS OF GASES

1. PROPERTIES OF GASES

135. Characteristics of Gases. — Gases, like liquids, possess the property of *fluidity*, *i.e.* they may be deformed by any force however small. But a gas differs from a liquid in that it has no definite size of its own; it not only fits itself to the shape of the vessel containing it, but always entirely fills it. On account of their common property of fluidity, liquids and gases are classed together as **fluids**. As a consequence of this property, the laws of pressure relative to liquids stated in 1, § 104 and in § 110 are equally applicable to gases. Other laws, however, arise in the case of a gas on account of the tendency to adapt its size to the capacity of the containing vessel.

136. Laws Common to Liquids and Gases. — (1) The force exerted against any surface is perpendicular to that surface. (2) The force at any point is the same in all directions (§ 104). (3) An immersed body is buoyed up by a force equal to the weight of the liquid or the gas that it displaces (§ 120).

137. Weight and Density of Air. — We are taught by everyday experience that a gaseous medium which we call **air** surrounds us on every hand. The bubbles that may be produced by blowing through a tube inserted into water, the process of breathing, the resistance that the air offers to a rapidly moving bicycle or train, the various effects of the wind, and many other phenomena demonstrate to us the presence of this medium.

It was shown by experiment in § 3 that air has weight. The same apparatus may be used in finding the density of air in the following manner:

Let the bulb be weighed carefully before and after admitting the air. The increase in weight will give the mass of the air contained in the bulb. The capacity of the bulb is next ascertained by filling it with water and weighing it. Dividing the mass of the air by the capacity of the bulb gives the density of the air.

The density of air at the temperature of freezing water and under the average sea-level pressure is 0.001293 gram per cubic centimeter. Hence a liter (1000 cm.^{3}) of air weighs nearly 1.3 grams. A cubic foot of air weighs about an ounce and a quarter; hence 12 cubic feet weigh nearly a pound. The amount of air in an ordinary schoolroom weighs more than half a ton. Thus we live submerged in an ocean of air which extends many miles above us and exerts a pressure of nearly 15 pounds per square inch.

2. PRESSURE OF THE AIR AGAINST SURFACES

138. Atmospheric Pressure. — Since air has weight and fluidity, the atmosphere must exert a force against all surfaces with which it comes in contact. The existence of such a force may be shown by the following experiments:

1. Fill a tumbler with water and invert it in a vessel of water. Lift the inverted tumbler until its opening is horizontal and just submerged, as in Fig. 101. The water remains in the tumbler because it is supported by the force exerted by the atmosphere downward against the free surface of water in the vessel.

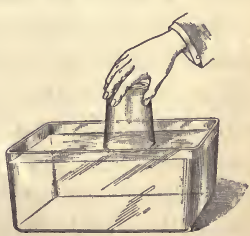


FIG. 101. — Atmospheric Pressure Supporting Water.

2. While the inverted tumbler is in the condition shown in Fig. 101, place a piece of cardboard across its mouth, pressing it close against the rim. Carefully lift the tumbler from the water, and the cardboard will not fall off. The force exerted by the air against the cardboard in an upward direction is sufficient to support the weight of the water in the tumbler, as in Fig. 102.

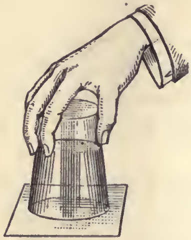


FIG. 102. — Action of Atmospheric Pressure Upward.

3. Tie a piece of sheet rubber over a glass vessel, as shown in Fig. 103. Place the vessel on an air pump and exhaust the air from the space beneath the rubber. The membrane, no longer supported by the air from below, is more and more depressed until it finally bursts under the pressure of the air above.

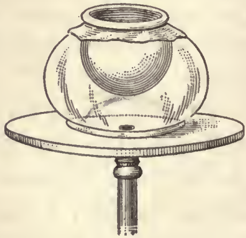


FIG. 103. — Atmospheric Pressure upon a Rubber Membrane.

4. If a glass tube 3 or 4 feet long (or even longer) can be secured, place it nearly vertical with one end in a tumbler of water. Try to elevate the water to the top of the tube by "sucking" on the upper end of it. Now insert the lower end of the tube in mercury and try to elevate it in the same manner. It is so much denser than water, that it can be lifted only a few inches.

139. Limitations of Atmospheric Pressure. — The attention of Galileo was called to the fact that in wells of unusual depth suction pumps (§ 155) were unable to lift water more than about 32 feet above the level of the water in the wells. At that time it was supposed that "nature abhorred a vacuum" and that she hastened to fill all such spaces with whatever material happened to be most available. Thus Galileo was led to believe that nature's abhorrence for a vacuum had its limitations, and he probably suspected that the rise of water in pumps was due to air pressure on the surface of the water. It remained for his pupil, Torricelli, however, to devise a suitable method for measuring the actual pressure of the atmosphere. He succeeded in doing this in the year 1643.

140. Torricelli's Experiment. — This important historical experiment is performed by making use of a strong glass tube about 80 centimeters long which is closed at one end. The tube is first filled with mercury in order to expel the air, after which it is closed by the finger and inverted in a vessel of mercury, Fig. 104, care being taken to prevent the entrance of air. The mercury falls at once and leaves a vacuum of several centimeters at the top of the tube. The force due to the atmosphere which is exerted downward against the free surface of mercury in the vessel is transmitted to the interior of the tube in which it is able to support a column of the liquid usually about 75 centimeters in height.

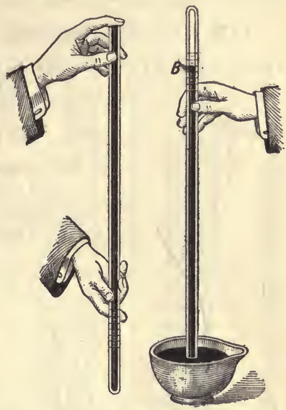


FIG. 104. — Torricelli's Experiment.

141. Pascal's Experiment. — Pascal reasoned that if the column of mercury in a Torricellian tube were indeed supported by the atmosphere, the column should become shorter at a high altitude. The column was accordingly measured at the top of a high tower in Paris and a small decrease detected. Five years after Torricelli's discovery, the tube was carried to the top of the Puy de Dome, a high mountain in Auvergne, France, where a test showed a decided decrease of about 8 centimeters in the height of the mercurial column.

142. Variations in Atmospheric Pressure. — The atmospheric pressure at a given place is far from being constant. The variations at any given place amount to about 3 centimeters. At sea-level the average height of the mercurial column is about 76 centimeters; hence this height is taken as the standard of pressure and is called **mean sea-level pressure.** Any pressure equivalent to a column of mercury 76 centimeters high is said to be "one atmosphere."

143. The Barometer. — The mercurial barometer (pronounced *ba rŏm′e ter*) is merely a mounted Torricellian tube for showing the pressure of the atmosphere. In order that it may give correct indications, the mercury must be pure and clean, and the space above the liquid in the tube as free as possible from the presence of air and other gases; *i.e.* the vacuum must be as nearly perfect as possible. *Atmospheric pressure is usually measured in inches or centimeters of mercury.* The space above the mercury in the tube is known as the **Torricellian vacuum.**

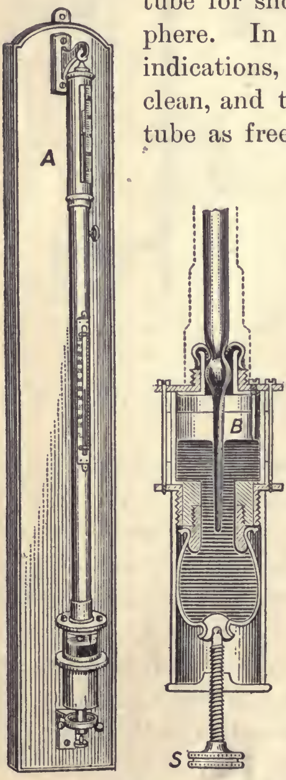


Fig. 105. — A Standard Mercurial Barometer.

As the height of the mercurial column changes with the pressure of the air, the surface of the liquid rises and falls in the reservoir, or cistern, below. For an accurate reading of the barometer the mercury column must be measured from the surface of the liquid in the reservoir. In the barometer shown in Fig. 105 the reservoir has a flexible bottom. By turning the screw S the surface of the mercury in the cistern is brought to the zero point of the scale, whose position is marked by an ivory index B. The height of the mercury

FIG. 106. — A Mercurial Barometer.

is then read by observing the position of the upper surface of the liquid in the tube at A.

Some mercurial barometers have the form shown in Fig. 106. The height of the mercury is found by reading the positions of the two liquid surfaces A and B, in which case the difference between the two readings gives the length of the mercury column supported by the atmosphere.

144. The Aneroid Barometer. — A barometer of the form shown in Fig. 107 is in common use. As the name indicates, the aneroid barometer is "without liquid," and depends for its operation on a circular chamber C made of thin metal with corrugated sides. The air in the chamber is partly removed, after which the chamber is hermetically sealed. An increase of atmospheric pressure forces the side of the box inward, but a decrease allows it to spring out. This motion, although very slight, is transmitted through the multiplying systems of levers L and A and the small chain B to the axle S which carries the index, or movable pointer, I. The scale is graduated to correspond to the readings of a standard mercurial barometer. These

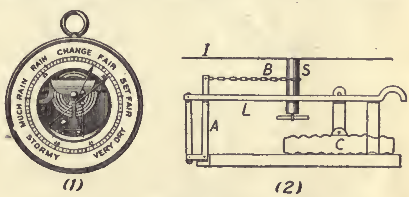


FIG. 107. — The Aneroid Barometer.

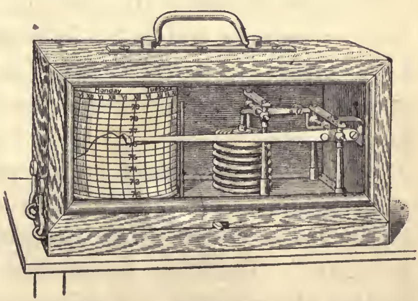


FIG. 108. — The Barograph, or Self-registering Barometer.

instruments are made of convenient size to be used by surveyors and explorers in ascertaining altitudes. It should be said that the words "Rain," "Fair," etc., printed on the dial of an aneroid barometer are merely indications of the general trend of weather changes.

The principle of the aneroid is employed in the "barograph," or self-recording barometer, shown in Fig. 108. This instrument is provided with an index which carries a pen that makes a continuous record of the atmospheric pressure on a revolving cylinder covered with suitable paper. A portion of such a record is shown in Fig. 109.

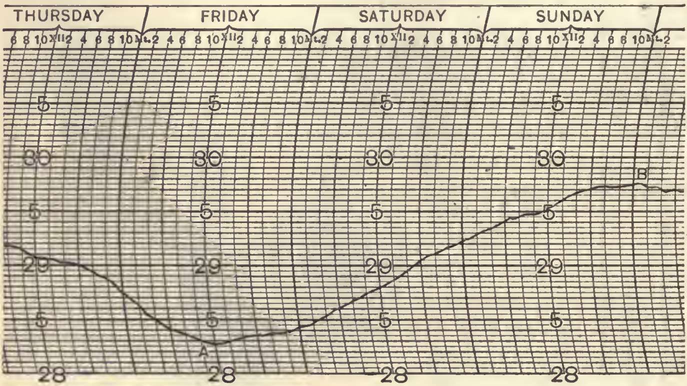


Fig. 109. — Portion of a Record Made by a Barograph.

145. Utility of the Barometer. — The barometer is an important laboratory instrument, inasmuch as the atmospheric pressure must be known in carrying out many experiments in both Physics and Chemistry. In this respect it ranks in usefulness with the thermometer, balance, etc.

From the readings of barometers taken simultaneously at many places of observation and telegraphed to central stations, the direction of atmospheric movements can be predicted. Thus the barometer becomes an aid in forecasting the weather. Furthermore, a "low" barometer, *i.e.* decreased pressure, usually accompanies or precedes stormy weather, while a rising barometer generally denotes the approach of fair weather. If a weather map be consulted, certain regions will be found marked "High" and others marked "Low." The places at which the pressures are equal are joined by curves called *isobars*, upon each of which is indicated the barometric reading, Fig. 110. The direction of the wind at each place of observation is indicated by an arrow. The general direction of the wind is always from places of "high" toward those of "low" pressure.

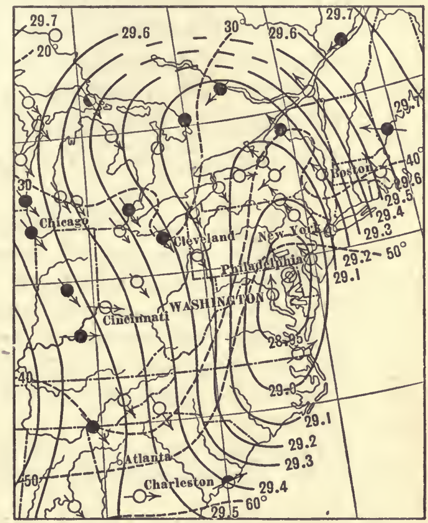


FIG. 110. — A Portion of a Weather Map.

Another important use of the barometer is made in measuring the difference in altitude of two places. This measurement depends upon the fact that the atmospheric pressure decreases with the elevation above sea-level. For places not far above the level of the sea the decrease is about 1 millimeter for every 10.8 meters of elevation, or 0.1 inch for every 90 feet of ascent. The decrease in pressure as one climbs a mountain is easily accounted for when it is recalled that it is the air above the level of a given place that produces the pressure. Descending a mountain or into a mine simply submerges one more deeply in the atmospheric ocean and thus puts above his level more air to be supported.

146. Atmospheric Pressure Computed. — In order to compute the pressure of the atmosphere in grams per square centimeter, we have only to find the pressure per unit area due to mercury when its depth is equal to the height of the barometric column. Consider a tube whose cross-sectional area is 1 square centimeter and in which the height of mercury is 76 centimeters. The pressure at the bottom of such a tube is the product of the area, height, and density of the mercury, as shown in § 106. We have, therefore, $1 \times 76 \times 13.6$, or 1033.6 grams. Hence, if the barometer reading is 76 centimeters, the atmospheric pressure is 1033.6 grams per square centimeter.

EXERCISES

1. The column of mercury in a barometer stands at a height of 74.5 cm. What is the height in inches?

2. How high a column of water could be supported by atmospheric pressure when the barometer reads 75 cm.?

3. When the barometer reads 74 cm., what is the atmospheric pressure expressed in grams per square centimeter? in dynes per square centimeter?

4. If the pressure of the air is 15 lb. per square inch, calculate the total force exerted upon a person the area of whose body surface is 16 sq. ft.

5. A soap bubble has a diameter of 4 in. Calculate the force exerted by the air against its entire surface when the barometer reads 29 in. A cubic inch of mercury weighs 0.49 lb.

FIG. 111.

6. What result would be obtained by performing Torricelli's experiment with a tube twice as long as the one described in § 140?

7. How would the height of mercury be changed if a tube of larger cross-sectional area were used?

8. Show why a change in the area of the mercurial surface in the cistern of a barometer has no effect on the height of the column.

9. What result would Torricelli have obtained in his experiment if he had used a tube only 70 cm. long?

10. Try to suck the water from a bottle (see Fig. 111) out of which a glass tube passes through a tightly fitting rubber stopper. Explain the results observed.

11. Inkstands are sometimes made in the form shown in Fig. 112. Explain why the ink in the reservoir can remain at a greater height than that outside.

FIG. 112. — Pneumatic Inkstand.

12. Why is it necessary to make a small vent hole in the upper part of a cask when it is desired to draw off the liquid in a steady stream from a faucet placed near the bottom?

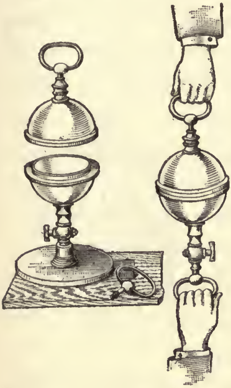


FIG. 113. — Magdeburg Hemispheres.

13. Explain why water will not run in a steady stream from an inverted bottle.

14. The Magdeburg hemispheres shown in Fig. 113 are closely fitting hollow vessels about 4 in. in diameter. By placing the hemispheres together and exhausting the air the pull of two strong boys is scarcely sufficient to separate them. Explain.

15. Assuming the atmospheric pressure to be 15 lb. per square inch, calculate the force required to separate the hemispheres shown in Fig. 113 when the air within them is completely exhausted.

3. EXPANSIBILITY AND COMPRESSIBILITY OF GASES

147. Compressibility of Air and Other Gases. — Gases, unlike liquids, are easily reduced in volume by increasing the pressure under which they exist. This is evident from the fact that the quantity of air in the pneumatic tire of a bicycle, for example, may be increased to double or triple the original mass. Again, the air in a pneumatic cushion is compressed into a smaller space when one sits upon it, but it springs back to its original volume when the pressure is relieved. Thus air and other gases manifest the property of *expansibility* as well as *compressibility.* The popgun and air rifle make use of these properties of air: first the air is compressed in the cylinder of the gun, then as the pellet moves, the force of expansion drives the missile with great acceleration from the barrel.

1. Place a partially inflated balloon under the receiver of an air pump. As soon as the pump is set in action, the swelling of the balloon will indicate the expansive tendency of the air within it.

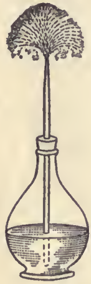


Fig. 114. — Illustrating Expansibility of a Gas.

2. Arrange a bottle as shown in Fig. 114. Blow forcibly into the tube, thus causing some bubbles of air to pass into the bottle above the liquid. Quickly remove the lips from the tube, and water will be driven out by the expanding air within the bottle.

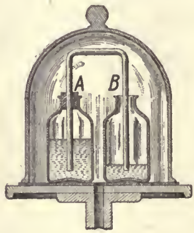


Fig. 115. — Air in *A* Forces Water into *B*.

3. Place two bottles under the receiver of an air pump as shown in *A* and *B*, Fig. 115. *A* is tightly corked and about two thirds full of water. *B* is uncorked and contains air only. When the pump is put into operation, the air pressure on the surface of the water in *B* is reduced, and the consequent expansion of the air in *A* above the liquid forces the water through the tube into *B*. If air be now admitted into the receiver, the water will be driven back into *A*. Why?

The expansibility of gases is explained by assuming that their molecules (§ 129) are in rapid motion. As a consequence of this motion, innumerable molecules strike against every part of the walls of the containing vessel. Although the mass of a molecule is extremely small, its speed is so enormous (equaling or exceeding that of a cannon ball) that it strikes the side of the vessel with an appreciable force. Thus a continuous storm of such blows results in the production of a steady outward pressure against the walls of the vessel inclosing the gas.

148. Pressure and Elastic Force in Equilibrium. — The experiments just described teach us why hollow bodies, such as balloons, cardboard boxes, etc., are not crushed by atmospheric pressure. The crushing force exerted against the external surface of a balloon only 10 centimeters in diameter is more than 300 kilograms. This force, however, is counteracted by the expansive force of the gas within it. If the external force is decreased, the gas expands until the forces are again in equilibrium. On the other hand, if the external force is increased, the gas is compressed until the inward and outward forces balance each other. Thus the human body is able to withstand the enormous force exerted by the atmosphere upon it. All the cavities that might otherwise collapse are prevented from doing so by the expansive force of the gases which they contain.

149. Law of Expansion and Compression. — The relation which exists between the pressure to which a gas is subjected and its volume was discovered by Robert Boyle (1627–1691) of England, in 1662, and by Mariotte, of France, fourteen years later. In France this relation is usually called "Mariotte's Law," but we speak of it as **Boyle's Law.** An experiment to illustrate Boyle's method of discovery may be performed as follows:

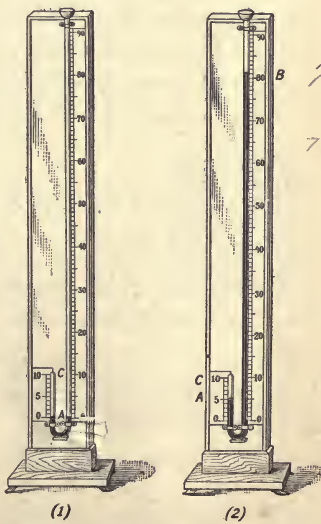


Fig. 116. — Illustrating Boyle's Law.

Take a bent glass tube (1), Fig. 116, of which the shorter arm is hermetically sealed. The long arm, left open at the top, should be about 90 centimeters long. The length of the short arm should be at least

10 centimeters. Pour a small quantity of mercury into the tube so that the liquid stands at the same level in both sides. We thus have a quantity of air confined in the space AC under one atmosphere of pressure owing to the transmission of pressure by the mercury. Measure the length of the space AC, and then pour mercury into the tube (see (2), Fig. 116), until the surface of the liquid in the long arm is as far above that in the short arm as the height of mercury in the barometer. If the new volume of air in the short arm be measured, it will be found to be just one half the original volume. The new pressure is due both to the atmospheric pressure on the mercury at B and the column of mercury in the tube. It is, therefore, two atmospheres.

This experiment teaches us that by increasing the pressure upon a confined gas from one atmosphere to two atmospheres, the volume is caused to be only one half as great. This is only a special instance, however, of a more general law. If p is the pressure of a gas whose volume is v, then

under a pressure $2p$ the volume of the gas will be $\frac{1}{2}v$;
under a pressure $3p$ the volume of the gas will be $\frac{1}{3}v$;
under a pressure $\frac{1}{2}p$ the volume of the gas will be $2v$;
under a pressure $\frac{1}{3}p$ the volume of the gas will be $3v$, etc.

Now, since

$$pv = 2p \times \frac{1}{2}v = 3p \times \frac{1}{3}v = \frac{1}{2}p \times 2v = \frac{1}{3}p \times 3v,$$

we are able to state the relation as follows:

The product of the pressure and volume of a given mass of gas at a constant temperature is constant.

Where P is the pressure of the gas when its volume is V, and p the pressure when the volume is v, the law may be expressed algebraically in the forms:

$$\mathbf{PV = pv, \text{ or } P : p :: v : V.} \qquad (1)$$

EXAMPLE. — The observed volume of a gas is 30 cm.^{3} when the barometer reads 74.5 cm. What volume will it occupy under a barometric pressure of 76 cm.?

SOLUTION. — According to Boyle's Law the product of the pressure and volume under the first condition will be equal to their product under the second condition. Hence, if x represents the volume required, we have

$$74.5 \times 30 = 76 \times x.$$

Solving this equation for x, we obtain

$$x = 29.4 \text{ cm.}^3.$$

EXERCISES

1. If the volume of a certain gas is 200 cm.^{3} when its pressure is 1000 g. per square centimeter, what volume will it occupy when its pressure has been increased to 1200 g. per square centimeter?

2. The volume of an air bubble at a depth of 1 m. of mercury is 1 cm.^{3} What will be its volume when it reaches the surface if the barometer reading is 75 cm.?

SUGGESTION. — The first pressure is equal to the sum of the atmospheric pressure and that due to the mercury in which it is immersed, or 175 cm.

3. A gas is often confined in a tube, as shown in A, Fig. 117, whose open end is beneath the surface of some liquid. How much is the pressure of the gas confined in such a tube in a vessel of mercury when the surface in the tube is 25 cm. below the level of the liquid outside, the barometer reading 75 cm.?

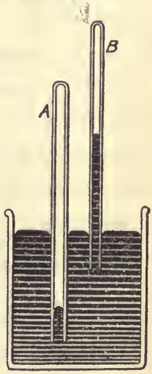


FIG. 117.

4. If the volume of the gas under the conditions given in Exer. 3 is 15 cm., what will be its volume if the tube is elevated until the surfaces are at the same level?

5. In B, Fig. 117, the surface of the mercury in the tube is 25 cm. above that of the mercury on the outside. If the atmospheric pressure is 75 cm., what is the pressure of the confined gas?

6. Under the conditions given in Exer. 5 the volume of the gas confined in the tube is 50 cm.^{3}. What volume will the gas occupy when the surfaces are brought to the same level?

7. The volume of an air bubble 136 cm. under water is 0.5 cm.^{3}. Barometer reading = 75.4 cm. Calculate (1) the pressure to which the bubble is subjected, and (2) the volume it will have as it emerges from the water.

SUGGESTION. — Reduce the depth of water to its equivalent in terms of mercury, and compute the first pressure as in Exer. 2.

8. To what depth would the inverted tumbler shown in Fig. 1 have to be taken in order to become half filled with water if the barometer reading is 75 cm.?

9. A gas tank whose capacity is 3.5 cu. ft. is filled with illuminating gas until the pressure is 225 lb. per square inch. How many cubic feet of gas at atmospheric pressure will be required in the filling of the tank? (Assume one atmosphere to be 15 lb. per square inch.)

10. Why is it safer to test an engine boiler by pumping in water rather than air?

150. Changes in Density. — Since a change in the volume of a given mass of gas occurs whenever the pressure is changed, it follows that there will also be a change in its density. Forcing the gas into one half its original volume doubles the amount in each cubic centimeter; making the volume one third multiplies the mass in each cubic centimeter by three; and so on. Therefore, doubling the pressure of a gas, thus reducing the volume one half, doubles the density; tripling the pressure multiplies the density by three; and so on. Hence, we may state these relations as follows:

The density of a gas at constant temperature is directly proportional to the pressure.

Expressed algebraically,

$$\mathbf{D} : \mathbf{d} :: \mathbf{P} : \mathbf{p}, \qquad (2)$$

where D is the density of the gas when the pressure is P, and d the density when the pressure is p.

EXERCISES

1. Hydrogen, whose density is 0.09 g. per liter under one atmosphere of pressure, is condensed in a steel cylinder until the pressure is 15 atmospheres. Calculate the density of the gas in the cylinder.

2. Illuminating gas is condensed in a reservoir until its density has increased from 0.75 g. per liter to 4.5 g. per liter. Calculate the pressure in the reservoir. Express the result in atmospheres.

3. If 4 liters of air at ordinary atmospheric pressure are admitted into a vacuum of 10 liters capacity, what will be the pressure and density of the air? (Under one atmosphere the density of air is 1.29 g. per liter.)

4. What is the weight of the quantity of illuminating gas condensed in a cylindrical tank of 3 cu. ft. capacity until the pressure is 225 lb. per square inch? (The density of the gas under one atmosphere of pressure is 0.75 g. per liter.)

5. If the gas shown in the tubes in Fig. 117, *A* and *B*, is air, what is the density of it under the conditions given in Exercises 3 and 5 on page 151?

4. ATMOSPHERIC DENSITY AND BUOYANCY

151. Atmospheric Density Changes with Altitude. — The air, unlike the water of the ocean, which is practically incompressible, diminishes in density as one ascends a mountain or rises in a balloon. As the pressure becomes less, the density of the air decreases proportionally. Thus at the summit of Mont Blanc in Switzerland, an altitude of three miles, the barometer indicates only one half as much atmospheric pressure as at the sea-level. Hence the density of the air at this altitude is only one half as great.

Aëronauts have succeeded in ascending to an altitude of about 7 miles, where the pressure is only 18 centimeters, or about a quarter of sea-level pressure. Greater altitudes have been explored by the aid of balloons equipped with self-registering instruments until a height of about 14 miles has been attained. Figure 118 shows the changes in the pressure and density of the atmosphere at various altitudes. The numbers at the left indicate altitudes in miles above sea-level, those at the extreme right the densities of the air compared with the density at sea-level, while the next column gives the barometric readings in inches. From the figure it may be seen that at an elevation of 15 miles, for example, the density of the air is only one thirtieth of

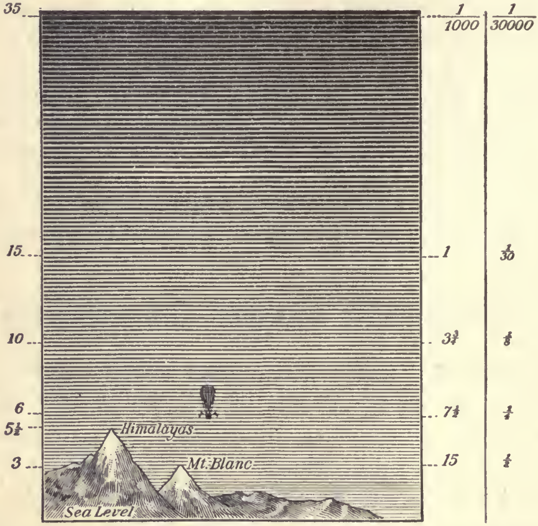


FIG. 118. — Showing the Decrease of Atmospheric Pressure with Altitude.

its sea-level density, while the barometer would read about one inch of mercury.

152. Buoyancy of Air. — On account of the fact that the pressure of the air decreases as the altitude increases, its pressure downward upon the top surface of a box, for example, is less than its upward pressure against the bottom. It follows, therefore, as for bodies immersed in water (§ 121), that an object is buoyed up by a force equal to the weight of the air that it displaces. In general, bodies are so heavy in comparison with the amount of air displaced that the consequent loss of weight is not taken into account. A man of average size, for instance, is buoyed up by a force equal to about 4 ounces. If, however, the air displaced by a body should weigh more than the body itself, it would be lifted by the air just as a piece of wood is lifted when immersed in water. This is the case of a balloon, in which the weight of the material composing the bag, together with the gas used to fill it, ropes, basket, ballast, etc., is less than that of the air which it displaces. In order that this may be so, it is necessary to inflate the balloon with a gas lighter than air, which weighs 1.29 kilograms per cubic meter. Hydrogen gas is sometimes used whose density is 0.09 kilogram per cubic meter, but more frequently the material is common illuminating gas weighing about 0.75 kilogram per cubic meter. The balloon *United States* which won the first international race at Paris in 1906 was filled with over 2000 cubic meters of illuminating gas, thus creating a lifting force of more than a ton.

EXERCISES

1. A balloon whose capacity is 1000 m.^{3} is filled with hydrogen. If the weight of the bag, basket, and ropes is 235 kg., what additional weight can the balloon lift?

SUGGESTION. — Find the difference between the weight of the air displaced by the balloon and the entire weight of the balloon and its equipment.

2. What will be the lifting capacity of the balloon in Exer. 1 when filled with illuminating gas?

3. When will a balloon cease to rise? When will it begin to fall?

4. A kilogram weight of brass (density 8.3 g. per cm.^{3}) will weigh how much in a vacuum?

SUGGESTION. — Add to the weight of the body the weight of the air that it displaces.

5. APPLICATIONS OF AIR PRESSURE

153. The Air Pump. — The air pump is used to remove the air or other gases from a closed vessel called a *receiver*. It was invented about 1650 by Otto von Guericke (1602–

1686), burgomaster of Magdeburg, Germany. A simple form of the air pump is shown in Fig. 119. *C* is a cylinder within which slides the tightly fitting piston ***P***. ***R*** is the receiver from which the air is to be exhausted. The receiver is connected with the cylinder by the tube ***T***. *s* and *t* are valves opening upward. The operation of the pump is as follows:

When the piston is pushed down, the valve *s* permits the air in the cylinder to escape, but closes to prevent its return when the piston is lifted. Raising the piston tends to produce a vacuum in the cylinder; but the air in the receiver and connecting tube expands, lifts the valve *t*, and fills the cylinder. Thus each down-and-up stroke of the piston results in the removal of a portion of the air in the receiver. After several strokes of the piston, the air in the receiver becomes rarefied to such an extent that its expansive force is no longer sufficient to lift valve *t*, and no further exhaustion can be produced.

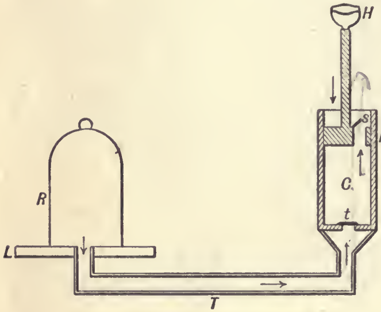


FIG. 119. — Air Pump.

Some air pumps are so constructed that the valves are opened and closed automatically at the proper moment so that a greater degree of rarefaction can be reached.

154. The Condensing Pump. — The condensing pump is used to compress illuminating gases in cylinders for use in lighting vehicles, stereopticons, etc., and further for inflating pneumatic tires, operating drills in mines, air-brakes on railway cars, and for many other purposes. The most common condensing pump is that used for inflating bicycle tires. Figure 120 shows the construction of such a pump. When the piston P is forced down, the air in the cylinder of the pump is driven into an air-tight rubber bag within the tire T. The small valve s opens to admit the air into the tire, but closes to prevent its return. On lifting the piston a partial vacuum is produced in the cylinder, and the air from outside finds its way into the cylinder past the soft cup-shaped piece of leather attached to the piston. During the downward stroke of the piston, however, this leather is pressed firmly against the sides of the cylinder and thus prevents the escape of the air. Repeated strokes of the piston add to the mass of air already in the tire, and the process of pumping may be continued until the tire is sufficiently inflated.

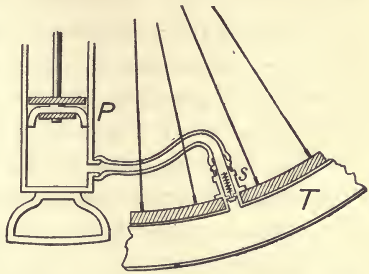


Fig. 120. — Inflating a Tire T by Means of a Condensing Pump P.

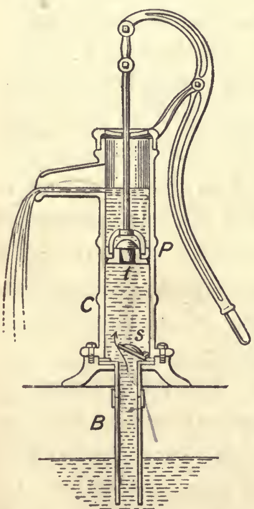


Fig. 121. — Common Lift Pump.

155. The Common Lift Pump. — The simplest pump for raising water from wells is the common lift pump. This consists of a barrel, or cylinder, C, Fig. 121, connected with a well or other source of water by a pipe B. The entrance of this pipe into the cylinder is covered by a valve s, opening upward. In the cylinder is a closely fitting piston P which can be raised or lowered by means of a rod which is usually connected with a lever for convenience in operating. The piston contains a valve *t*, also opening upward. The action of the pump is as follows:

When the piston is raised, the pressure of the air in the tube and lower part of the cylinder is diminished. The atmospheric pressure on the surface of the water in the well then forces water into the pipe. As the piston is lowered, valve *s* closes, and *t* allows the air in the cylinder to escape, but closes again when the piston begins to ascend. The second stroke again reduces the pressure below the piston, and water is forced still higher in the pipe. At last the water reaches the piston, passes through during the downward stroke, and is lifted toward the spout of the pump when the piston moves upward.

It will be seen that the action of the common lift pump is dependent on the pressure of the atmosphere to elevate the water to a point just high enough to come within reach of the piston; *the piston must not be farther above the water in the well than the height of the water column that the air can support.* When water is to be lifted from a well more than 33 or 34 feet in depth, the cylinder is placed low enough to enable the piston to move within that distance of the surface of the water.

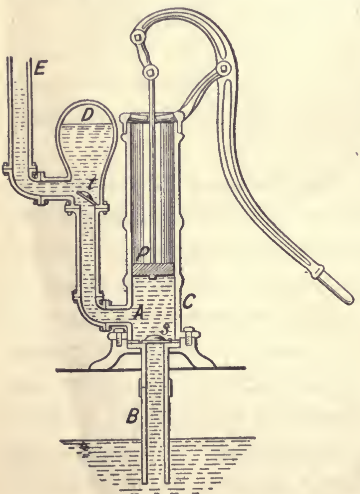


FIG. 122. — A Force Pump.

156. The Force Pump. — The force pump is used to deliver water under considerable pressure either for spraying purposes or in order to elevate it to a reservoir placed some distance above the level of the pump. It is made in many different forms. Usually the force pump has no opening or valve in the piston. The water escapes from the cylinder through a side opening *A*, Fig. 122, past the valve *t*, thence upward to the spout or reservoir. In other respects the description of the action of the lift pump applies equally well to the force pump. In order to obtain a steady stream of water, a force pump is often provided with an air chamber *D*. The entrance of the water through *t* serves to compress the air in the chamber, which by its expansive force maintains the current in pipe *E* while the piston is moving upward.

157. The Siphon. — The siphon is a bent tube with unequal arms. It is used for removing liquids from tanks or reservoirs that have no outlet, or for drawing off the liquid from a vessel without disturbing a sediment lying upon the bottom.

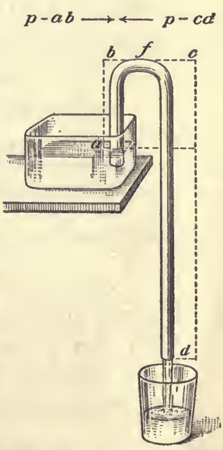


Fig. 123. — The Siphon.

Let a glass tube bent in the form shown in Fig. 123 be filled with water, and the ends closed while the tube is inverted and placed in the position shown. On opening the ends water will flow through the tube from the vessel in which the liquid has the higher level.

The action of the siphon is explained as follows: the upward pressure at *a* due to the atmosphere is only partly counterbalanced by the pressure of the liquid column *ab*. If the atmospheric pressure is *p*, the resultant force acting toward the right is $p - ab$. Again, the upward pressure of the atmosphere at *d* is opposed by the pressure due to the liquid column *cd*. Hence the resultant pressure acting toward the left is $p - cd$.

Since the pressure due to the liquid in the long arm exceeds that of the liquid in the short arm, the excess of force in the tube tends to move the water toward the lower vessel. The resultant of all the forces is equal to the difference of pressure due to the liquids in the two arms, *i.e.* to a column of liquid equal to $cd - ab$. The liquid will therefore continue to flow until this difference is 0; or, in other words, until the liquid reaches the same level on the two sides. A second condition necessary to the action of a siphon is that *for water the height of the bend* f *must not be greater than* 33 *or* 34 *feet above* a, since its elevation to the point f is due to the atmospheric pressure.

158. Work done by Compressed Gases. — Since a gas exerts a force against the sides of the vessel containing it, it is obvious that work will be done by the gas if the side of the vessel is made movable and the gas allowed to expand (§ 55). Imagine air to be compressed in the cylinder C shown in Fig. 124, in which P represents a movable piston. If the external resistance offered to the piston is not as great as the force exerted by the air inside, work will be done by the expanding gas, and the piston will move. If the pressure remains constant during the process, the work done will be measured by the product of the *force* exerted by the air against the piston and the *displacement* produced.

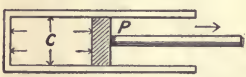


FIG. 124. — An Expanding Gas in C does Work on Piston P.

EXAMPLE. — A tube leads from a cylinder to a reservoir where air is stored under a pressure of 5000 g. per cm.^{2}; the area of the piston is 50 cm.^{2}. Find the work done by the gas when the piston moves 20 cm.

SOLUTION. — The total force exerted by the gas against the piston is 50×5000, or 250,000 g. Hence the work done is $250{,}000 \times 20$, or 5,000,000 gram-centimeters.

Since energy can be transferred from place to place in a compressed gas, air under great pressure is often conducted through pipes over long distances from a condensing apparatus (§ 154) to a point where the energy is to be utilized. In this way a locomotive engineer is able to control a train by means of compressed air conducted from the engine, where it is compressed, to suitable apparatus in connection with the brakes under each car (§ 159). In mining operations and stone quarrying, pneumatic machines utilize compressed air for drilling the holes in rocks where explosives used in blasting are to be placed. Many other applications of the power of compressed gases have found a place in modern engineering.

159. The Air Brake. — Compressed air is widely used by the railroads of many countries in the operation of the Westinghouse air brake, Fig. 125, which works as follows: — The locomotive is provided with a condensing pump which keeps the air in a large reservoir at a pressure of about 80 pounds per square inch. From this reservoir the compressed air is conducted through the train pipe P to an auxiliary reservoir R placed under each car. As long as the pressure is maintained in P, air is allowed to enter R and at

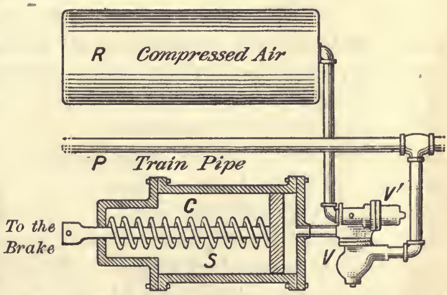


FIG. 125. — The Westinghouse Air Brake.

the same time is prevented from entering the cylinder C by a complicated automatic valve V, and the brakes are held "off" by the spring S. If, however, the engineer by moving a lever in the cab, allows the pressure in the pipe P to fall, the passage between R and P is at once closed by the valve V, and the compressed air in R is admitted into C. The pressure of the air forces the piston to the left and sets the brakes against the wheels. By admitting air from the reservoir on the engine into the pipe P, the valve V again establishes a communication between P and R and allows the air in C to escape. The spring S forces the piston back and releases the brakes. The great advantage of this brake is that in case of any accidental breaking of the train pipe P, the brakes are automatically set. They are often arranged to be operated from any coach in case of emergency.

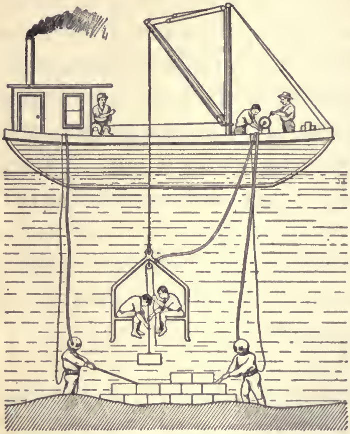


FIG. 126. — Apparatus Used for Work under Water.

160. Subaqueous Operations. — Important use is made of compressed air in various engineering operations performed under water, as laying the foundations of bridges, excavating for tunnels, recovering the cargoes from sunken vessels, etc. Figure 126 shows the most important apparatus for subaqueous work, the diving bell, and the submarine diver. A condensing pump on board the vessel forces air through a tube into the bell, thus supplying the workman with oxygen and preventing the rise of water in the working chamber. Air is supplied to the workman in diving armor in the same manner. The foul air escapes from the diving suit through a valve above the chest. In many cases the air is supplied from a reservoir carried on the back of the diver.

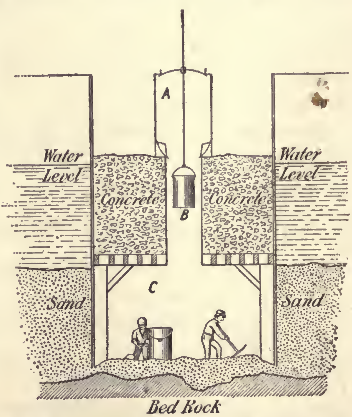


FIG. 127. — Excavating below the Water Level by the Aid of Compressed Air.

Deep excavations are frequently made and foundations built up from bed rock by the aid of the pneumatic caisson (pronounced *kās′son*). See Fig. 127. The working chamber C is gradually lowered through soft soil by removing the earth from within after loading the roof of the chamber with heavy masonry above. When the caisson sinks below water level, air under suitable pressure is forced into *C* to prevent the influx of water. The excavated earth is removed, and the foundation material introduced through the air lock *A*. The bucket is lowered into *A*, after which the opening around the wire cable is closed air-tight. Air is then allowed to flow into *A* until the pressure there is equal to that in *C*. The semicircular doors separating *A* and *C* are now thrown open, and the bucket lowered into the caisson. As the caisson is forced more and more below water level, the pressure of the air is correspondingly increased in order to prevent the water and mud from crowding in.

EXERCISES

1. If the pressure against the 8-inch piston of an air brake is 75 lb. per square inch, how much force drives the piston forward?

2. When a train is broken in two, the cars are brought quickly to rest. Explain.

3. What are the advantages gained by the use of air brakes?

4. A diver sinks 68 ft. below the surface of water. Under how many atmospheres is he working? Explain why his body is not crushed by this force.

5. A caisson is sunk until the bottom is 51 ft. below water level. Under what pressure must the laborers work?

6. Explain how a bucket of earth is removed from a caisson through the air lock.

SUMMARY

1. The laws of pressure relative to liquids are equally applicable to gases, except that the pressure is not proportional to the depth (§§ 135 and 136).

2. The density of the air under standard conditions of pressure and temperature (*i.e.* 76 cm. and 0° C.) is 1.293 g. per liter or about 1.25 oz. per cubic foot (§ 137).

3. The pressure of the atmosphere is measured by the barometer. At sea level the average height of the mercurial column supported by the air is 76 cm. or very nearly 30 in. Expressed in units of force, the average sea-level pressure is 1033.6 g. per square centimeter or 14.7 lb. per square inch. The atmospheric pressure decreases with the altitude, but not proportionally (§§ 139 to 146).

4. Gases adapt themselves to the form and capacity of the vessels containing them. The expansive force of a gas confined in a receptacle tends to prevent the collapse of the vessel under atmospheric pressure (§ 147).

5. The product of the pressure and volume of a given mass of a gas at a constant temperature is constant. This is known as *Boyle's Law*, or Mariotte's Law (§ 149).

6. The density of a gas at a constant temperature is directly proportional to the pressure to which it is subjected (§ 150).

7. A body in air is buoyed up by a force equal to the weight of the air that it displaces (§ 152).

8. The air pump is used in the rarefaction of gases. The condensing pump is used in compressing air and other gases (§§ 153 and 154).

9. The action of "suction" pumps is dependent upon the pressure of the atmosphere on the water in the well or cistern (§§ 155 and 156).

10. The siphon is a bent tube having unequal arms used for conveying a liquid over the side of a reservoir to a lower level than that in the reservoir. Its action is dependent on atmospheric pressure (§ 157).

11. Compressed air possesses energy. This energy is used in many important mechanical devices, as the air brake, rock drills, etc. (§ 158).

CHAPTER IX

SOUND: ITS NATURE AND PROPAGATION

1. ORIGIN AND TRANSMISSION OF SOUND

161. Cause of Sound. — Whenever the sensation of sound is traced to its external cause, we find that its source is always something which is in a state of vibration. Sometimes the vibration of the body emitting the sound is sufficiently great to be visible, *i.e.* to give a certain blurred indistinctness to the outline of the body. This is easily perceived when a stretched wire is plucked or a tuning fork is sounded.

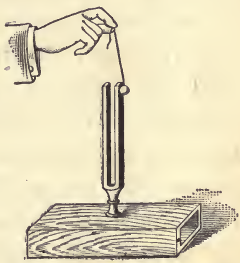


Fig. 128. — Demonstrating the Motion in a Sounding Tuning Fork.

Let a small pith ball suspended on a thread be allowed to touch a tuning fork that is emitting a sound. (See Fig. 128.) It will be thrown violently away. Touch one of the prongs lightly to the water in a tumbler. A ripple is produced, or perhaps a spray is thrown from the prong. Again, attach a fine wire or bristle to the prong of a tuning fork with a small quantity of sealing wax. Sound the fork, and draw the bristle across a piece of smoked glass. A wavy line, Fig. 129, results, showing the existence of a *back-and-forth* motion in the fork.

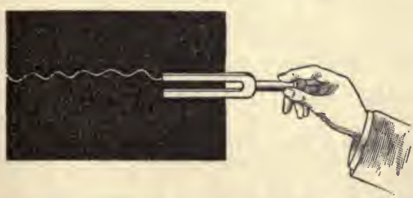


Fig. 129. — The Motion in the Prong of a Fork is Vibratory.

162. The Nature of a Vibration. — The vibration of a pendulum has been studied in § 80. While sounding bodies vibrate in a manner similar to that of the pendulum, they vibrate from a different cause. When a pendulum is drawn aside and then set free, a component of the force of gravity moves the pendulum bob back toward its original position at the center of its arc. When, however, we pluck the string of a guitar, for example, or set the prongs of a tuning fork in vibration, the motion is due to the elasticity of the material of which the body is composed. The string, on being drawn aside, is stretched slightly. Now, on being released, its tendency to resume its original length causes it to straighten. When we set a tuning fork in vibration, the prongs are bent. Since they are made of elastic steel, they immediately tend to resume their original shape. Hence the prongs move back to their initial positions. Again, the string of the guitar, like the pendulum, is in the state of motion when it reaches its original position. Hence it must continue to move until some resistance checks it. Therefore it swings beyond this position to a point where its velocity is zero, whence it returns in the same manner as before. The phenomenon is repeated by the string until, like the pendulum again, its energy is expended in overcoming the resistance of the air and the friction of its own molecules.

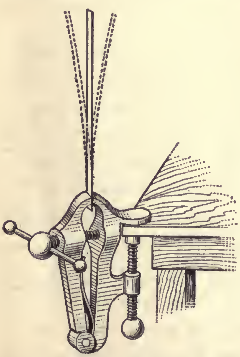


FIG. 130.—Thin Strip of Wood in Vibration.

Clamp a thin strip of wood, as a yardstick, by one end, Fig. 130, and set it in vibration. As it is being drawn aside, its tendency to move back toward its original position is very apparent. Let the stick move very slowly back to the original position, and it will stop there; but if it is set entirely free, it will continue to move until it is some distance beyond the center. Why? Attach a weight of 100 or 200 grams near the free end of the stick, and it will be found to vibrate much more slowly than before. The force due to the elasticity of the wood cannot bring the increased mass so quickly to the center nor stop it so quickly when the center is passed on account of the increased inertia. Make a comparison between the vibrating yardstick and the sounding tuning fork. The yardstick does not vibrate with sufficient rapidity to produce sound.

163. Transmission of Sounds to the Ear. — Sounds reach the ear through the air as the transmitting medium. Many other substances may, however, be the means of propagation, as the following experiments will show:

1. Let the ear be held against one end of a long bar of wood, and let the shank of a small vibrating tuning fork be brought against the other end. A loud sound will be heard. The scratch of a pin at one end can easily be heard at the other.

2. Place the shank of a tuning fork in a hole bored in a large cork. Set a tumbler full of water upon a resonance box, and bring the cork in contact with the surface of the water. If the fork is in vibration, the water will transmit the motion to the box, as shown by the increased intensity of the tone. Again, as most boys have found experimentally, if the ear be held beneath the surface when two stones are struck together under water, a loud sound results even at some distance from the stones.

164. Medium Necessary for Propagation of Sound. — That a sounding body cannot be heard without the presence of some transmitting medium may be shown by the aid of the air pump.

Let an electric bell be placed upon a thick pad of felt or cotton, suspended by flexible wire springs under the receiver of an air pump, as shown in Fig. 131. Make the connection with a battery, so that the bell can be rung from the outside. Set the bell ringing, and begin to exhaust the air from the receiver. The sounds coming from the bell become less and less distinct until the greatest possible exhaustion has been produced. If now the air is slowly

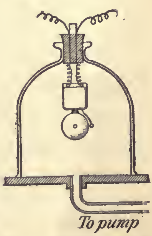


Fig. 131. — Bell Ringing in a Partial Vacuum.

admitted into the receiver, the loudness of the sound increases until its full intensity is reached.

Although the sound of the bell used in this experiment will never become entirely inaudible, mainly on account of the transmission of sound by the vibration of the supporting wires, we are given reason to believe that with complete exhaustion and the removal of all other transmitting media, no sound would be heard.

165. Velocity of Sound Transmission. — Every one is familiar with the fact that it requires time for sound to travel over a given distance. If we watch a locomotive from a distant point as the engineer blows the whistle, we first observe the jet of steam as it issues from the whistle, and a few seconds later we hear the sound. The interval of time that often elapses between a lightning flash and the peal of thunder is further evidence that the velocity of sound is not exceedingly great.

Many investigators during the nineteenth century gave their attention to the accurate determination of the velocity of sound. For the most part their experiments consisted in measuring the interval of time between the flash of a gun and its report heard at some distant station of observation. The result obtained by Regnault, a French physicist, gives sound a velocity, in air, of 1085 feet per second at the temperature of freezing water. This velocity is equivalent to 331 meters per second. At higher temperatures the velocity is somewhat greater, the increase being 2 feet, or 0.6 meter, per second for each degree centigrade.

166. Velocity of Sound in Various Mediums. — The velocity of sound in solids has been the subject of many investigations. The accepted value found for iron is about 5100 meters per second at 20° C. The velocity of sound in wood depends greatly upon the kind of wood.

The average value, however, is approximately 4000 meters per second.

The most exact measurement of the velocity of sound in water was made in 1827 by Colladon and Sturm in Lake Geneva, Switzerland. Two observers stationed themselves in boats at opposite sides of the lake. At one of the stations a bell was sounded beneath the water and a gun fired on deck at precisely the same instant. The sound was received by means of a large ear trumpet held under water at the other station. The time intervening between the stroke of the bell and the report transmitted by the water could thus be ascertained and the velocity computed. The results of many observations gave an average of 1400 meters per second as the velocity of sound through water.

EXERCISES

1. The flash of a gun is seen 3.5 seconds before the report is heard. If the temperature is 20° C., what is the distance between the observer and the gun?

2. A locomotive whistle was sounded 3 mi. from an observer. If the temperature of the air was 10° C., how long was the sound in traversing the distance?

3. The distance between two stations is 12 mi. If the interval of time between the flash and the report of a gun was found by experiment to be 56 seconds, what was the speed of the sound?

4. A bullet was fired at a target 500 m. away, and in 3 seconds was heard by the gunner to strike. The temperature of the air being 20° C., what was the velocity of the bullet?

5. When one end of an iron pipe is struck a blow with a hammer, an observer at the other end hears two sounds, one transmitted by the iron, the other by the air. If the pipe is 1500 m. long, and the temperature 25° C., what is the interval of time between the two sounds?

2. NATURE OF SOUND

167. Sound a Wave Motion. — We have seen in § 164 that a medium is essential for the transmission of sound, but thus far no explanation has been given of the manner in which this transmission takes place. Sounds continue to come from an electric bell even though we cover it tightly with a glass jar, but it is very plain that nothing, *i.e.* no material thing, can pass through the glass. The whole process will become clear, however, if we consider that *a sound is transmitted through the air and glass in the form of waves.* Hence, the further study of *Sound* will be a study of waves and wave motion.

168. Two Kinds of Waves. — We are all familiar with the waves that move over a surface of water. We have only to observe a small boat as it rises upon the crests and sinks down into the troughs to realize that it is *not carried along* by the wave. Again, a wave is often seen to pass over a field of grain. While the wave moves rapidly across the field, each spear of grain simply bends with the pressure of the wind and then rises again. The following experiment may be used to show this manner of wave propagation:

Let one end of a soft cotton clothesline about 25 feet long be attached to a hook in the wall, while the other end is held in the hand. Give the end of the rope a quick up-and-down motion, and a wave will be seen to run along the rope from one end to the other.

In the experiment just described it is clear that the forward motion of the waves produced in the rope is at right angles to the direction of the motion of the particles composing the rope, as shown in Fig. 132. On this account such waves are called **transverse waves.**

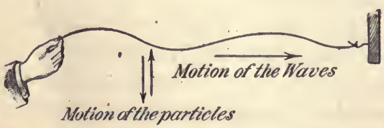


FIG. 132. — Transverse Waves in a Rope.

A second kind of wave motion takes place in bodies that are elastic and compressible, as in gases, wire springs, etc. We are able to make a study of waves of this kind by letting a coil of wire, Fig. 133, represent the medium through which such waves are transmitted:

Let us imagine a blow is given the spring at A that quickly compresses a few turns of the spiral near the end. On account of the elasticity and inertia of these parts of the spring, they will move forward slightly and compress those just ahead. These in turn will compress the coils still farther along, and thus a pulse, or wave, is carried along the spring from A to B.

FIG. 133. — Waves Transmitted by a Spring. The Individual Turns Move back and forth while the Waves Progress along the Spring.

Again let the end of the spring at A be given a very sudden pull. The turns near the end will be drawn apart for an instant, but the adjacent turns will be drawn toward A, *one after another*, until the end B receives the impulse. A blow against A is transmitted by the spring to the point B, and a sudden pull at A is likewise transmitted as a pull against the fastening at B. Waves of this kind, in which the motion of the parts of the medium are parallel to the direction of propagation, are called **longitudinal waves.**

169. Transmission of a Sound Wave. — The manner in which a sound is transmitted by the air becomes clear when we compare the process with that which occurs when a wave is transmitted by a spiral spring.

Imagine a light spring, (1), Fig. 134, to be attached at one end to one of the prongs of a vibrating tuning fork F and at the opposite end to a diaphragm G. Each vibration of the fork will alternately compress and separate the spirals of the spring near the end. These pulses will be transmitted by the spring in the manner described in § 168, and will cause the diaphragm at G to execute as many vibrations per second as the tuning fork, and the diaphragm will give out a sound corresponding to that emitted by the fork.

Let the air take the place of the spring and the ear *E* replace the diaphragm. When the prong of the vibrating fork, (2), Fig. 134, moves to the right, a compression (*c*) of the air is produced in front of it. This compression, or **condensation**, moves to the right with the velocity of sound, or at about the rate of about 1120 feet per second. When the prong of the vibrating fork moves to the left, the air just at the right is rarefied, and the adjacent portions of the air move in to fill this **rarefaction** (*r*), which

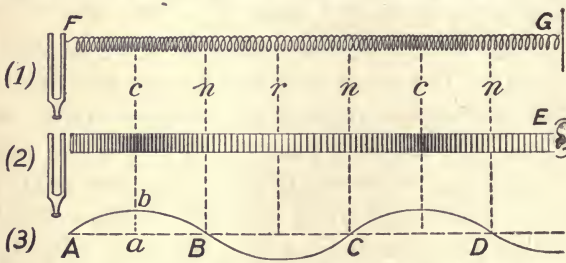


Fig. 134.—Illustrating the Corresponding Parts of Transverse and Longitudinal Waves.

travels to the right immediately following the condensation. Condensation and rarefaction thus follow one another as long as the fork continues to vibrate, and the drum of the ear at *E* receives as many pulses per second as the tuning fork emits. It is therefore caused to vibrate as many times per second as the fork. Between *c* and *r* is *n*, which marks the region where there is neither condensation nor rarefaction.

The curve in (3), Fig. 134, is drawn to show the parts of a transverse wave which is sometimes used to represent diagrammatically waves of other kinds. The **crest** *AB* corresponds to the *condensation*, or compression, of the longitudinal waves shown in (1) and (2). The **trough**

BC corresponds to the *rarefaction.* The distance *ab* represents the distance that each particle in the wave moves from its original position and is called the **amplitude** of the wave. A **wave** includes a complete crest and trough, or a condensation and a rarefaction. The distance between the corresponding points on any two adjacent waves, as *AC*, *BD*, etc., is the **wave length.**

170. Velocity, Wave Length, and Vibration Frequency. — Imagine a tuning fork whose rate of vibration, or *vibration frequency*, is 256 per second. When the fork is set in vibration, it sends out 256 complete longitudinal waves during each second. At the completion of the 256th vibration, the first wave has progressed a distance numerically equal to the velocity of sound, or about 1120 feet. This space, therefore, contains 256 waves, and the length of each wave can be found by dividing 1120 feet by 256. Hence the length of each wave is about 4.4 feet. Letting n be the frequency, v the velocity of sound, and l the wave length, we have

$$\mathbf{l} = \frac{\mathbf{v}}{\mathbf{n}}, \text{ or } \mathbf{nl} = \mathbf{v}. \tag{1}$$

3. INTENSITY OF SOUND

171. Sound Waves Spread in all Directions. — If a bell be struck, the sound is heard as readily in one direction as another if no obstruction intervenes and other conditions are equally favorable. Obviously a wave emitted by the bell spreads out in all directions from it. Figure 135 illustrates this fact. Each condensation has the form of a hollow spherical shell that

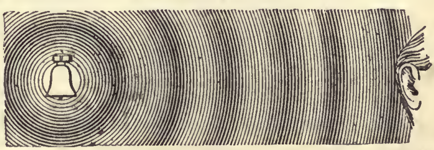


FIG. 135.—Sound Waves Spread in all Directions from their Source.

continually enlarges as the wave advances. This is followed by a rarefaction of a similar form. It is plain that the energy imparted by the bell to a single wave is carried from the source by particles of air composing a hollow spherical shell. At a given distance from the bell this shell will have a certain area which at twice the distance will be four times as great, since the area of a sphere is directly as the *square* of its radius. Thus at twice the distance from the bell the energy will be imparted to four times as many particles, hence the energy of each will be one fourth as great. At three times the original distance, the energy will be imparted to nine times as many particles, and the energy of each will be one ninth as much. Hence, *the intensity of sound is inversely proportional to the square of the distance measured from the source of the sound.*

172. Intensity and Amplitude. — The energy imparted to each wave by a sounding body will depend upon the intensity of the vibration of that body, *i.e.* upon the *amplitude* of vibration. An increase in the amplitude of vibration of the sounding body produces a corresponding increase in the amplitude of each wave emitted. Thus each wave is caused to carry away from the sounding body an increased quantity of energy. When the waves fall upon the tympanum of the ear, a corresponding increase in its amplitude of vibration is produced.

Let one of the prongs of a tuning fork be struck lightly against a soft pad or cork. A tone is emitted that is scarcely audible. Now let the fork be struck more forcibly. On account of the greater amplitude of vibration the intensity of the tone will be much greater than before.

Like the pendulum (§§ 79 and 80), a sounding body completes its vibrations in equal times, the period of vibration being practically independent of the amplitude (*i.e. isochronous*). Therefore when the fork is struck forcibly against the pad, it still performs the same number of vibrations per second as before.

173. Intensity and Density of the Medium. — The experiment with the bell and air pump (§ 164) shows that *the intensity of a sound depends upon the density of the medium in which the sound is produced.* As the air in the receiver becomes rarer, the intensity of the sound grows less.

Support two similar bell jars with the mouths opening downward. Keep one filled with illuminating gas and the other with air. Set an electric bell ringing, and place it first in the jar containing gas, then in the one filled with air. It will be readily observed that the sounds emitted in the rarer medium, the illuminating gas, are less intense than those produced in the jar filled with air.

At the top of a high mountain a gun makes only a small report when fired. At high elevations explorers and aëronauts converse with difficulty. The denser the gas, the greater the energy imparted to each wave by the vibrating body. For this reason a sounding body will cease to vibrate sooner in the denser of two media.

174. Intensity and Area. — When the area of a sounding body is small, as that of a small tuning fork, for example, the condensations and rarefactions produced are not well marked. It is obvious that of two vibrating tuning forks of the same frequency, the smaller will have the effect of cutting through the air without producing more than a slight compression, while the one of larger area will compress a greater volume of air, and thus give out more of its energy to each wave emitted.

Set a tuning fork in vibration, and hold the shank against the panel of the door or table. A loud sound will be heard coming from the large area that is set in vibration by the fork.

In the construction of many musical instruments use is made of this relation between the intensity of sound and the area of the sounding body. Thus a piano utilizes two or three wires to produce a given tone when the area of a single wire is not sufficiently great. Again, a sounding board of large area is often placed beneath the strings of instruments to form a large surface of vibration when the strings are excited.

4. REFLECTION OF SOUND

175. Reflection of Sound Waves. — It is a fact frequently illustrated in nature that sound waves may be reflected. As long as waves proceeding from a source of sound pass through a homogeneous medium, no reflection takes place; but when the density of the medium is disturbed, the waves suffer partial or total reflection.

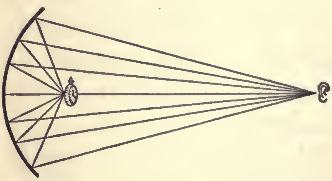


Fig. 136. — Sound Reflected by a Concave Surface.

Let a watch be placed a few inches in front of a concave reflector, as shown in Fig. 136. By moving the ear from point to point a place may be found, sometimes several feet from the reflector, where the sound of the watch may be distinctly heard.

The experiment may be varied by setting a similar reflector at a distance of several feet from the first and facing it, as shown in Fig. 137. If the watch is placed slightly nearer the reflector than in the preceding experiment, a point may be found a few inches in front of the second at which the sound is focused. This point, or *focus*, may be found by using an ear trumpet made by attaching a piece of rubber tubing to a glass funnel. When the open funnel is placed at the focus of sound, a distinct ticking of the watch will be heard by holding the end of the rubber tubing in the ear.

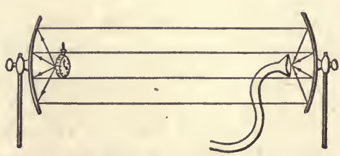


Fig. 137. — Sound Undergoes Two Reflections from Concave Surfaces.

176. Echoes. — The familiar phenomenon of echoes is due to the reflection of sound. When one speaks in a room of moderate size, the waves reflected from the walls reach the ear so quickly that they combine with the direct waves and produce an increase of the intensity of the sound. But when the walls are one hundred feet or more from the speaker, he hears a distinct echo of each syllable he utters. Sometimes a reflecting surface is so distant that several seconds may intervene before an echo returns. Between two reflecting surfaces the echoes sent back and forth are often remarkable. It is related that at a point in Oxford County, England, an echo repeats a sound from fifteen to twenty times. Extraordinary echoes are also found to occur between the walls of deep cañons.

In so-called "whispering galleries" we have illustrated the formation of a sound focus due to curved surfaces. In the crypt of the Panthéon in Paris there is a place where the slight clapping of hands at one point gives rise to sounds of great intensity at another. In the Mormon Tabernacle at Salt Lake City, Utah, a whisper near one end can be distinctly heard at the other, so perfect is the reflection from the ellipsoidal surfaces of the walls.

EXERCISES

1. A hunter fires a gun and hears the echo in 5 seconds. How far away is the reflecting surface, the temperature being 20° C.?

2. How far is a person from the wall of a building, if, on speaking a syllable, he hears the echo in 4 seconds, the temperature being 15° C.?

3. A gun is fired, and the echo is returned to the gunner from a cliff 250 ft. away in 4.5 seconds. Calculate the velocity of sound.

SUMMARY

1. Sounds are produced by bodies in a state of rapid vibration (§ 161).

2. Sounding bodies vibrate as a consequence of the elasticity and inertia of the material composing them (§ 162).

3. Sounds are transmitted from their sources by solids, liquids, and gases. The air is the propagating medium in most cases (§§ 163 and 164).

4. The velocity of sound waves in air at the freezing temperature is 1085 ft. (331 m.) per second. The velocity is increased 2 ft. (0.6 m.) per second for each degree centigrade. In wood and iron the velocity of sound is respectively 3 and 4 (approximately) times the velocity in air (§§ 165 and 166).

5. Sound is a wave motion. Wave motion may be either *transverse* or *longitudinal* (§§ 167 and 168).

6. The energy of a sounding body is transmitted by longitudinal waves. The parts of such waves are called *condensations* and *rarefactions*. The corresponding parts of transverse waves are *crests* and *troughs* (§ 169).

7. The relation between velocity, wave length, and number of vibrations per second is given by the equation $v = ln$ (§ 170).

8. The intensity of a sound depends on the *amplitude* and *area* of the sounding body, the *density* of the medium where the sound is produced, and is inversely proportional to the square of the *distance* from its source (§§ 171 to 174).

9. *Echoes* are sounds reflected from walls, woods, hills, cliffs, etc. (§§ 175 and 176).

CHAPTER X

SOUND: WAVE FREQUENCY AND WAVE FORM

1. PITCH OF TONES

177. Musical Sounds and Noises. — In order that a sound may be pleasing to the ear, it is essential that the vibrations be made in precisely equal intervals of time; in other words, the vibrations must be isochronous. Any device that produces isochronous pulses emits a **musical** sound. If the pulses are not isochronous, the result is a **noise.**

Rotate on a whirling table a metal or cardboard disk provided with two or more circular rows of holes, Fig. 138. Let the holes in one row be equidistant, while those in the other rows are placed at irregular intervals. Blow a stream of air through a rubber tube against the row of equidistant holes, and a pleasant musical sound will result. Now direct the stream against another row of holes, and it will be observed that the sound produced is of an unpleasant character.

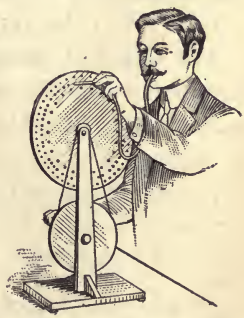


Fig. 138. — Musical Sounds Produced by Isochronous Pulses in the Air.

178. Pitch. — Probably the most striking difference in musical sounds is in respect to that which we call **pitch,** a term applied to the degree of highness or lowness of a sound. We are accustomed to sounds varying in pitch from the low, rumbling thunder to the shrill, piercing creak of small animals and insects. How this wide difference in sounds is brought about may be shown by the following experiment:

Rotate a metal or cardboard disk in which there are several circular rows of holes perforated at regular intervals, but having a different number in each row. See Fig. 139. Keeping the speed of rotation constant, blow a stream of air forcibly against one row and then another. A distinct difference in pitch will be observed. Again, keeping the stream of air directed against one of the rows, change the speed of rotation from very slow to very fast. The pitch will rise from a low tone to one that is very shrill.

Fig. 139. — The Siren Disk.

These facts teach that *the pitch of a tone depends upon the number of wave pulses per second sent from a sounding body to the ear.* The stream of air blown against the disk is alternately transmitted and interrupted by the motion of the holes. The succession of pulses thus produced constitutes the musical tone that is heard. The vibration frequency of the tone emitted may be found by multiplying the number of revolutions of the disk per second by the number of holes in the circle. An instrument of the kind used in the experiment is called a *siren.*

179. Musical Scales Produced by a Siren. — The most common series of tones of different pitches is the **musical scale.** It is possible with the help of a siren designed in a manner similar to the one shown in Fig. 139 to determine the relation of the several tones that comprise the ordinary musical scale. The construction and use of the siren is as follows:

On a circular disk of cardboard or metal, about 10 inches in diameter, draw eight concentric circles about $\frac{1}{2}$ inch apart. Upon the circles thus drawn, beginning with the smallest, drill the following numbers of equidistant holes: 24, 27, 30, 32, 36, 40, 45, and 48. Mount the disk upon a whirling table and rotate with a uniform speed. Beginning with the smallest circle, blow a stream of air against each row of holes in succession. The tones produced will be recognized at once as those belonging to the major scale. Increase the speed of rotation, and again direct the current of air against the several rows in succession. Although the pitches are higher than before, yet the scale is produced as perfectly as at first.

The experiment teaches that the major scale is a series of tones whose vibration frequencies have the same relation as the numbers 24, 27, 30, 32, 36, 40, 45, and 48. To these tones we give the names *do*, *re*, *mi*, *fa*, *sol*, *la*, *ti*, *do*. These tones form the foundation upon which has been built up our musical system.

180. The Major Diatonic Scale. — Any series of tones whose vibration frequencies bear the relations given in the preceding section constitute a **major diatonic scale.** The first tone of such a series is the **key tone.** The various scales in use are named according to their key tones; for example, *the scale of C*, *the scale of G*, etc. Physicists assign to the tone called *middle C*, 256 vibrations per second.[1] Hence the second tone of the major scale of C must be produced by $256 \times \frac{27}{24}$, or 288 vibrations per second. This tone is called D. The next tone, or E, must have $256 \times \frac{30}{24}$, or 320 vibrations per second, etc. The following table, Fig. 140, shows the manner in which these tones are expressed on the musical staff, their *relative* and *absolute vibration numbers*, and the *vibration ratios.*

Scale of C	Staff								
	Letters	C	D	E	F	G	A	B	C′
	Absolute Vib. No.	256	288	320	341.3	384	426.6	480	512
For all Scales	Tone Number	1	2	3	4	5	6	7	8
	Syllables	*do*	*re*	*mi*	*fa*	*sol*	*la*	*ti*	*do*
	Relative Vib. No.	24	27	30	32	36	40	45	48
	Vibration Ratios	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

FIG. 140. The Major Scale.

The nature of the seven definite intervals leading from C to C', or one octave, may be seen from the table. Start-

[1] The international standard of pitch in this country and Europe is based upon $A = 435$ vibrations per second.

ing with $C = 256$ vibrations, the vibration numbers of the successive tones of the scale, as shown by the experiment in the preceding section, bear the ratios $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{15}{8}$, and 2 to the vibration number of C, the key tone. *These ratios are the same for all major scales, no matter what the key tone may be.*

181. Intervals. — *A* **musical interval** *refers to the relation between the pitches of two tones.* The experiment with the siren in § 179 shows that the value of an interval depends upon the *ratio*, not the *difference*, between the vibration numbers of the tones; for example, when this ratio is $\frac{48}{24}$, or $2:1$, the interval is an *octave*, as the interval between C' and C. Other important intervals are the *sixth*, *i.e.* the interval between the first tone and the sixth of a major scale, given by the ratio $\frac{40}{24}$, or $5:3$; the *fifth*, $\frac{36}{24}$, or $3:2$; the *fourth*, $\frac{32}{24}$, or $4:3$; the *major third*, $\frac{30}{24}$, or $5:4$; and the *minor third*, $\frac{36}{30}$, or $6:5$. In order to determine the interval between two tones, the ratio of their vibration frequencies is first computed and then compared with the ratios given for the several cases. Hence two tones of 500 and 300 vibrations per second respectively are a sixth apart, because the ratio of their vibration numbers is $5:3$. If these two tones were sounded together or in succession, the interval would be recognized at once by a musician.

182. The Major Chord. — A careful inspection of the vibration numbers of the tones of the major scale will show the presence of three groups consisting of three tones each, whose frequencies bear the ratios $4:5:6$. Such a group of tones is a **major chord.** Beginning with $C = 256$ vibrations per second, we have for the first chord C, E, and G (*do-mi-sol*) whose vibration frequencies are 256, 320, and 384. This chord is called the *tonic triad* of the scale of C. The second of these chords, called the *subdominant triad*, is formed in the same manner and includes

F, *A*, and *C′* (*fa-la-do*). The third is formed likewise by making use of the tones *G*, *P*, and *D′* (*sol-ti-re*) and is called the *dominant triad*. Since these triads include all the tones of the major scale, it may be said that this scale is founded upon these three major chords. The following table shows these relations:

Tonic Triad	4 : 5 : 6 :: 256 : 320 : 384, ***C***, ***E***, *and* ***G***.
Subdominant Triad	4 : 5 : 6 :: 341 : 427 : 512, ***F***, ***A***, *and* ***C′***.
Dominant Triad	4 : 5 : 6 :: 384 : 480 : 576, ***G***, ***B***, *and* ***D′***.

183. Sharps and Flats.—The introduction of the black keys on the organ or piano keyboard is (1) for the purpose of accommodating the instrument to the range of the voice in the case of songs and (2) to give variety to selections designed for instrumental performance. That it is necessary to insert additional tones becomes apparent at once when we consider the tones that are required to form a major scale beginning upon B_1, just below middle *C*, having 240 vibrations per second. The keys that are used in the scale of *C* are all white keys, Fig. 141, and have the frequencies indicated. In the major scale of *B* the vibration frequencies must be successively 240, 270, 300, 320, 360, 400, 450, and 480. It will be observed that the only white keys that satisfy this scale are *E* = 320 and *B* = 480 vibrations per second. Since the number 270 lies about midway between the frequencies of *C* and *D*, the black key *C*♯ (read "*C* sharp") is introduced. Others must be placed between *D* and *E*, *F* and *G*, *G* and *A*, and *A* and *B*. These are called respectively *D*♯, *F*♯, *G*♯, and *A*♯.

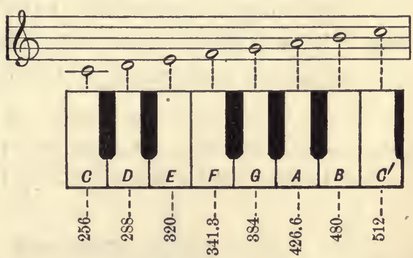


FIG. 141.—Illustrating the Scale of *C* on Staff and Keyboard.

These four tones are also called $E\flat$, $G\flat$, A, and B, and are read "*E* flat," "*G* flat," etc.

184. Tempered Scales. — In the preceding section is shown the necessity for introducing additional tones in order that a piano, for example, may be used to produce the major scale of *B*. These new vibration numbers, however, will not satisfy scales which begin on other key tones, for every change to another key tone increases the demand for small changes in the vibration numbers of the tones. The difficulty is surmounted by sacrificing the perfect, simple ratios found in §§ 180 to 183 and substituting others that are sufficiently near to satisfy a musical ear. See the table given below. This method of tuning an instrument is called **tempering**, and the scales derived by the process are called **tempered scales.** *The desired result, which is to permit the execution of musical compositions written in any key, is secured by making the interval between any two adjacent tones the same throughout the length of the keyboard.* By this process the octave is divided into twelve precisely equal intervals called **half steps.** The imperfection introduced by equal temperament tuning is illustrated by the following table:

	C	*D*	*E*	*F*	*G*	*A*	*B*	*C*
Perfect scale of *C*	256.0	288.0	320.0	341.3	384.0	426.6	480.0	512.0
Tempered scale	256.0	287.3	322.5	341.7	383.6	430.5	483.3	512.0

EXERCISES

1. Taking *G* as the key tone of a major scale, compute the vibration frequency of each of the tones contained in an octave.

2. The vibration frequency of a tone is 264. Calculate the frequencies of its third, fourth, and octave.

3. Calculate the vibration frequency of the tone one octave below middle *C*.

4. What is the wave length of a tone whose vibration frequency is 256 when the temperature is 15° C.? (See §§ 165 and 170.)

5. If middle C were given the frequency 260, what would be the frequencies of D, A, and B?

6. A tone two octaves above middle C has how many vibrations per second?

7. If the keyboard of a piano extends three and a half octaves in each direction from middle C, calculate the vibration number of the lowest and the highest C on the instrument.

8. Ascertain the interval between the pitches of two tuning forks making 256 and 192 vibrations per second.

9. The tones of three bells form a major chord. One makes 200 vibrations per second, and its pitch lies between the pitches of the others. Calculate the frequency of the bells.

10. A tone is produced by a siren revolving 300 times per minute. What is the name of the tone produced, if the number of holes in the row is 24?

2. RESONANCE

185. Sympathetic Vibrations. — Vibrations that are produced in one body by another near by which has the same vibration period are called **sympathetic vibrations.** The following experiments will serve to illustrate the case:

1. Tune two wires of a sonometer, Fig. 150, so that they emit tones of the same pitch when plucked. Place a Λ-shaped paper rider astride one of the wires and then pluck the other. The rider will be thrown off, and if the wire that was plucked be stopped, a tone will be heard coming from the other. Throw the wires slightly out of unison and repeat the experiment. Only very slight vibrations will be imparted to the second wire.

2. Let two mounted tuning forks having the same pitch be placed near together, as shown in Fig. 142. Set one of the forks in vibration by bowing it, or by striking it with a rubber stopper attached to a wooden or glass handle. On checking its vibrations a sound will be heard coming from the second fork.

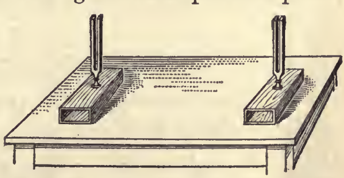


Fig. 142. — Sympathetic Vibrations between Forks in Unison.

Students will find it interesting to experiment in a similar manner with a piano. For example, let the key C be depressed so carefully that no tone is produced. The damper is thus lifted from the wires of a certain vibration frequency. Now if the tone *C* be sounded by a voice, the corresponding tone will be heard coming from the instrument.

These experiments show the facility with which a body is put in vibration by another having the same frequency. The principle may be stated as follows:

Sound waves sent out from one vibrating body impart vibrations to others, provided the vibration frequencies are the same.

186. Sympathetic Vibrations Explained. — The phenomenon of sympathetic vibrations is readily accounted for when we consider that the body first set in vibration sends out periodic impulses through the air or the wood connecting the two bodies. The first impulse received by the second body gives it a very small amplitude of vibration. It returns at precisely the instant to receive the second impulse, and the amplitude of the second vibration is caused to be greater than that of the first. The third impulse, in like manner, adds energy to that already transmitted to the body. Thus, after the arrival of several impulses the amplitude of vibration of the second body is great enough for the production of an audible tone.

FIG. 143. — Reënforcement of a Sound by an Air Column of Suitable Length.

187. Resonance. — A vibrating tuning fork when held in the hand produces a sound that is scarcely audible. Its tone may be augmented, however, by placing it above a properly adjusted air column that is caused to vibrate sympathetically with it.

Let a vibrating tuning fork be held over a tall cylindrical jar, as shown in Fig. 143. By pouring water into the jar while the fork is in this position, a condition will be reached such that an intense sound may be heard proceeding from the jar. Pouring in more water destroys the effect.

Let the experiment be repeated with a fork of a different pitch. If the pitch is higher, the air column will need to be shortened by pouring in more water; if lower, the column must be longer.

188. Resonance Explained. — Let *a* and *b*, Fig. 144, represent the two extreme positions of the prong of a vibrating tuning fork. Just as the prong begins a downward swing from *a* it starts a condensation downward in the tube. When the prong begins its upward swing from *b*, it starts a condensation in the air above it. Now, in order to have a reënforcement of the tone emitted, the condensation started downward in the tube must be reflected by the water and return in time to unite with the condensation produced above the prong as it moves upward. Hence the condensation must travel down the tube and back while the prong of the fork moves from *a* to *b*, *i.e.* during one half a vibration of the fork. In a similar manner the rarefaction started in the tube as the prong leaves *b* must return from the bottom of the tube in time to unite with the rarefaction produced above the prong.

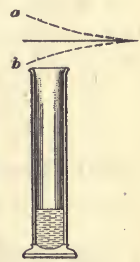


FIG. 144. — The Cause of Resonance.

189. Resonance Tubes and Wave Length. — In the explanation of the phenomenon of resonance given in the preceding section, it is clear that the waves in the air of the resonance tube travel twice the length of the air space within the tube while the fork is making one half a vibration. The wave therefore goes four times the length of the air column during a complete vibration of the fork. But a wave progresses one wave length during a complete vibration of a sounding body. Hence the length of the vibrating air column is one fourth of the length of the air wave produced by the fork.

Experiments performed with tubes of different sizes have shown that the diameter of the tube influences the length of tube required for the best effect. It has been found that the length of the air column must be increased by two fifths of its diameter in order to equal a quarter of a wave length.

Tuning forks are often mounted on properly adjusted boxes whose air spaces act as resonators when the forks are sounded, as shown in Fig. 142.

3. WAVE INTERFERENCE AND BEATS

190. Destruction of One Wave by Another. — Let us imagine two trains of sound waves, (1) and (2), Fig. 145. Let the waves in the two trains be of the same length, *i.e.* produced by sounding bodies of the same pitch. If the condensations in one train at a given instant are at c, c and those in the other train at c', c', it is obvious that the two trains cannot move through the same air simultaneously. For, if the compressions are equal, the condensations of the first train will unite with the rarefactions r', r' of the second, and the result will be no change in the density of the air. In other words, one train of waves will destroy the other. Thus two sounds may combine and produce silence. A study of the analogous case of transverse waves will help to make the matter clearer.

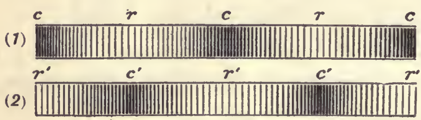


FIG. 145. — Two Longitudinal Waves in the Condition Necessary for Interference.

Let (1) and (2), Fig. 146, represent two trains of transverse waves. Let their lengths and amplitudes be equal.

If, now, the crests *c*, *c*, *c* of one train unite with the troughs *t*, *t*, *t* of the other, then one train will obviously destroy the other, since the crests of the first train will just fill the troughs of the second and *vice versa*. Thus two trains of water waves, for example, may combine and produce a level surface.

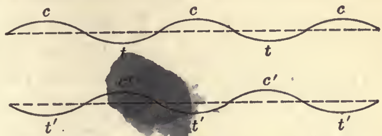


Fig. 146. — The Interference of Transverse Waves.

The destruction of one wave train by another similar train is called **interference.**

191. An Example of Interference. — Let *f*, *f*, Fig. 147, represent the ends of the two prongs of a tuning fork. At the moment the prongs vibrate toward each other a condensation is produced in the region *a* and rarefactions at *b*, *b*. When the prongs move outward, a rarefaction is produced at *a* and condensations in the regions *b*, *b*. Now since a condensation starts from the region *a* at the instant a rarefaction starts from *b*, and *vice versa*, there will be places in the air around the tuning fork where the parts of the waves coming from *a* will unite with the unlike parts of those from *b*, and continuous interference will result. The regions of interference near the prongs of a fork are shown by the dotted lines in the figure. These places may be found by rotating a tuning fork held near the ear. Positions in which the sound becomes almost inaudible can thus be easily located.

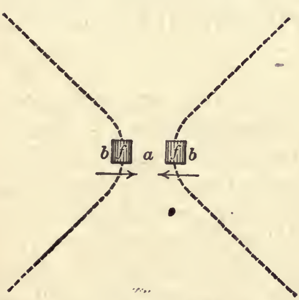


Fig. 147. — Regions of Silence near a Tuning Fork.

Hold a vibrating tuning fork above a resonance tube tuned to reënforce the tone emitted. Rotate the fork slowly, and a position will be found, Fig. 148, in which the sound becomes practically inaudible; for, in this position, the mouth of the jar receives the unlike parts of waves sent out from the opposite sides of the nearer prong of the fork.

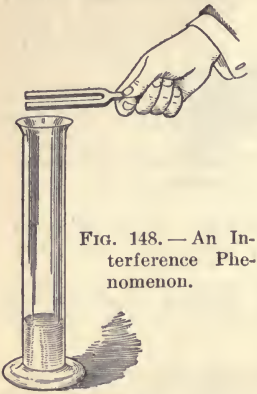


Fig. 148. — An Interference Phenomenon.

192. Alternate Interference and Reënforcement. — When it is understood how two trains of sound waves may combine and produce an intensified sound (§ 188) or interfere and produce silence (§ 190) the interesting phenomenon of **beats** is readily explained. Beats are always produced when two tones that differ slightly in pitch are sounded at the same time. The effect is obtained as follows:

Tune two resonance jars to reënforce two tuning forks of the same pitch. Load the prongs of one of the forks with pieces of tin bent in such a form as to cling firmly to the fork while it is in vibration. This will make the pitch of this fork slightly lower than that of the other. Now sound the two forks simultaneously and hold them over the resonance tubes. Fluctuations in the intensity of the sound (*i.e. beats*) will be distinguishable at a distance of several meters. Load the prongs of the fork more heavily, and more rapid beats will be observed.

Since the forks used in the experiment differ in pitch, the waves sent out will differ in length, — the longer waves being produced by the fork with the weighted prongs. Since the pitches differ but a small amount, the wave lengths will be almost equal. Inasmuch as the vibrations of the weighted fork are continually falling behind those of the other, at certain periods the two forks will be producing condensations and rarefactions simultaneously; hence reënforcement results. A little later the weighted fork will have fallen one half a vibration behind the other, and while one fork is producing a condensation, the other will be sending out a rarefaction. Hence, at this instant, there is interference between the two trains of waves.

Figure 149 shows portions of the two trains of waves sent out from the forks. It will be observed that at *r*, *r*

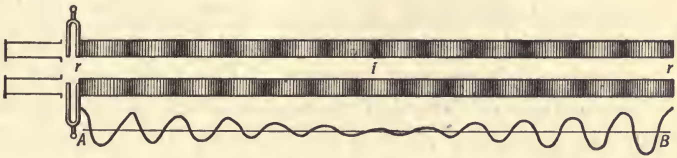


FIG. 149.—The Production of Beats Illustrated.

the two trains unite to produce the greatest reënforcement of the sound, while at *i* unlike portions of the waves unite and thus destroy the sound. The resultant wave is represented by the line *AB*.

193. Law of Beats.—Let one tuning fork make 100 vibrations per second, and another, 101. At a given instant imagine each fork to be sending out a condensation. Hence, at this instant, reënforcement takes place. Just one half of a second later, one fork has made 50 complete vibrations, and the other $50\frac{1}{2}$. Therefore, at this instant one fork is producing a rarefaction, and the other a condensation, and thus the resulting sound is weakened. Hence, the intensity of the sound will increase and decrease once per second. This effect constitutes one beat. When the difference in vibration frequency is two, the phenomenon occurs twice per second. In every case *the number of beats produced per second is equal to the difference between the vibration frequencies of the two sounding bodies.*

4. THE VIBRATION OF STRINGS

194. The Pitch of Strings. — Many musical instruments employ vibrating strings or wires on account of the fact that such bodies emit tones of a rich quality. The wide range of pitch necessary is obtained by varying the *length*, *tension*, and *mass* of the strings used. Thus the lengths of the wires of a piano, for example, vary from those that are about 2 inches long to others whose lengths are 4 or 5 feet. The short wires are of a small diameter, while the long ones are large and massive. The violin makes use of four strings of different masses, and the performer obtains the necessary pitches by fingering the strings so as to allow the proper length to be put in vibration.

195. Law of Length. — The laws governing the vibration of strings may be shown by means of an instrument called

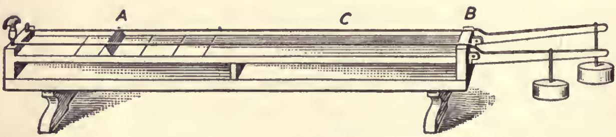


FIG. 150. — A Sonometer.

the *sonometer* (pronounced *so nŏm′e ter*), Fig. 150. The relation between the pitch of a string and its length may be shown by the following experiment:

Adjust the tensions of the two similar wires of a sonometer until they emit tones of the same pitch. Set up a bridge A and place a second bridge under the middle of one of the wires and pluck first one wire and then the other. It will be observed that the tones produced are an octave apart; *i.e.* one half the original length of the wire produces a pitch of twice (§ 180) as many vibrations per second. Another test may be made by placing a bridge at C just two thirds of the distance AB from A. By plucking both wires as before the interval *do-sol* is easily recognized.

This experiment may be extended to the production of any interval that can be recognized. In any case it will be found that the ratio of the length of the vibrating portion AC to the original length AB is the inverse of the vibration ratio given in § 180. Thus one half the length produces a pitch of twice as many vibrations per second, and two thirds of the original length emits a tone of three halves of the vibration frequency. Hence, *the vibration frequencies of strings or wires are inversely proportional to their lengths.*

196. Law of Tensions. — Let the two weights used to produce the tensions in the wires of a sonometer be moved as far out as possible on the lever arms. Set the bridges so that the wires emit tones of the same pitch. Now move one weight back, thus shortening the lever arm (§ 93) until it produces only one fourth as much tension on this wire as before. The pitch will now be found to be an octave lower when the two wires are plucked.

This experiment may be applied to other intervals; for example, four ninths of the original tension will cause the vibration frequencies to have the ratio of 2 : 3. Hence, *the vibration frequencies of strings or wires are directly proportional to the square roots of their tensions.*

197. Law of Masses. — Let a sonometer be equipped with two wires of the same material, lengths, and tensions, but of different sizes. If possible, make the diameter of one twice that of the other. Since the masses are as the squares of the diameters, the mass of a given length of the larger will be four times that of the smaller. If now the wires are plucked, the pitch of the smaller will be an octave above that of the other.

The vibration frequencies of strings or wires are inversely proportional to the square roots of their masses per unit length.

198. Vibration of a String in Parts. — The vibration of a stretched string or wire is more complicated than at first appears. When a string is bowed or plucked, the tone

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emitted is a compound tone produced by the string vibrating as a whole *simultaneously* with its vibration in parts. The readiness with which a string vibrates in parts may be shown by the following method:

Attach one end of a white silk cord about 1 m. in length to one of the prongs of a large tuning fork attached firmly to the table and whose frequency is not more than 100. Let the other end pass over

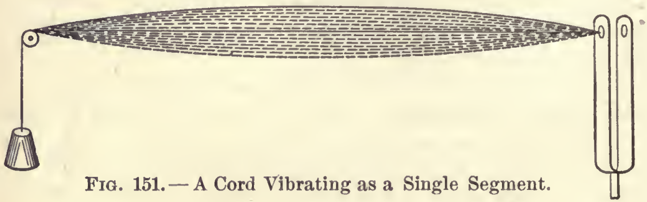

Fig. 151. — A Cord Vibrating as a Single Segment.

a smooth hook or pulley and support a weight, as in Fig. 151. Set the fork in vibration and adjust the tension and length of the cord until it vibrates as a single segment, as shown. Now reduce the weight

Fig. 152. — The Vibration of a Cord in Two Parts.

used until the cord vibrates in two parts, Fig. 152, when the fork is set in motion. Under suitable tensions the cord will vibrate in any desired number of equal parts up to six or seven.

The tone emitted when a string vibrates as a whole is called its **fundamental.** The fundamental is the lowest tone that a string can produce. By the *pitch of a string* is meant the pitch of its fundamental. The vibrating portions of the string are called **loops,** or **segments,** and a point where the amplitude of vibration is zero is called a **node.** Loops are often called *ventral segments* and *antinodes.*

199. Overtones of Strings. — The character of the tone produced by a vibrating string is complicated by the division of the string into equal parts, as shown in the preceding section. Each vibrating segment of a string produces a tone that is higher in pitch than the fundamental. *The tones emitted by the vibrating portions of any sounding body are called* **overtones,** *or* **partial** *tones.*

The presence of overtones emitted when a wire is plucked may be detected by the sympathetic vibrations set up in a neighboring wire.

Let the two wires of a sonometer be tuned in unison. Place a bridge under the center of one wire and set Λ-shaped paper riders near the middle of each half of this wire. If the longer wire is now plucked near one end, the shorter wires will be thrown into vibration, as the riders will show. Replace the riders, and pluck the longer wire at the center. Only feeble vibrations are now set up in the shorter wires. Again, place the bridge under one wire just one third of the original length from one end, and place a rider astride the shorter portion. Now if we pluck the longer wire near one end as before, the rider will be thrown off as in the former case. Replace the rider and pluck the longer wire one third of its length from one end. The rider will remain stationary.

In this experiment we have used the shorter wire to show when certain overtones are present in the tone emitted by the longer wire. The results indicate that when a wire is plucked near one end, the tone produced contains an overtone an octave higher than the fundamental. The wire not only vibrates as a whole, but divides into two parts, each part having a frequency double that of the fundamental. This condition is illustrated in Fig. 153. When the wire is plucked near the center, this overtone is not present, since a node would be required at this point.

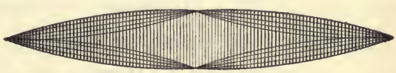

FIG. 153. — A Wire Emitting its Fundamental and First Overtone.

200. A Series of Overtones. — The second part of the experiment in the preceding section shows that the wire divides into three equal parts, each of which vibrates with three times the frequency of the fundamental. In a similar manner it is possible to detect the presence of still higher overtones, all of which are produced simultaneously when the string is plucked or struck near one end. The following table shows the positions and frequencies of the overtones produced when a middle *C* string is plucked near one end.

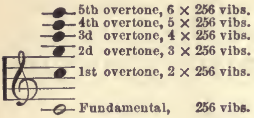

FIG. 154. — Table Showing the Overtones Produced on a Middle *C* String.

When the vibration frequency of an overtone is 2, 3, 4, 5, etc., times that of the fundamental, it is called an **harmonic**. The relative intensity of the various overtones produced by a wire depends chiefly upon the manner in which it is set in motion, the point where it is struck or plucked, and the rigidity, density, and elasticity of the wire.

5. QUALITY OF SOUNDS

201. Overtones and Tone Quality. — It is a familiar fact that two tones of the same pitch and intensity do not necessarily sound alike. The tones produced by a violin, for example, are readily distinguished from those of the piano or flute. The tones of one violin may differ from those of another. This difference is one of **quality**. The cause of this difference between tones was long a matter of study and investigation. It was finally explained fully by the German physicist Helmholtz[1] (1821–1894). He tells us that *the quality of a sound depends upon the overtones produced by the sounding body and their relative intensities.*

Let a wire be plucked at one end and the tone compared with that emitted when the wire is plucked in the middle. Again, let the wire be struck with a soft rubber hammer and then with something hard. The tones produced will be of the same pitch but of different quality. This last experiment may also be made with either a bell or a tuning fork.

[1] See portrait facing page 196.

HERMANN VON HELMHOLTZ (1821–1894)

A new era in the history of Sound was created in 1862 by the publication of Helmholtz's *Lehre von den Tonempfindungen,* a work which has been translated into English under the title *Sensations of Tones.* The author recognizes musical tones as periodic motions of the air and distinguishes the three characteristics of tones as intensity, pitch, and quality. He finds that tone quality is due to the number and relative intensity of the upper partials, or overtones. Helmholtz devised hollow spherical resonators of a variety of sizes by the aid of which he was able to analyze the human voice and other musical tones. By means of a large series of tuning forks of different pitches which were operated electrically, he produced tones of such a composition as to imitate the vowel sounds of the human voice, the tones of organ pipes, etc.

Helmholtz received a medical education at Berlin, and from 1855 to 1871 was professor of physiology at Bonn and Heidelberg. He was led to the study of physics in his endeavor to understand the principles involved in the eye and ear. Later he became an accomplished mathematician. In 1871 Helmholtz was appointed professor of phyics at Berlin, and in 1888 became the first director of the well-known *Reichsanstalt* (Imperial Physico-Technical Institute) in Charlottenburg. His death occurred in 1894.

The first contribution of Helmholtz to the science of physics is his famous treatise on the *Conservation of Energy* published in 1847, a publication which was of great influence in establishing this doctrine. He is known throughout the medical world as the inventor of the ophthalmoscope, an instrument used in examining the interior of the eye.

These experiments teach that the quality of the sound produced by a body depends upon the manner in which the body is set in motion. As shown in § 199, the sound given out by a wire is rich in overtones when plucked near one end; but when plucked at the center, all the overtones that require a node at that point are absent. When a tuning fork is struck with a hard substance, high ringing overtones are plainly heard, while the fundamental is extremely weak. A very different quality is produced, however, by bowing the fork or striking it against a soft pad.

202. Stringed Instruments. — Among the most common musical instruments which depend on the vibration of strings and wires are the piano, violin, guitar, mandolin, and banjo. The piano consists of a series of tightly stretched wires, which in the lower octaves are very massive and vary in length from three to five feet. The shortest wires of the instrument are often not more than two inches in length. When a key is struck, the motion is transmitted through a system of levers to a padded hammer which strikes the wires tuned to give the corresponding pitch. The tone emitted is greatly intensified by the vibrations produced in the sounding-board. When the key is released, a padded damper falls against the vibrating wires, thus checking the motion. A pressure of the foot upon the right-hand pedal of the instrument raises all the dampers and thus leaves the vibration of every wire unchecked. Changes of temperature and humidity of the atmosphere gradually put the wires out of tune. The necessary tuning is accomplished by a suitable adjustment of the tension of each wire.

Instruments of the violin type which are played with a bow, as well as the guitar, mandolin, and banjo, derive their musical tones from properly tuned wires or strings whose lengths may be changed at will by the performer. The violinist by long practice learns the exact position at which to press the strings against the finger board to produce the desired tones. By this method any pitch from that of the lowest string to the one derived from the shortest possible portion of the string of highest fundamental can be produced. The difficulty is reduced in the case of the guitar, mandolin, and banjo by the slightly raised *frets*, or bridges, placed across the finger board to indicate the position of the finger. These instruments owe their wide difference in tone quality (1) to the nature of the string, (2) to the manner in which the strings are put in vibration, and (3) to the size, shape, and material of their sounding-boards.

EXERCISES

1. How are the different pitches produced which are necessary in rendering a selection on a guitar, harp, mandolin, or cello?

2. The bridges under a stretched wire are 4 ft. apart. Where must a third bridge be placed to raise the pitch a major third? a minor third?

SUGGESTION. — See § 181 for the vibration ratios for these intervals.

3. A wire 180 cm. long produces middle *C*. Show by a diagram where a bridge would have to be placed to cause the string to emit each tone of the major scale of *C*.

4. What is the vibration frequency of the tone three octaves above middle *C*? What length of the wire in Exer. 3 would be required to produce this tone?

5. The tension of a string is 9 kg. What tension must it have in order to produce a tone an octave higher? an octave lower? a fifth higher?

6. Write the vibration frequencies of the tones one, two, and three octaves both above and below middle *C*.

7. Write the vibration frequencies of the first four overtones of a *G* string. Name these tones.

8. The density of steel is 7.8, and that of brass 8.7. What is the vibration frequency of a steel wire, if that of a brass wire of equal length, diameter, and tension is 280?

SUGGESTION. — Let x be the vibration frequency of the steel wire and then write the proportion based on the law of masses, § 197.

9. The vibration frequency of two equal wires 155 cm. long is 300. How many beats per second will be heard when one of the wires has been shortened 5 cm.?

10. Two middle *C* forks were placed near together and the prongs of one of them weighted with bits of sealing wax; when both forks were sounded, 4 beats per second were heard. Find the frequency of the weighted fork.

6. VIBRATING AIR COLUMNS

203. Organ Pipes. — The pipe organ, flute, clarinet, cornet, etc., employ resonant air columns for the production of musical tones. The pitch of the tones emitted by such columns depends mainly upon the length of the column. The relation between the length of a column and its pitch is shown by the following experiments:

1. Measure the length of various organ pipes that are available for experimental purposes. Compute the ratio of the length of each pipe to the length of pipe giving the key tone of a major scale. This ratio will be found to be the inverse of the vibration ratio for the corresponding tone of the scale as given in § 180.

2. Prepare a set of glass or metal tubes about 1 centimeter in diameter of the lengths 10, 15, 20, and 30 centimeters. Leaving the tubes open at both ends, blow across one end of the tube 10 centimeters long in such a manner as to produce an audible tone. Even if the sound is not loud, its pitch can usually be recognized and sung by members of the class. Again, blow across the end of the 20-centimeter tube, and compare its pitch with that of the first tube. The interval between the two tones will be practically an octave, the shorter tube having the higher pitch. Verify this relation by using the tubes whose lengths are 15 and 30 centimeters. In a similar manner ascertain the interval between the pitches of the tones emitted by the tubes whose lengths are 20 and 30 centimeters. This interval will be about a fifth.

Organ pipes are either *open* or *stopped*. A stopped pipe is formed by closing one end of a tube. The experiment teaches that when the length ratio of two open pipes is 1:2, the vibration ratio is 2:1; and when the former ratio is 2:3, the latter is 3:2. Hence, *the vibration frequencies of open pipes are inversely proportional to their lengths.*

204. Open and Stopped Pipes. — Close an end of one of the tubes used in Experiment 2 of the preceding section, and cause it to emit its tone as before. Produce also the tone of the pipe when both ends are open, and compare the pitches. The pitch of the closed tube is an octave lower than that of the open one. This result may be verified by using any of the tubes. If an organ pipe is available, blow it first while open and then while closed tightly at one end. The best results are obtained by blowing it moderately.

The pitch of a closed pipe is an octave lower than that of an open one of the same length.

205. Mechanism of an Organ Pipe. — Sectional views of open organ pipes made of metal and wood are shown in Fig. 155. Air is blown from a wind chest into the small chamber *c* and flows in a thin stream through the narrow orifice *i* against the lip *a*. At *a* the air is set in vibration, and this in turn puts the air contained in the tube in vibration. Since the pitch of the pipe is determined by its length, it can reënforce only those vibrations at the lip *a*, whose vibration frequency corresponds to its own. The tube may be regarded as a resonator for reënforcing a tone of a particular pitch. A vibrating tuning fork whose pitch is the same as that of the tube would also set the air column in vibration when held near the open end.

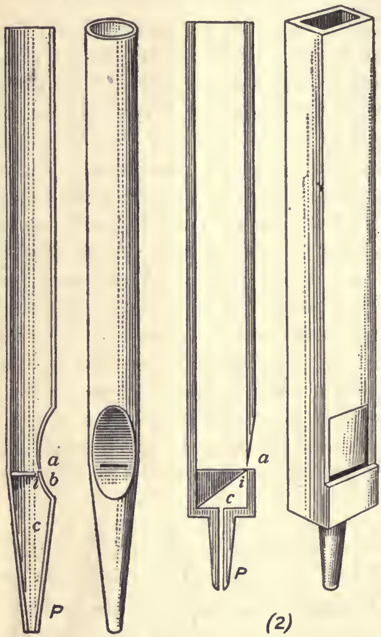

FIG. 155. — Open Organ Pipes. (1) A Metal Pipe. (2) A Wooden Pipe.

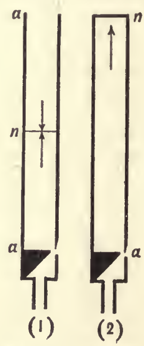

FIG. 156. — Nodes in Open and Stopped Pipes.

206. Nodes in Vibrating Air Columns. — When an open pipe is yielding its fundamental, or lowest tone, a node *n*, (1), Fig. 156, is formed at the center. The arrows indicate that the vibratory motion of the air on opposite sides of a nodal point is always in opposite directions. At the open ends of the tube *aa* the air is free to vibrate and the density of the air remains practically unchanged; hence these points are often called *antinodes* or *loops*. Since the distance from a node to an antinode is always equal to a quarter of a wave length (§ 189), the length of the entire wave is four times *na*, or twice the length of the tube.

The case of stopped pipes is different. A node is always formed at the closed end of the tube *n*, (2), Fig. 156, and an antinode near *a*. The wave length is four times the distance *an*, or four times the length of the tube. It thus becomes clear why the fundamental of a closed pipe is an octave below that of the open one.

207. Overtones Produced by Organ Pipes. — By blowing an organ pipe forcibly, tones of a higher pitch than the fundamental will be produced. This is also true of the tubes used in Experiment 2, § 203. Just as in the case of vibrating strings (§ 198), the vibrating body, which is the air column in this instance, divides into parts according to existing conditions. It is by the vibration of these parts that the higher tones, or *overtones*, are produced. Figure 157 shows the position of the nodes and antinodes when the pipe is sounding its first and second overtones. In (1) it is shown that the tube contains four quarter waves; hence the wave length is equal to the length of the tube. Similarly for the second overtone (2), the division of the vibrating air column forms six quarter waves. Now since the vibration frequency increases as the wave length decreases, the first overtone has twice the frequency of the fundamental; the second, three times the frequency; and so on. Thus the overtones of an open pipe are the same

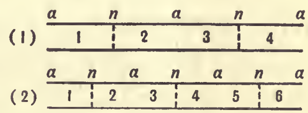

FIG. 157. — Overtones of Open Pipes.

as those of vibrating strings (§ 200), and are therefore *harmonics*.

The division of the air column in stopped pipes is otherwise. As we have already seen, a node is always formed at the closed end and an antinode at the open end. Hence, in the production of the fundamental tone, only one quarter wave is contained in the pipe. But when the first overtone is emitted, a node must again be formed at the closed end n, and an antinode at the open end. Evidently, if only one other node is to exist it must be at n' (1), Fig. 158, one third of the length of the pipe from the open end. The pipe then contains three quarter waves. Similarly, when yielding its second and third overtones, the column must divide into five and seven quarter waves respectively in order to retain a node at the closed end and an antinode at the open end. This is shown for the second overtone in (2). Hence in a stopped organ pipe only those overtones can be produced whose frequencies are *odd multiples* of the vibration frequency of the fundamental.

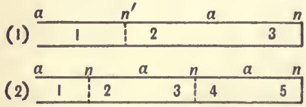

Fig. 158. — Overtones of Closed Pipes.

208. Wind Instruments. — Under this title are classed all instruments whose tones are emitted by air columns. The most important instrument of this kind is the *pipe organ*. The wide range of pitch is derived from organ pipes (see Fig. 155) of different lengths. A great variety in the quality of tones is secured by the use of pipes of

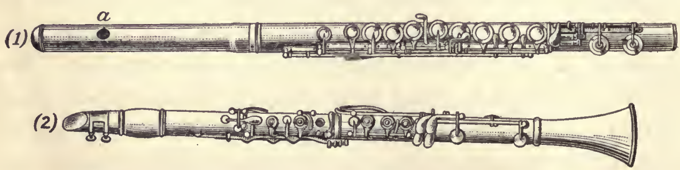

Fig. 159. — (1) The Flute. (2) The Clarinet.

different kinds, as open and closed wooden pipes, metal pipes, reed pipes, etc.

The *flute*, (1), Fig. 159, corresponds to an open organ pipe whose length can be changed by the aid of openings along the side controlled by the fingers. The air in the instrument is set in vibration by blowing a current from the lips forcibly across the opening at a.

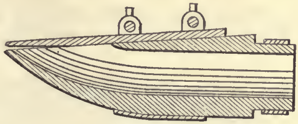

FIG. 160. — Mouthpiece of the Clarinet.

The *clarinet*, (2), Fig. 159, is provided with a mouthpiece, Fig. 160, which contains a reed or tongue of light, flexible wood, which alternately opens and closes the aperture. The action of the instrument resembles that of the so-called "squawker," made of the stem of the dandelion or by cutting a tongue on the side of a quill. The current of air blown by the lungs causes the reed to close the opening momentarily. There is thus started through the tube a wave, the returning reflected pulse from which forces the reed outward. A series of rapid puffs is in this manner maintained at the end of the tube. The performer secures the various pitches by openings in the side of the instrument, as in the case of the flute.

The *cornet*, Fig. 161, is provided with a mouthpiece of the form shown in Fig. 162, within the large opening of which the lips are caused to vibrate. The necessary range of pitch is obtained by blowing overtones and controlling the length of the tube by the valves. (See Fig. 154.) The trombone, Fig. 163, is an instrument of the cornet type, in which the pitches are

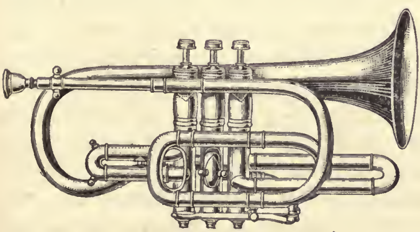

FIG. 161. — The Cornet.

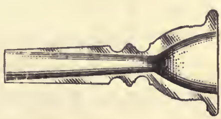

FIG. 162. — Mouthpiece of the Cornet.

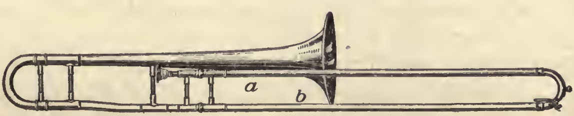

FIG. 163. — The Slide Trombone.

produced by sliding the portion *ab* to the proper positions for changing the length of the vibrating air column.

209. Vibrating Reeds and Diaphragms. — Reed organs, accordions, and mouth organs are provided with reeds similar to that shown in Fig. 164. A current of air blown in the direction of the arrow suffices to set the tongue *a* in vibration. Different pitches are obtained by making the reed of suitable length and rigidity.

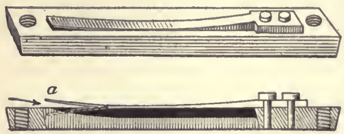

FIG. 164. — An Organ Reed.

A vibrating disk, or diaphragm, is employed in the telephone and phonograph. The description of the *electric* telephone will be deferred until a study has been made of its underlying principles. The *mechanical* telephone may be made by simply connecting two metal diaphragms with a strong cord or wire. The vibrations set up at one end by speaking against the diaphragm are transmitted by longitudinal waves in the wire which set up corresponding vibrations at the second instrument. Such telephones are used for short distances only.

The *phonograph* affords an interesting application of the recording and reproduction of sounds by the aid of a small diaphragm. In the process of making a "record" a smooth, or blank, cylinder of soft material, as wax, is placed on the rotating shaft of the instrument, and against its surface is carefully adjusted a sharp cutting point attached to the back of the diaphragm. The tones of a voice or instrument produce sound waves which are collected by the horn and transmitted to the diaphragm. When the diaphragm vibrates, its up-and-down motion causes the point to engrave in the surface of the rotating cylinder a spiral groove of ever-varying depth corresponding to the complex form of the sound waves which fall upon it. In the reproduction of sound the record is mounted on the revolving shaft, and a delicate stylus which is attached to the back of the diaphragm is adjusted in the spiral groove on the surface of the cylinder. When the cylinder rotates, the stylus rises and falls with the little irregularities of the groove and thus sets the diaphragm in vibration. Since the manner in which the diaphragm vibrates is governed wholly by the nature of the engraved record, the sounds which it emits resemble very closely those by which the record was produced. In many instruments of this type, the records are engraved on disks instead of cylinders.

EXERCISES

1. What is the wave length of the tone produced by an open pipe 2 ft. long? by a closed pipe of the same length?

2. Compute the length of a middle *C* open pipe, the temperature being 20° C.

SUGGESTION. — See § 170 for the relation of wave length and pitch.

3. A whistle may be regarded as a stopped pipe. If the cavity of a whistle is 1 in. long, find the vibration frequency of its tone when the temperature of the air with which it is blown is 25° C.

4. By blowing across the end of a tube 6 in. long, closed at one end, the first overtone is emitted. Find its frequency, the temperature being 18° C.

5. Find the vibration frequencies of the first four overtones of a middle *C* pipe, (1) when the pipe is open at both ends and (2) when it is stopped at one end.

6. When a small stream of water is allowed to run into a bottle, a sound is heard. Does the pitch of the tone rise or fall? Explain.

7. If the ear is held near the mouth of a tall jar or the open end of a tube, a tone will be heard. Perform the experiment, and then explain how the so-called "sound of the sea" is heard coming from large sea-shells.

SUMMARY

1. A *musical sound* is distinguished from a *noise* by the isochronism of the vibrations. *Pitch* is governed by the number of vibrations per second (§§ 177 and 178).

2. A *major diatonic scale* is a series of eight tones whose vibration numbers are to each other in the relation of the numbers 24, 27, 30, 32, 36, 40, 45, and 48, or which bear the following ratios to the first, or key tone: 1, $\frac{9}{8}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{15}{8}$, and 2 (§§ 179 and 180).

3. An *interval* is the relation between the pitches of two tones. It depends entirely upon the ratio of their vibration numbers. The common intervals are the *octave*, *sixth*, *fifth*, *fourth*, *major third*, and *minor third*. The intervals are expressed by definite, simple ratios (§ 181).

4. A *major chord* consists of three tones whose vibration numbers are as 4 : 5 : 6 (§ 182).

5. Sharps and flats are used for the purpose of supplying the necessary tones for scales other than the scale of *C*. Scales are *tempered* in order that an instrument with fixed tones may produce all scales equally well (§§ 183 and 184).

6. Resonance depends upon the fact that a vibrating body will impart vibrations to another near by, whose natural vibration frequency is the same as its own (§§ 185 to 189).

7. Two trains of waves will weaken or destroy each other if the condensations in one train coincide with the rarefactions of the other. The cancellation of sound is complete when the amplitudes and wave lengths are equal (§§ 190 and 191).

8. The coincidence of two trains of waves which differ slightly in length results in the alternate reënforcement and weakening of the resultant tone. The fluctuations of intensity produced in this manner are called *beats*. The number of beats per second is equal to the difference of the vibration numbers of the two tones (§§ 192 and 193).

9. The vibration frequency of a stretched string or wire depends on its length, mass, and tension. The frequency is inversely proportional to the length and the square root of the mass per unit length and directly proportional to the square root of the tension (§§ 194 to 197).

10. A string as a rule vibrates as a whole and at the same time in parts. The tone produced by the vibration as a whole is its *fundamental*, and the tones emitted by the vibrating parts are its *overtones* (§§ 198 and 199).

11. Since a string divides into *equal* parts, *i.e.* into halves, thirds, quarters, etc., the vibration numbers of the overtones are 2, 3, 4, 5, etc., times the frequency of the fundamental. Such overtones are called *harmonics* (§ 200).

12. The *quality* of a sound is dependent on the overtones present and their relative intensities (§ 201).

13. The pitch of an organ pipe depends upon its length. The vibration numbers of the fundamentals emitted by pipes are inversely proportional to their lengths. The fundamental of a stopped pipe is an octave lower than that of an open pipe of equal length (§§ 203 to 205).

14. The wave length of the fundamental of an open pipe is twice, and of a stopped pipe four times, the length of its air column (§ 206).

15. The overtones of open pipes are all the harmonics, as is the case for strings; but the overtones of stopped pipes are only those that have respectively 3, 5, 7, etc., times the vibration frequency of the fundamental (§ 207).

CHAPTER XI

HEAT: TEMPERATURE CHANGES AND HEAT MEASUREMENT

1. TEMPERATURE AND ITS MEASUREMENT

210. Temperature. — Among the most common experiences of everyday life are the sensations of warmth and coldness as we come near or in contact with objects around us. These sensations enable us to distinguish between different bodies with respect to that condition called **temperature.** If several vessels of water or pieces of iron, for example, are placed before us, we are able by the sense of feeling to arrange them in the order of their various temperatures. On this account, we find in common language many terms made use of to express what may be called the *degree of hotness* of bodies; as hot, warm, tepid, lukewarm, cool, and cold. We say that one body is *warmer than another;* or, to use another expression, *has a higher temperature than the other.*

Although our temperature sense is of great value to us at all times, it does not afford an infallible guide in every instance, as the following experiments will show:

1. Place the hand against the wooden portion of the class-room seat, and then transfer it to the iron part. State which feels the warmer.

2. Let three vessels of water be prepared: the first very warm, the second very cold, and the third lukewarm. Place the right hand in the cold water and the left hand in the warm water, and hold them there about a minute. Now transfer the left hand to the third vessel. The water will feel cold. Now remove the right hand from the cold water, and place it in the third vessel. The same water feels warm.

In the first experiment the temperatures of the iron and wood are practically the same; but the hand which is warmer than either the wood or iron falls most rapidly in temperature when in contact with the iron than when touching the wood. Furthermore, it is a well-known fact that a room may be considered warm by a person who has been running, and cold by another who has been sitting quietly in the house.

211. Relation between Temperature and Heat. — When a body which has a certain temperature becomes hotter, we ascribe the cause of this change to the acquisition of *heat*. We say that heat has been added to it. When such a body becomes colder, we say that heat has been taken from it. The rise in the temperature of a body may be produced in many different ways, *e.g.* an iron is heated in a fire, a piece of steel by its friction with a moving grindstone, an electric lamp by a current of electricity, or the earth by the rays of the sun.

It is necessary for us to distinguish carefully between the *temperature* of a body and its *heat*. The fact is well known that a cup of water taken from a boiling kettle is just as hot as the water remaining in the kettle; or a pail of water dipped from a pool has at that instant precisely the same temperature as the pool. But it is also very evident that of two hot-water bottles filled from the same kettle, the one containing the greater quantity of water will give out more heat and for a longer time than the smaller bottle. Hence, while the temperatures of two bodies may be equal, the amounts of heat contained within them may be very different. Thus a thermometer placed in a vessel of water, for example, indicates the temperature of the water, but in no way does it show the amount of heat which the water contains.

212. Nature of Heat. — In order to investigate the nature of that which brings about the changes in the temperatures of bodies, let the following experiments be performed:

Fig. 165.

1. Rub the face of a coin or button against a hard-wood board for a minute or two. The metal will become very hot.

2. Bend a piece of iron wire back and forth, and then feel of the place where the bending occurred. The wire becomes too hot to hold in the fingers.

3. Give the end of a nail or a piece of lead a dozen rapid blows with a hammer. A decided rise in temperature will be detected.

4. Place some tinder in the end of the piston of a "fire syringe," Fig. 165, and then force the piston into the cylinder. When the piston is withdrawn, the tinder will be found to be burning.

These cases are alike in that the heat required to bring about the rise in temperature is produced in every instance at the expense of work on the part of the person performing the experiment. Thus *energy is given up by the person, and heat appears.* Again, a moving train apparently loses its kinetic energy when the brakes are applied; but an examination of the brakes and wheels reveals the fact that the energy has been converted into heat, as is shown by a large increase in temperature.

On the other hand, heat is constantly used in steam engines for hauling trains, running machinery, and for performing work in many other ways.

The conclusion to which experimental results lead is that *heat is a form of energy.* The steam engine is simply a device for converting heat energy into a form of energy which can be utilized, *i.e.* into the energy of mechanical motion. When, however, this motion is checked, the mechanical energy is changed back into heat energy.

213. Molecular Theory Applied to Heat. — The explanation of such phenomena as we have before us is greatly aided by the so-called *molecular theory of matter* which was briefly stated in § 129. It is assumed that *all matter is composed of small particles called molecules. The molecules of a body are separated by small spaces within which they move rapidly about, probably with frequent collisions.* In solids each molecule is restricted in its motion to a certain space which it does not leave. When this restriction is removed, the body assumes the liquid state and the only constraint is the mutual attraction between the molecules (cohesion). In gases even this last limitation is practically removed, and hence the space occupied by a given mass of gas is governed only by the size of the vessel containing it.

With the help of the molecular theory it is now possible to give a more definite idea of the nature of heat. If m represents the mass of a molecule and v the average velocity of the molecules of a body, it is plain that each molecule possesses kinetic energy of the amount $\frac{1}{2}mv^2$ (Eq. 2, § 62), and the entire body will have within it as many times this amount as there are molecules. This energy is called heat. Hence, *we may define* **heat** *as the kinetic energy of molecular motion.*

214. Some Effects of Temperature Changes. — When by increasing or decreasing the amount of heat in a body its temperature is raised or lowered, one or more resulting changes may take place: (1) the body may expand or contract; (2) the body may undergo a change in its properties, as in hardness, elasticity, ductility, etc.; (3) the body may change its pressure against other bodies, *e.g.* a gas may increase or decrease its pressure against the walls of the containing vessel.

The application of heat to a body, however, does not always change its temperature. The body may be changed from a solid to a liquid, or from a liquid to a gas while its temperature remains constant, *e.g.* melting ice, boiling water, etc. These different classes of phenomena are commonly known as *heat-effects*.

FIG. 166. — Tube and Bulb of a Mercurial Thermometer.

215. Measurement of Temperatures. — The instrument most widely used for the determination of temperatures is the *mercurial thermometer*. It is constructed upon the principle that mercury expands when warmed. This thermometer consists of a capillary tube at the lower end of which is a bulb containing mercury, Fig. 166. The mercury completely fills the bulb and extends some distance into the tube. Since the expansion of mercury is greater than that of glass, the thread of mercury in the tube rises when the temperature of the mercury in the instrument is increased, and falls when it is decreased. Before the tube of a thermometer is sealed, the mercury in the bulb is heated until it entirely fills the instrument, in which condition the glass at *A* is sealed off in a hot flame. When the mercury cools and contracts, it leaves a good vacuum above it in the tube.

216. The Fixed Points of a Thermometer. — In order to make it possible to compare the temperature measurements of one thermometer with those of another, two *fixed points* that are easily obtained are located on the tube of the instrument. The first of these is the *freezing point of pure water*. This point is found by packing the thermometer in ice or snow, as shown in Fig. 167. When the mercury has ceased to fall, the position of the end of the column is marked upon the tube.

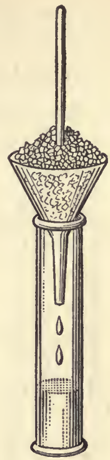

FIG. 167.—Locating the Freezing Point on a Thermometer Tube.

The second fixed temperature point is the *boiling point of pure water.* The thermometer is suspended over boiling water in a tall vessel so that the thread of mercury in the tube is completely enveloped by the steam, as in Fig. 168. The mercury rises for a time, but finally comes to a position at which it remains stationary. Since, however, this temperature changes with the pressure of the atmosphere, it should be taken under normal atmospheric pressure (*i.e.* 760 millimeters of mercury). Otherwise a correction must necessarily be made.

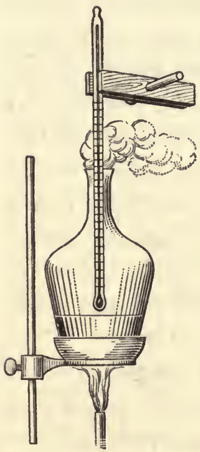

FIG. 168.—Determining the Boiling Point on a Thermometer Tube.

The points thus obtained on a thermometer scale are often marked with the words "freezing" and "boiling," these being the names given to the fixed points of temperature.

217. Graduation of Thermometer Scales.—The space between the freezing and boiling points is now divided into temperature units called *degrees.* According to the *centigrade*[1] scale the freezing point is marked 0°, and the boiling point 100°. The interval between the two points is then divided into 100 equal parts. Similar divisions are produced on the tube above the boiling point and below the freezing point. Centigrade ther-

[1] From *centum* and *gradus*, meaning a hundred degrees.

mometers are almost exclusively used for scientific purposes.

The *Fahrenheit* thermometer scale was introduced by a German physicist of that name about 1714. On this scale the freezing point is marked 32°,[1] and the boiling point 212°. The interval between these two points is therefore divided into 180 equal parts, and similar divisions are laid off both above the boiling point and below the freezing point. The Fahrenheit thermometer is the household instrument in use among most English-speaking people, and is that employed by the United States Weather Bureau and by physicians.

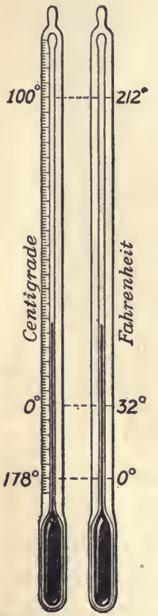

Fig. 169. — Centigrade and Fahrenheit Thermometer Scales.

On all thermometers, temperatures below the zero point are read as negative quantities. In every case the initial letter of the name is affixed to indicate the scale used. For example, 25° C., 100° F.

218. Thermometer Scales Compared. — It is obvious that any thermometer can be provided with both the centigrade and the Fahrenheit scales, as shown in Fig. 169. It will readily be observed that 100 centigrade degrees measure the same interval as 180 Fahrenheit degrees. Hence,

$$100 \textit{ centigrade degrees} = 180 \textit{ Fahrenheit degrees,}$$

or

$$1 \textit{ centigrade degree} = \tfrac{9}{5} \textit{ Fahrenheit degree.}$$

But in order to change a temperature reading from one system into the other, it is necessary to take account of the fact that 0° *F. is* 32 *Fahrenheit degrees below* 0° *C.*

[1] Fahrenheit placed 0 at the temperature which he produced by a mixture of ice, water, and salammoniac.

For example, 68° F. is 68 – 32, or 36 Fahrenheit degrees above the freezing point. But 36 Fahrenheit degrees are equivalent to $\frac{5}{9} \times 36$, or 20 centigrade degrees. Now a temperature that is 20 centigrade degrees above the freezing point is 20° C. Therefore the temperature 68° F. is equivalent to 20° C.

Letting F represent a Fahrenheit reading and C the corresponding reading on the centigrade scale, we have:

$$\mathbf{F - 32 = \tfrac{9}{5}\,C.} \qquad (1)$$

219. Range of a Mercurial Thermometer. — The use of a mercurial thermometer in the measurement of high and low temperatures is limited by the boiling and freezing points of mercury. The former is about 350° C., and the latter – 38.8° C. The boiling of the mercury can be prevented by increasing the pressure upon it by the presence of nitrogen gas above it in the tube. Such thermometers may register temperatures up to about 500° C. For temperatures below – 39° C. liquids having low freezing points must be used. Such a liquid is alcohol, which freezes at – 111° C. Many alcohol thermometers are in common use.

220. Galileo's Air Thermometer. — The first instrument designed for the measurement of temperatures was Galileo's air thermometer, Fig. 170. The use of this instrument dates from 1593. The device consists of a vertical glass tube of small bore, on the upper end of which is a large bulb containing air. This air is warmed slightly, and the end of the tube is placed in some liquid, such as colored water. When the air cools, the liquid rises in the tube, being forced up by the atmospheric pressure. Obviously the liquid column will rise and fall

FIG. 170. — Galileo's Air Thermometer.

according as the temperature of the air in the bulb is reduced or increased. On account of its sensitiveness to small changes of temperature, this thermometer is frequently used for experimental purposes.

EXERCISES

1. Would the range of a mercurial thermometer of given length be increased or decreased by reducing the size of the bulb? by making the bore of the tube smaller? Would the distance representing a degree be increased or decreased?

2. If the bulb of a mercurial thermometer should permanently contract after its graduation, how would the fixed points be affected?

3. In a certain experiment only the bulb of a thermometer is exposed to the temperature that it is desired to measure. If this temperature is above that of the room, will the reading of the instrument be too large or too small?

4. How would you proceed to test experimentally the points on a mercurial thermometer?

5. The boiling point of water falls 0.1 of a centigrade degree for a decrease of 0.27 cm. in the atmospheric pressure. What is the boiling point when the barometer reads 73.3 cm.? *Ans.* 99° C.

6. Reduce the following centigrade readings to the corresponding values on the Fahrenheit scale: 20°, 35°, 50°, −20°, −40°.

7. How many centigrade degrees lie between the Fahrenheit and centigrade zero marks?

8. The following temperature measurements were taken with a Fahrenheit thermometer: 77°, 41°, 14°, −4°, −40°. What would a centigrade thermometer have indicated in each case?

9. The difference in temperature of two vessels of water is 25 centigrade degrees. Express the difference in Fahrenheit units.

10. One room is 18 Fahrenheit degrees warmer than another. What is the difference between their temperatures on the centigrade scale?

11. The boiling point of water at a certain place was found to be 98.8° C. What was the atmospheric pressure at the time? See Exer. 5. *Ans.* 72.76 cm.

2. EXPANSION OF BODIES

221. Linear Expansion. — We have already observed the use made of the expansion of mercury when heated.

It is generally true of all bodies that an increase in size accompanies a rise in temperature. Thus a metal rod, for example, undergoes an increase in length, which, although usually small, must be taken into account in planning bridges, laying railroad tracks, etc.

1. Figure 171 illustrates the well-known ring and ball. When both are of the same temperature, the ball passes readily through the ring. When, however, the ball is heated, it becomes too large to fit the ring. If cooled, it will be found to resume its original size.

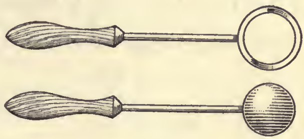

FIG. 171. — Ring and Ball.

2. Let a metal bar, preferably of brass, rest at one end upon a block of wood, as A, Fig. 172, and at the other end upon a round glass or metal rod B about 2 millimeters in diameter placed

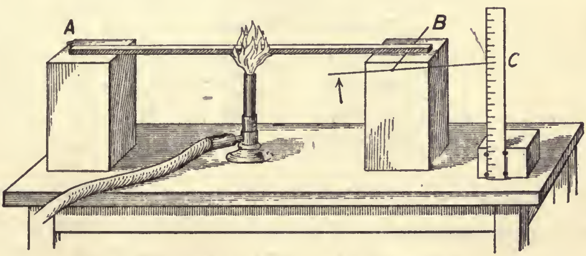

FIG. 172. — Expansion of a Metal Rod.

upon a smooth block of wood. Attach a very light index of glass or paper about 20 centimeters long to the small rod in such a manner that it can move over a scale C, as shown. Now heat the bar by moving a flame along it. The movement of the index will indicate an elongation of the bar. When the bar cools, the index returns to the original position.

222. Coefficients of Linear Expansion. — Not all solids expand equally. For example, a bar of copper a meter long expands more than a bar of iron of equal length for the same increase in temperature. *The ratio of the increase in length of a metal bar for an increase of one degree* *in temperature to its length at 0° C. is called the* **coefficient of linear expansion** *of the metal.*

Thus a rod of metal one meter long that expands 1 millimeter when the temperature rises from 0° C. to 100° C. has a coefficient of linear expansion equal to 1 ÷ (1000 × 100), or 0.00001. The coefficient of linear expansion of a substance is expressed by the equation

$$k = \frac{l_2 - l_1}{l_1(t_2 - t_1)}, \tag{2}$$

where l_1 and l_2 are the lengths before and after heating, and t_1 and t_2 are the initial and final temperatures.

COEFFICIENTS OF LINEAR EXPANSION

Aluminium	0.000023	Iron	0.000012
Brass	0.000018	Lead	0.000027
Copper	0.000017	Platinum	0.000009
Glass	0.000009	Steel	0.000011
Invar	0.00000087		

223. Applications of Unequal Expansion. — By referring to the table of the coefficients of expansion of metals given in § 222, it will be seen that the common metals, brass and iron, expand unequally. Hence if two flat bars of these metals are riveted together, as shown in (*a*), Fig. 173, an increase in temperature will expand the brass more than the iron and cause the bar to bend, as in (*b*).

FIG. 173.—Compound Bar of Brass and Iron.

The bending of a compound bar of two metals is employed in the *dial thermometer* in common use. The bar in this case is made circular in form and has one end fixed, as at *A*, Fig. 174. The other end is attached by means of a cord or chain at *B* to a small axle *C*, which carries the pointer *D*. A rise in temperature causes the free end of the bar *B* to move inward and the pointer to register a higher temperature on

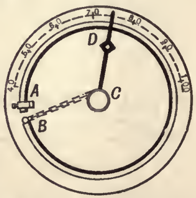

FIG. 174.—Dial Thermometer.

the graduated scale. For a fall in temperature the reverse movement takes place.

The same principle is ingeniously applied to the balance wheel of a watch, in order to make the period of vibration independent of temperature changes. An increase in temperature weakens the hairspring which controls the vibration of the wheel and lengthens its spokes, the effect in each instance being to make the watch lose time. In order to correct this tendency, the balance wheel is constructed as shown in Fig. 175. The outer rim of each of the compound bars A and B is made of brass, and the inner part of iron. An increase in temperature causes the portions A and B to bend toward the center just enough to keep the period of vibration constant.

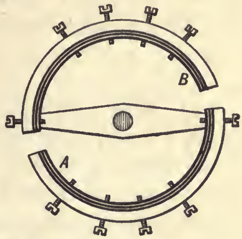

FIG. 175. — Balance Wheel of a Watch.

EXERCISES

1. Ascertain how steel tires are tightened or "set" by a blacksmith, and explain the various steps of the process.

2. In what manner do engineers take account of the expansion and contraction of the rails when laying a railroad track?

3. Consult the table of linear coefficients of expansion, and ascertain why platinum wires can be sealed in glass without danger of breakage when the glass cools. Examine an incandescent lamp bulb, and see that this is the case.

4. Why is one end of a long steel bridge often supported on rollers?

5. Invar is an alloy of nickel and steel. Consult the table of linear coefficients of expansion, and show why it is a valuable metal from which to make tapes for measuring, standards of length, and pendulum rods.

6. Glass stoppers can often be loosened by carefully heating the neck of the bottle. Explain.

7. How much does the length of a 90-foot steel rail vary if the extremes of temperature are $-24°$ C. and $35°$ C.?

8. At the temperature of $0°$ C. an iron pipe is 100 ft. long. What will be its length when steam at $100°$ C. is passing through it?

9. A metal rod is 60 cm. long and expands 1.02 mm. when the temperature is raised from $0°$ C. to $100°$ C. Compute the coefficient of linear expansion of the metal.

224. Cubical Expansion. — In general, substances when heated expand in all directions, *i.e.* a rise in temperature is accompanied by an increase in volume. This may be shown to hold true for water by the following experiment:

Fill a flask with water, and insert a rubber stopper through which passes a small glass tube about 40 centimeters long. Some of the liquid will rise in the tube. Mark the position of the top of the liquid column, and set the flask in a vessel of warm water. The liquid at first falls slightly in the tube as the flask expands and then rises slowly because of the increase in volume of the water.

225. Coefficients of Cubical Expansion. — The cubical expansion of a substance is related to volumes in the same manner as linear expansion is related to lengths. Thus *the* **coefficient of cubical expansion** *of a substance is the ratio of the increase in volume for a change of one degree in temperature, to the volume at* 0° *C.*

The cubical expansion of a substance may be expressed by the equation

$$\mathbf{k} = \frac{\mathbf{v}_1 - \mathbf{v}_0}{\mathbf{v}_0 \mathbf{t}}, \tag{3}$$

where v_1 and v_0 are the volumes at the temperatures t° C. and 0° C. respectively. This equation is easily reduced to the form

$$\mathbf{v}_1 = \mathbf{v}_0(1 + \mathbf{kt}). \tag{4}$$

The coefficient of cubical expansion of any substance is three times the coefficient of the linear expansion of that substance. Hence the cubical expansions for the substances given in the table in § 222 are easily computed.

226. Abnormal Expansion of Water. — If a quantity of water at freezing temperature is warmed, its volume *decreases* until it reaches a temperature of 4° C. Upon being heated above this point it expands as do other liquids. This exception to the general rule is of vast importance in nature. As the atmosphere grows colder, the water at the surface of lakes and ponds falls in temperature, and its density at first increases. The surface layers of water then sink and are replaced by the warmer water from below. This continues until the temperature of 4° C. is reached, when further cooling makes the surface water less dense than that underneath. Hence the water that has cooled below 4° C. does not sink, but remains at the top and is frozen. Thus, since practically the entire quantity of water in a lake must be reduced to 4° C. before the surface is frozen, ice is much slower in forming on deep bodies of water than on shallow ones. Furthermore, on account of the fact that the colder ice and water at the top do not transmit the heat rapidly away from the warmer layers of the liquid below, the unfrozen water remains at a temperature of about 4° C. Hence, animal life flourishes at the bottom of a lake in winter, even when a little lower temperature would prove fatal.

227. Cubical Expansion of Gases. — When the temperature of a gas is raised, its volume is increased, unless the expansion is prevented by an increase of the pressure upon the gas. Under a constant pressure the expansion of gases is much greater than that of liquids or solids. When the temperature of a gas is increased, the change is simply an increase in the average speed of its molecules. As a consequence, there results an increase in the force delivered by each molecule in its collision with the sides of the containing vessel. There will also be an increase in the number of blows delivered per second. Now the pressure of the gas outward is nothing more than the result of this continuous bombardment of molecules; and thus an increase in the average molecular speed produces a corresponding increase in the pressure of the gas. By permitting the gas to expand, the pressure may be kept constant. Under a constant pressure, *all gases have the same coefficient of cubical expansion* (**Law of Charles**[1]). This number has been found experimentally to be $\frac{1}{273}$, or 0.00366, of the volume of the gas at 0° C. Thus a gas whose volume at 0° C. is 100 cubic centimeters, when heated to 25° C. increases in volume $\frac{25}{273}$ of 100 cubic centimeters. Its volume becomes, therefore, $100 + \frac{25}{273} \times 100$, or 109.1 cubic centimeters.

228. The Absolute Scale of Temperatures.—If a body of air, for example, at 0° C. is kept at a constant pressure and heated, its volume will increase $\frac{1}{273}$ of its original volume for every degree that its temperature is raised. Thus, at 273° C. its volume will be doubled. On the other hand, if the original body of air is cooled below 0°, its volume will be diminished $\frac{1}{273}$ of its volume at 0° for every degree that its temperature is lowered. If, now, the volume were to continue to diminish at this rate until the temperature should reach $-273°$ C., mathematically it would become nothing. Practically, however, the air and other gases become liquids before reaching this temperature, and thus lose the properties of gases. A temperature 273 centigrade degrees below the freezing point of water is called *absolute zero*, and temperatures measured from this point as the 0 of the scale are called *absolute temperatures*. It is clear that temperatures on the centigrade scale can be reduced to the absolute by simply adding 273°. Thus 20° C. $= 293°$ Ab., and $-40°$ C. $= 233°$ Ab.

Although no one has ever succeeded in cooling a body to absolute zero, temperatures approaching within a very few degrees of this point have been attained by the evaporation of liquefied gases. The following table will serve

[1] Also called Gay-Lussac's Law.

to show some facts regarding the history of low temperature production:

DATE	TEMPERATURE	EXPERIMENTER
1714	− 17° C.	Fahrenheit
1778	− 40° C.	Van Marum
1823	− 102° C.	Faraday
1877	− 103° C.	Cailletet
1877	− 183° C.	Pictet
1898	− 262° C.	Dewar
1908	− 269° C.	Onnes

229. Laws of Gaseous Bodies. — Since the volume of a gas is doubled when its temperature is raised from 273° Ab. (0° C.) to 2 × 273, or 546° Ab. (273° C.), and the increase in volume is uniform (§ 227), it is clear that the following law may be stated:

The volume of a given mass of gas under constant pressure is proportional to its absolute temperature.

Again, if the body of gas is confined in a vessel of sufficient rigidity to keep the volume constant at all temperatures, the pressure of the gas against the walls of the vessel will increase as the temperature rises. Furthermore, the pressure will increase $\frac{1}{273}$ of the pressure at 0° C. for every degree that the temperature is raised, and will decrease $\frac{1}{273}$ of the pressure at 0° C. for every degree that the temperature is lowered. In other words, when the volume of a gas remains constant, the change in pressure takes place according to a law similar to that governing the change in volume. Hence

The pressure of a given mass of gas whose volume remains constant is proportional to the absolute temperature.

Example. — At 25° C. the volume of a certain mass of gas is 400 cm.^{3}. Compute its volume when the temperature is lowered to 0° C. and the pressure kept constant. If the original pressure is 740 mm., compute the pressure after the decrease in temperature, assuming that the volume remains constant.

Solution. — Letting x be the volume of the gas at 0° C., we have, by applying the law of volumes stated above,

$$400 : x : : 25 + 273 : 273;$$

whence, $$x = 366.4 \text{ cm.}^3.$$

In the second place, by applying the law of pressures, if x is the pressure at 0° C.,

$$740 : x : : 25 + 273 : 273;$$

whence, $$x = 677.9 \text{ mm.}$$

230. Laws of Charles and Boyle Combined. — Boyle's Law (§ 149) states that the product of the pressure and volume of a given mass of gas remains constant when the temperature remains the same. In practice, volume, pressure, and temperature may all vary. In such cases the following law will hold:

The product of the pressure and volume of a given mass of gas is proportional to the absolute temperature.

Example. — The volume of a gas collected in a vessel under a pressure of 740 mm. and at a temperature of 20° C. is 500 cm.^{3}. Compute the volume that the gas would have at 0° C. and under a pressure of 760 mm.

Solution. — Let x be the required volume. By combining the laws of Boyle and Charles we have

$$740 \times 500 : 760 \times x :: 20 + 273 : 273;$$

whence, $$x = 453.6 \text{ cm.}^3.$$

EXERCISES

1. What fractional part of its volume at 0° C. does a cubic meter of gas expand when warmed from that temperature to 50° C., the pressure remaining constant? What is the final volume of the gas?

2. The volume of a certain gas at 20° C. is 300 cm.^{3}. What is its volume when the temperature is reduced to 0° C., the pressure being constant?

3. The pressure exerted by a gas confined in a reservoir is 500 g. per square centimeter when the temperature is 10° C. What is its pressure when the temperature is raised to 40° C.?

4. The volume of a gas collected in a chemical experiment is 30 cm.^{3}, its temperature 25°, and its pressure 750 mm. Find the volume of the same gas at 0° C. and under a pressure of 760 mm.

5. If the mass of a cubic centimeter of air at 0° C. and under a pressure of 760 mm. is 0.001293, what will be its density in a room where the temperature is 22° C. and the barometer reads 745 mm.?

6. The quantity of air in a room $8 \times 12 \times 15$ ft. will contract to what volume when its temperature falls from 20° C. to 0° C.?

7. To what temperature would the air confined in a flask at atmospheric pressure and a temperature of 10° C. have to be heated in order to exert a pressure of 3.5 atmospheres?

8. Upon heating 300 cm.^{3} of a gas from 0° C. to 30° C., the volume was found to be 333 cm.^{3}. Ascertain the coefficient of expansion of the gas.

9. The capacity of a steel gas cylinder is 3 cu. ft. Illuminating gas is compressed in the cylinder until the pressure is 15 atmospheres at a temperature of 10° C. What volume will this quantity of gas assume when allowed to escape into a space where the pressure is 1 atmosphere and the temperature 25° C.?

3. CALORIMETRY, OR THE MEASUREMENT OF HEAT

231. The Unit of Heat. — The unit employed in the measurement of heat is called the **calorie**. *The calorie is the quantity of heat required to raise the temperature of a gram of water one centigrade degree.* When the temperature of a gram of water is raised from 0° C. to 100° C., 100 calories of heat are required. Again, to change the temperature of 1 kilogram of water from 20° C. to 45° C. requires that $1000 \times (45 - 20)$, or 25,000 calories of heat be taken on by the water. Thus, if a mass of water be changed in temperature a given amount, *the quantity of heat involved is measured by the product of the mass of water*

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and its change of temperature. Hence, to change the temperature of a mass of m grams of water t centigrade degrees, we may write for the required quantity of heat,

$$\mathbf{H} \text{ (in calories)} = \mathbf{m} \text{ (in grams)} \times \mathbf{t} \text{ (in centigrade degrees)}. \quad (5)$$

232. Specific Heat. — If a given quantity of heat be applied to equal masses of different kinds of matter, as lead, mercury, water, iron, and copper, the temperatures will not all be changed equally. The amount of heat that will warm a gram of water one degree (*i.e.* one calorie) will raise the temperature of an equal mass of lead or of mercury about 30 degrees and that of the iron or the copper about 10 degrees.

Place 100 grams each of lead shot, iron cuttings, and bits of aluminium wire in three large test-tubes. Set the tubes upright in a vessel of boiling water, and allow them to remain there several minutes while the water continues to boil. Also place 100 grams of water at the temperature of the room in each of three beakers. Now pour the lead shot whose temperature is 100° C. into the water in one of the beakers, stir the mixture thoroughly, and ascertain the rise in temperature of the water. Do the same with the other metals. Although the metals fall in temperature almost equal amounts, they deliver to the water very unequal quantities of heat. The aluminium will warm the water through about twice as many degrees as the iron and about six times as many as the lead.

Experiments like those just described lead to the conclusion that each gram of lead gives out, upon cooling one degree, about one sixth as much heat as one gram of aluminium. Also that a gram of iron delivers only one half as much heat as an equal mass of aluminium when cooled an equal amount. For this reason substances are said to differ in *thermal capacity*, or **specific heat.**

The specific heat of a substance is the ratio of the quantity of heat required to raise the temperature of a certain mass *of it one degree to the quantity of heat required to raise the temperature of an equal mass of water one degree.*

The specific heat of a substance is numerically equal to the number of calories received by one gram of the substance when its temperature rises 1° C. Thus the heat required to raise the temperature of 1 g. of iron 1° C. is 0.113 calories; of 1 g. of lead, 0.032 calories, etc.

233. Specific Heat Determined. — When a hot substance, as heated mercury, for example, is placed in cold water, the two bodies assume the same temperature. The heat given up by the substance which cools is utilized in raising the temperature of the water. In other words, *the quantity of heat gained by the cold body equals that lost by the warm body.*

Place 300 grams of lead shot in a large test-tube, and suspend the tube in boiling water for at least ten minutes. While the lead is heating, the tube should be kept closed with a cork. Place 100 grams of water in a beaker, and cool it a few degrees below the temperature of the room. Now pour the shot quickly into the water and stir carefully until the temperature of the mixture becomes stationary. Note and record the rise in temperature of the water. In order to complete the solution of the problem, we must form an equation that expresses the equality between the heat lost by the lead and that gained by the water. The following illustration will make the process clear:

In an experiment the initial temperature of the water was 17° C., the temperature of the mixture 24.2° C., and the boiling point 100° C.

Let x be the specific heat of lead.

The fall in temperature of the lead $= 100° - 24.2°$.
The heat lost by the lead $= (100 - 24.2)\ 300\,x$ calories.
The rise in temperature of the water $= 24.2° - 17°$.
The heat gained by the water $= (24.2 - 17)\ 100$ calories.

Hence, $(100 - 24.2)\ 300\,x = (24.2 - 17)\ 100;$
whence, $x = 0.0316.$

This process for determining the specific heat of a substance is known as the "method of mixtures." It can be applied successfully to a large number of substances that do not dissolve when placed in water. The specific heats of some common substances are given in the following table:

Specific Heats

Aluminium	0.212	Mercury	0.033
Copper	0.093	Platinum	0.032
Ice	0.502	Silver	0.056
Iron	0.113	Tin	0.055
Lead	0.032	Zinc	0.093

EXERCISES

1. A mass of 75 g. of water is cooled from 95° C. to 32° C. How much heat is given up?

2. A 100-gram mass of copper rises in temperature from 15° C. to 100° C. How much heat does it absorb?

3. If 500 calories are applied to 500 g. of mercury at 10° C., to what point will the temperature of the mercury rise?

Suggestion. — Let x be the final temperature of the mercury.

4. A mass of iron weighing 400 g. and having a temperature of 98° C. is placed in 100 g. of water at 14° C.; the temperature of the combined masses is 40° C. Compute the specific heat of the metal.

Suggestion. — See example given in § 233.

5. Find the resulting temperature when 400 g. of water at 90° C. are mixed with 150 g. of water at 10° C.

Suggestion. — If x is the resulting temperature, the former mass falls in temperature $90 - x$ degrees, while the latter rises $x - 10$ degrees. Express the equality between the heat lost by the former and that gained by the latter.

6. A vessel contains 250 g. of lead shot and 150 g. of water. Find the amount of heat required to raise the temperature of the mixture from 15° C. to 80° C. *Ans.* 10,270 calories.

7. The temperature of a block of ice weighing 100 kg. rises from −15° C. to the melting point. What quantity of heat is absorbed?

8. If 100 g. of aluminium at 97° C. were dropped into 50 g. of water at 10° C., what would be the temperature of the mixture?

SUMMARY

1. *Temperature* is the degree of hotness of a body and is the condition which determines in what direction a transfer of heat will take place (§§ 210 and 211).

2. *Heat* is a form of energy into which all other forms are convertible. It is the kinetic energy of the moving molecules of a body (§§ 212 and 213).

3. Changes in the temperature of bodies produce (1) expansion and contraction, (2) change in their properties, and (3) changes in pressure. The application of heat may change the state of matter without producing a change in temperature (§ 214).

4. The mercury thermometer makes use of the expansion of mercury in the measurement of temperatures. Each instrument is graduated according to the position of *two fixed points*, the boiling point and the freezing point of pure water (§§ 215 and 216).

5. On the *centigrade* scale the freezing point is marked 0°, and the boiling point (under a pressure of 760 mm.) is marked 100°. On the *Fahrenheit* scale the corresponding points are marked 32° and 212°. The relation between temperatures expressed on the two scales is

$$F - 32 = \tfrac{9}{5}\,C \text{ (§§ 217 and 218).}$$

6. The *coefficient of linear expansion* of a body is the ratio of its increase in length for an increase of 1° C. to its length at 0° C. (§§ 221 to 223).

7. The *coefficient of cubical expansion* of a substance is the ratio of its increase in volume for a change of 1° C. to its volume at 0° C. (§ 225).

8. The volume of a given mass of water when heated from 0° C. contracts until the temperature reaches 4° C. Further heating causes it to expand. Hence the greatest density of water is at 4° C. (§ 226).

9. The coefficient of cubical expansion of all gases is practically the same and is expressed by the fraction $\frac{1}{273}$ (§ 227).

10. *Absolute temperatures* are measured from absolute zero, which is the same as $-273°$ C. Hence temperatures measured on the centigrade scale are reduced to the absolute by adding 273° (§ 228).

11. The volume of a given mass of gas under constant pressure is proportional to its absolute temperature (§ 229).

12. The pressure of a gas under constant volume is proportional to its absolute temperature (§ 229).

13. The product of the pressure and volume of a given mass of gas is proportional to its absolute temperature (§ 230).

14. Heat is expressed in terms of a unit called the *calorie.* The calorie is the quantity of heat required to raise the temperature of 1 g. of water 1° C. (§ 231).

15. Like masses of different substances require different quantities of heat to produce equal changes of temperature, *i.e.* they differ in *specific heat.* The specific heat of a substance is the ratio of the quantity of heat required to raise the temperature of a certain mass of it 1° C. to the quantity required to raise the temperature of an equal mass of water the same amount (§ 232).

16. The quantity of heat required to raise the temperature of any mass of a substance a given amount is the product of the mass, the rise in temperature, and the specific heat of the substance (§ 233).

17. Specific heat is determined by the "method of mixtures" (§ 233).

CHAPTER XII

HEAT: TRANSFERENCE AND TRANSFORMATION OF HEAT ENERGY

1. CHANGE OF THE MOLECULAR STATE OF MATTER

234. Fusion. — If the motion of the molecules of a body is principally of a vibratory nature, we find difficulty in changing the form of the body and, therefore, call it a solid. When, however, by the application of heat, the molecular motion is so increased that the particles break away from the constraining forces and thus overcome their "fixedness of position," the mass assumes the property of *fluidity* and becomes a liquid. Hence the state in which a substance exists depends largely upon its temperature. Thus mercury, which is a liquid under ordinary conditions, changes into a gas at 350° C. and remains a solid at all temperatures below − 39° C. The temperature at which a solid changes into a liquid is called its *melting* or *fusing* point, and the process is called **fusion.** The reverse change, *i.e.* from a liquid to a solid, is called *solidification.* In the case of water, the process is called *freezing.*

Collect a quantity of snow in a vessel, preferably when the out-door temperature is several degrees below the freezing point. In summer broken ice must be used. Place a thermometer in the snow so that it can be read easily and set the vessel outside the window. When ready to proceed with the experiment, bring the snow into the warm room, and read the thermometer at brief intervals. The temperature of the snow will be found to rise gradually to 0° C., where the mercury remains while the snow is melting. The vessel of snow may be gently heated, and, if the mixture is kept thoroughly stirred, the temperature will remain 0° C. Continued heating after the snow has melted will raise the temperature of the resulting liquid.

The melting point of a crystalline substance, as snow, is well marked, but amorphous bodies, as glass, tar, pitch, etc., pass through a semiliquid state several degrees below the temperature of liquefaction. It is due to this property that glass can be formed into vessels of any desired shape, and that wrought iron can be fashioned according to the demands of the blacksmith.

235. Laws of Fusion. — The laws governing the fusion of crystalline substances and the reverse change of **solidification** are as follows:

(1) *A crystalline substance under a constant pressure has a definite fusing point which is also the temperature at which solidification takes place.*

(2) *When a crystalline substance begins to melt, its temperature remains constant until all of it is liquefied.*

(3) *A substance that contracts while melting has its fusing point slightly lowered by increased pressure, but a substance that expands while melting has its fusing point slightly raised by an increase of pressure.*

(4) *In the presence of a dissolved substance, the solid forms by crystallization at a temperature below the freezing point of the pure solvent.*

MELTING POINTS

Mercury . . .	− 39.° C.	Tin	232° C.
Ice	0.° C.	Lead	325° C.
Benzine	7.° C.	Silver	954° C.
Acetic acid . . .	16.° C.	Copper	1100° C.
Paraffin	55.° C.	Cast iron	1200° C.

236. Effect of Pressure on the Fusing Point of Ice. — It is well known that water expands when it freezes. This is shown by the floating of ice and the bursting of frozen water pipes. Ice therefore contracts when it melts. Hence, according to the third law of fusion stated in § 235, an increase of pressure lowers the melting or fusing point. The lowering is very slight, amounting to about 0.0075° C. for an increase of one atmosphere.

1. Let two pieces of ice be pressed firmly together. When the pressure is removed, the pieces will be found to be frozen together.

2. Connect two heavy weights by means of a strong wire, and hang them over a block of ice, as shown in Fig. 176. In a short time the wire will cut into the block and, at last, entirely through, leaving the ice still in one piece.

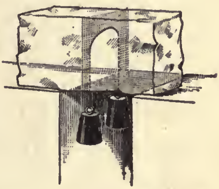


FIG. 176. — Wire Cutting through a Block of Ice.

In each of these cases the melting point at the places where the pressure is applied is slightly lowered; hence some of the ice melts, forming a film of water a little below 0° C. When the pressure is removed the film, which is at a temperature below the freezing point, solidifies, cementing the two pieces of ice together. In Experiment 2 the melting under pressure takes place below the wire; then, as the liquid flows up above the wire, the pressure is removed, and it freezes. This process is known as **regelation** (pronounced *rē ge lā′ tion*).

237. Heat of Fusion. — In general, the fusion of any solid requires the application of heat. If the substance is of a crystalline structure, as ice, the heat energy imparted to it does not sensibly raise its temperature during the melting process. In all such cases the energy supplied to the solid is used in producing the change of state. Non-crystalline solids, such as waxes, iron, glass, etc., become plastic when heated, and have no definite melting point.

1. Note the temperature of snow or finely chipped ice placed in a metal vessel. Place a flame under the vessel, and allow some of the ice to melt. Remove the flame, stir the contents of the vessel well, and again take the temperature. Apply more heat, and note the temperature after stirring. In every case the temperature will be found to be the same.

2. Place equal quantities of ice and ice water in two similar vessels, and set both in a large vessel of hot water placed over a flame. In one vessel heat is changing ice into water; in the other the temperature of the water is being raised. If the contents of the vessels are continually stirred, it will be found that when the ice is melted, the temperature of the water will have risen to about 80° C.

Heat energy is applied about equally to the ice and ice water in Experiment 2. That applied to the cold water increases the average kinetic energy of the molecules, (§ 213), and thus the temperature is raised. The heat applied to the ice, however, does not produce any increase in the kinetic energy, but suffers a transformation into potential energy by producing molecular separation in opposition to the mutual attraction (cohesion) between the particles that compose the body. In other words, *the heat energy expended in melting the ice has ceased to be heat* and simply represents the work necessary to change the ice from the solid to the liquid state. *The number of calories per gram required to liquefy a substance without producing any change in its temperature is called the heat of fusion*[1] *of that substance.*

238. Heat of Fusion of Ice Measured. — The quantity of heat required to melt a gram of ice can be measured by the method of mixtures as shown in the following experiment:

Let 300 grams of water at about 35° C. be placed in a beaker and its temperature accurately noted. Prepare also a quantity of ice in lumps about as large as walnuts. Dry the pieces of ice with a towel, and drop them in small quantities into the warm water. Stir thoroughly to melt the ice. When enough ice has been added and

[1] Sometimes called "latent" heat of fusion.

melted to make the temperature of the water about 5° C., weigh the contents of the beaker and compute the mass of ice melted. An equation is now formed between the heat lost by the water and that gained by the ice. An illustration will make the process clear.

In an experiment the temperature of the warm water was 36.5° C. After adding 110 grams of ice the resulting temperature of the water was 5.5° C.

Let x be the heat of fusion of ice.

The heat lost by the warm water = 300 (36.5 − 5.5) calories.

The heat required to warm 110 grams of water formed by the melted ice from 0° C. to 5.5° C. = 110 × 5.5 calories.

The heat required to melt the ice = 110 x calories.

Hence, $$110\,x + 110 \times 5.5 = 300\,(36.5 - 5.5);$$

whence, $$x = 79.9 \text{ calories.}$$

Careful investigation has shown that the heat of fusion of ice is 80 calories.

239. Heat Given out by Freezing Water. — According to the doctrine of the Conservation of Energy (§ 64), heat energy equivalent to that which is required to change a solid into a liquid must be given out when the reverse change (*i.e.* solidification) takes place.

Make a freezing mixture of snow and salt stirred well together. Into this mixture, which will be several degrees below 0° C., set a test-tube containing water and a thermometer. If the water in the tube is not disturbed, it may reach a temperature below 0° C. without freezing. If, however, the thermometer be moved gently against the wall of the test-tube, the water quickly begins to freeze and the temperature rises at once to 0° C.

In the freezing mixture of snow and salt used in the experiment, both solids pass into the liquid state. Just as in the case of fusion (§ 238) heat is absorbed by both the salt and snow during the change. The heat energy (kinetic energy) acquired by the solids in liquefying is converted into potential energy in giving their molecules the molecular freedom which exists in a liquid. In fact the solids cannot liquefy unless they can acquire heat somewhere. In this case the heat is taken from the water in the test-tube, and thus its temperature is lowered and a portion solidified. On the other hand the rise in temperature indicated by the thermometer which was placed in the test-tube shows that heat is given out when the water changes to ice.

When a gram of water freezes, 80 *calories are given out to its surroundings.* The enormous amount of heat evolved by the freezing of water in large lakes is of great economic importance, since it prevents large and sudden falls of temperature in the vicinity.

EXERCISES

1. How does the presence of tubs of water in a cellar tend to prevent the freezing of vegetables?

2. What has the large heat of fusion of ice to do with the rapidity with which snow and ice disappear on a warm day?

3. In freezing cream a metal vessel B (Fig. 177) containing it is surrounded by a rapidly liquefying mixture of ice and salt, A. Give a complete explanation of the process.

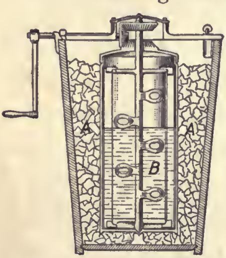


FIG. 177. — Liquid *B* Frozen by the Melting of Ice in *A*.

4. Can a piece of ice be warmed above 0° C.? Can it be cooled below 0° C.?

5. Find the amount of heat required to melt 50 g. of ice at 0° C. and to raise the temperature of the resulting water to 15° C.

6. A kilogram of ice at 0° C. is placed in an equal mass of water at 100° C. Find the resulting temperature. *Ans.* 10° C.

7. A piece of ice weighing 100 g. and having a temperature of – 15° C. is brought into a room where the temperature is 30° C. What thermal processes take place? What quantity of heat is involved in each process?

8. What mass of ice at 0° C. will be required to reduce the temperature of a kilogram of water from 100° C. to 20° C.?

SUGGESTION. — Let x be the required mass, and form an equation similar to that used in § 238.

240. Evaporation and Ebullition. — We have seen in § 235 that the change from the solid to the liquid state takes place *at a definite temperature* in crystalline substances. The change, however, from the liquid to the gaseous state occurs *at all temperatures* by the slow process of **evaporation.** The gas that rises from a liquid substance is called the **vapor** of that substance. Even ice and ice water evaporate. But by sufficient heating a liquid reaches a certain temperature at which the familiar process of boiling, or **ebullition,** begins. This temperature, which is called the *boiling point*, varies greatly with different substances and with the atmospheric pressure.

241. Evaporation Explained. — The process of evaporation is made clear by the help of the molecular theory (§ 213). At the exposed surface of a liquid the velocity of many of the molecules is sufficient to enable them to break through the surface beyond the range of attraction of the molecules of the liquid. If the temperature be raised, the average velocity of the molecules in the liquid is increased, and a more rapid surface loss will result. The removal of a large number of the most rapidly moving molecules in this manner decreases the *average kinetic energy* of those that are left behind. Consequently, the temperature of the remaining liquid is lowered by the process. Evaporation always takes place at the expense of the heat energy contained in the liquid.

242. Laws of Evaporation. — (1) *The rate of evaporation becomes greater as the exposed or free surface of the liquid is increased.*

A pint of water, for example, will evaporate faster when placed in a broad, shallow pan than when left standing in a pitcher. When spread over the floor, it disappears in a short time. Hence a wet cloth will dry faster when spread out than when left folded.

(2) *The rate of evaporation becomes greater as the temperature of the liquid and vapor is increased.*

As the temperature rises, the increased molecular motion enables molecules to break away from the surface at a greater rate.

(3) *The rate of evaporation is increased by the removal of the vapor from the space above the liquid.*

When the space above the liquid contains a quantity of the vapor, a great number of the vapor molecules moving about in all directions by chance strike the surface and reënter the liquid. The greater the number of molecules present in the vapor, the larger will be the number which return to the liquid. The return of the molecules which have once detached themselves from the liquid can be prevented by removing the vapor as fast as it is formed. This accounts for the fact that roads dry quickly on windy days, and ink is frequently evaporated by blowing upon the paper.

243. Vapor Pressure. — When a liquid is placed in a vacuum, it rapidly evaporates until a condition is reached when the quantity of vapor present becomes constant, *i.e.* when the number of molecules leaving the liquid per second is just equaled by the number which reënter it in the same length of time. The vapor is then said to be **saturated.** A vapor, like any other gas, exerts a pressure against the walls of the containing vessel. The amount of pressure exerted depends upon the temperature; if the temperature rises, more of the liquid evaporates and the pressure increases; if the temperature falls, some of the vapor condenses and the pressure decreases. *The pressure exerted by a saturated vapor above its liquid is called the* **maximum vapor pressure** *at the existing temperature.* For example, if a vessel of water be placed in a vacuum, it vaporizes at 0° C. until the vapor pressure is 4.6 millimeters of mercury; at 10° C., 9.16 millimeters; at 20° C., 17.39 millimeters, etc.

It is a peculiar fact that the maximum pressure exerted by a particular vapor in a closed space is independent of the pressure of other vapors that may be present. In other words, *the quantity of vapor required to produce saturation in a given space is the same whether that space is a vacuum at the beginning or is occupied by other vapors.*

244. Unsaturated Vapors. — When the vapor present in a given space is not enough to produce the condition of saturation, *i.e.* to produce the maximum vapor pressure at that temperature, the vapor is called **unsaturated vapor.** This will always be the case in a closed space in which an insufficient quantity of liquid is placed. On the other hand, when a vapor is kept in contact with its liquid, it is always saturated. For example, the vapor above a liquid in a tightly corked bottle is saturated, and evaporation cannot occur; but when the bottle is open, the vapor is always slightly unsaturated, and therefore a continual change of the liquid into a vapor takes place.

245. Atmospheric Humidity. — Since evaporation of water is always taking place at the surface of lakes, rivers, and other bodies of water, and also from the soil and vegetation, there is always more or less water vapor present in the atmosphere. That this is the case may be shown by the following experiment:

Fill a polished vessel or a glass beaker with ice water, and allow it to stand exposed to the air. In a short time drops of moisture will be seen forming on the exterior surface of the vessel.

Ordinarily the air does not contain saturated water vapor. But since the quantity of vapor necessary to produce the condition of saturation in a given space is less at low temperatures, the air in contact with the cold vessel soon reaches a temperature at which the water vapor already present becomes saturated. When the temperature is reduced below the point at which the water vapor is saturated, the vapor is condensed, and moisture is deposited upon the cold surface of the vessel. *The temperature at which moisture begins to form from the atmospheric water vapor is called the* **dew-point.**

We think of the air as being *dry* or *moist* (*i.e. arid* or *humid*) according as we feel that it contains little or much water vapor. These conditions of the air, however, involve (1) *the amount of vapor actually present and* (2) *the quantity necessary to produce saturation under the given conditions.* It is upon the relation of these two elements that the sensations of dryness and moisture depend. *The condition of the air, in regard to the water vapor which it contains, is expressed by the ratio of the mass of water vapor in a given volume of air to the mass of vapor required to produce the condition of saturation at the same temperature.* This ratio is called the **relative humidity** of the air. For example, if the quantity of water vapor actually present in a given space is 15 grams, and the amount required to produce saturation at that temperature is 20 grams, the relative humidity is $\frac{3}{4}$, or 75 %.

If air containing water vapor is caused to undergo a decrease of temperature, the relative humidity increases since the cool air is nearer to its point of saturation. If the cooling is carried far enough, moisture which we call *dew* is deposited on solid objects. The experiment of the beaker of ice water described above is an illustration of this effect. If the temperature at which the moisture is deposited is below 0° C., it is frozen as fast as it is formed and is called *frost*. Similarly, when any region of the air cools below the dew-point, particles of water produced by slow condensation collect about dust particles and produce *fogs* and *mists*. When a condensation takes place at high altitudes, *clouds* are formed. The slowly falling cloud particles may unite to produce a drop of *rain*. In cool seasons condensation may take place at temperatures below the freezing point. In this case the result is *snow*, *sleet*, or *hail*.

246. Ebullition. — It has already (§ 243) been stated that the saturation pressure (maximum vapor pressure) of water increases with the temperature. While at 10° C. it is only 9.16 millimeters of mercury, at 90° C. it is 525 millimeters, and at 100° C., 760 millimeters. It is clear, therefore, that at some definite temperature (viz. 100° C. for water) the maximum vapor pressure must be equal to a pressure of one atmosphere, or 760 millimeters. At this temperature the average speed of the molecules of the liquid becomes so great as to render the cohesive force between them unable longer to retain them. Hence small groups of molecules nearest the heated areas assume greatly enlarged volumes (bubbles) within which practically no cohesion exists, because of the vastly increased distance between the particles. Thus at this temperature ebullition, or boiling, takes place. This phenomenon is marked by the formation of bubbles of saturated vapor that rise to the surface and burst. The temperature at which this condition of the liquid is reached is the *boiling point* of the liquid.

247. Laws of Ebullition. — **1.** Fit a 2-hole rubber stopper in a test-tube. Thrust a thermometer through one of the holes and an open glass tube through the other. Place a small quantity of sulphuric ether in the test-tube, and hold it in a vessel of water at a temperature of about 70° C. Soon the ether will begin to boil, and the thermometer will indicate a steady temperature of about 35° C. (Caution. — On account of the high inflammability of ether vapor, the tube containing it should not be brought near a flame.) Place a finger over the end of the open glass tube, and thus carefully allow the pressure of the ether vapor to increase. The boiling point will be observed to rise several degrees.

2. Let a round-bottomed flask be half filled with water, and the water boiled for two or three minutes to enable the steam to expel the air. Close the flask with a rubber stopper, and invert it on a stand, as shown in Fig. 178. Although the temperature of the water will fall rapidly, the water can be made to boil vigorously by pouring cold water upon the flask.

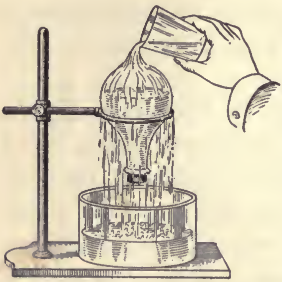


Fig. 178. — Water Boiling under Reduced Pressure.

3. Set a beaker of water at 90° C. or less under the receiver of an air pump and begin to exhaust the air. The water will boil vigorously as long as the pressure is kept sufficiently reduced.

The experiments just described illustrate the following general laws of ebullition:

(1) *Every liquid has its own boiling point, which is invariable under the same conditions.*

(2) *The boiling point of a liquid rises or falls as the pressure upon the liquid increases or decreases.*

The cold water poured upon the flask containing steam causes a portion of the vapor to condense. This reduces the pressure within the flask, thus lowering the boiling point to the temperature of the water. If the air has been very thoroughly expelled from the flask, the water may be kept boiling until it is scarcely lukewarm.

Because of the decrease of atmospheric pressure with an increase of altitude, the boiling point of water at Altman, Colo., probably the highest town in the United States, is about 88.5° C. Cooking processes at such heights are frequently accompanied by many difficulties. In a steam boiler, however, where the pressure is 125 pounds per square inch, the boiling point of water reaches 170° C.

Solid substances dissolved in a liquid raise its boiling point. A saturated solution of common salt boils at about 109° C. But the vapor rising from boiling brine is pure water vapor and condenses at 100° C. under an atmospheric pressure of 760 millimeters.

TABLE OF BOILING POINTS

Pressure 760 millimeters

Ether	35° C.	Benzine	80° C.
Chloroform	61° C.	Turpentine	160° C.
Alcohol	78.2° C.	Mercury	350° C.

248. Distillation. — When a liquid is vaporized in one vessel and the vapor afterwards condensed in another, the process is known as **distillation**. By this process pure water can be obtained from water containing dissolved substances and other foreign matter. The liquid to be distilled is placed in a vessel *A*, Fig. 179, and boiled. The vapor is conducted through the tube *B*, which is surrounded by a larger tube *C* containing a stream of cold water. The vapor is condensed on the cold walls of the small tube, and the resulting liquid runs out at the lower end into the vessel *D*.

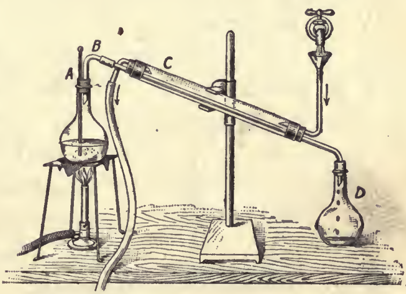


FIG. 179. — Illustrating the Process of Distillation.

If two liquids are mixed together and heated in vessel *A*, the one having the lower boiling point will be vaporized first. Its vapor can be condensed and collected in a separate vessel *D*. Alcohol is thus separated from fermented liquors, and gasoline and kerosene from crude petroleum.

249. Heat of Vaporization. — Fill a glass flask about half full of water, and place it over a flame to boil. Suspend one thermometer in the liquid and another a little above it. While the water is boiling, read both thermometers. They will continue to read practically alike.

Continue to apply heat until it is evident that the temperature of the water and steam is not raised above the boiling point.

The experiment shows clearly that *the heat applied to the flask is not utilized in raising the temperature of the water or of the steam.* As in the process of melting a solid (§ 238), heat is here transformed from kinetic into potential energy while changing the molecular condition of the water. For every gram of water vaporized a definite quantity of heat disappears. *The amount of heat required to change a gram of any liquid at its boiling point into vapor at the same temperature is called the* **heat of vaporization**[1] *of that liquid.* It represents the work that has to be done in producing a separation of the molecules of the liquid against their mutual attractions.

When *condensation*, the reverse of vaporization, takes place, an amount of energy equal to the heat of vaporization is given up by the condensing vapor. Thus the quantity of heat required to vaporize a certain mass of water at 100° C. is all delivered up by the steam when it returns to the liquid state.

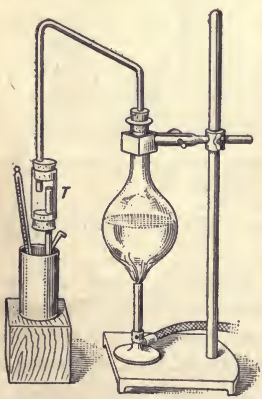


Fig. 180. — Measuring the Heat of Vaporization of Water.

250. Heat of Vaporization of Water Measured. — The heat of vaporization of water can be readily measured by allowing a known mass of steam to condense in, and deliver its heat to, a known mass of water.

Allow steam from a flask of boiling water, Fig. 180, to pass through a tube into a beaker containing, say, 400 grams of cold water. A trap T should be introduced in order to insure a flow of dry steam into the cold water. During the experiment note

[1] Sometimes called "latent" heat of vaporization.

the initial and final temperatures of the water. From 5° C. to 35° C. is a good range over which to work. The mass of steam condensed is found by ascertaining the gain in the mass of water in the beaker during the process.

For example, let the initial temperature of the water in an experiment be 5.6° C., the final temperature 35° C., and let the mass of water at the beginning be 400 grams, and at the close 419.5 grams.

Let x be the heat of vaporization of water.

The heat gained by the cold water $= 400\,(35 - 5.6)$ calories.

The heat lost by the 19.5 grams of water which was formed by the condensed steam on cooling from 100° C. to 35° C. $= 19.5\,(100 - 35)$ calories.

The heat delivered by the steam at 100° C. in changing to water at 100° C. $= 19.5\,x$ calories.

Hence, $$19.5\,x + 19.5\,(100 - 35) = 400\,(35 - 5.6);$$

whence, $$x = 538 \text{ calories.}$$

The heat of vaporization of water accepted by physicists is 536 calories.

251. Artificial Ice. — Certain substances that are gases under ordinary conditions of temperature and pressure *become liquids when the pressure is sufficiently increased.* This is the case of *ammonia* gas, the gas that is given off from common *aqua ammonia.* The pressure required to liquefy this gas under ordinary temperatures is about 10 atmospheres, or 150 pounds per square inch. On the other hand, liquefied ammonia returns to the gaseous state when the pressure is reduced, and *for each gram that vaporizes a quantity of heat equal to its heat of vaporization* (§ 249) *is abstracted from its surroundings.* Upon these principles is based the operations of the artificial-ice machines in common use.

An artificial-ice machine consists of three essential parts: (1) the compressor, (2) the condenser, and (3) the evaporator. See Fig. 181. The compressor is a pump which is run by an engine or motor whose function is to force ammonia gas under a pressure of about 10 atmospheres into the coils of the condenser C. Here the gas liquefies and gives up heat to the surrounding water which carries it away. From the condenser coils the liquefied ammonia passes through the regulating valve V into the coils of the evaporator E, where the pressure is kept below two atmospheres by the continual removal of ammonia gas by the compressor. The rapid vaporization of the liquid ammonia under the reduced pressure in these coils causes it to take heat from the surrounding brine. By this abstraction of heat the temperature of the brine is reduced to a point several degrees below the freezing point of water.

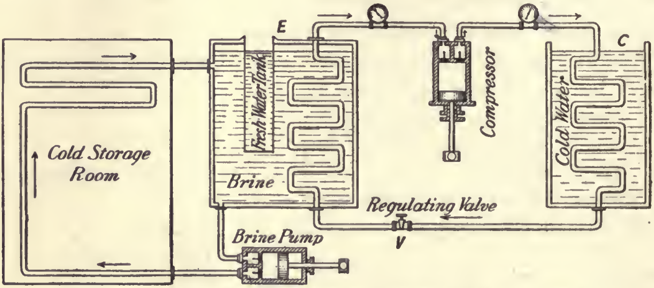


FIG. 181. — Artificial Cooling and Ice-making Apparatus.

In the production of ice the evaporator E is so constructed that metal vats of pure water of the desired size can be lowered into the cold brine and left until frozen. These vats are then withdrawn from the brine, and the ice removed to some place of storage.

The artificial cooling of storage rooms is brought about by cooling brine, as in the manufacture of ice. From the evaporator the cold brine is forced through coils of pipe placed about the walls of the rooms to be cooled. Inasmuch as there is no chance for the ammonia to escape, it can be used repeatedly with very little loss. The pressures are controlled by the regulating valve and may be read at any time from the gauges placed as shown in the figure.

EXERCISES

1. Explain the formation of moisture on the interior surface of windows.

2. The temperature of blades of grass and leaves of trees falls rapidly on cloudless evenings. What has this to do with the formation of dew?

3. Does heating the air in a room remove the water vapor? Why is the air in an artificially heated room usually "dry"?

SUGGESTION. — It is shown in § 245 that the dryness of the air does not depend wholly upon the water vapor present in a given space. The student should try to write out in full the entire explanation.

4. Heat a beaker of water over a flame, and observe that small bubbles rise to the surface long before the boiling point is reached. Compare this with the phenomenon of boiling.

5. When steam is allowed to flow through a tube into cold water, a loud sound is produced. Explain.

6. What becomes of the cloud that one sees near the spout of a teakettle? Is it steam?

7. Will clothes dry more quickly on a still or a windy day? Why?

8. How much heat is required to raise the temperature of 30 g. of water from 0° C. to 100° C. and convert it into steam?

9. If 50 g. of steam at 100° C. change into water at 40° C., how much heat is given out?

10. If the heat delivered by 10 g. of steam in condensing at 100° C. and cooling down to 0° C. were all applied to ice at 0° C., how many grams of ice would be melted? *Ans.* 79.5 g.

11. A vessel containing 600 g. of water at 20° C. is heated until it is one half vaporized. How many calories have been received?

12. The boiling point of water falls 1 centigrade degree for a decrease in pressure of 2.7 cm. Find the boiling point when the barometer reads 74.5 cm.

13. The boiling point of water falls 1 centigrade degree for an elevation of 295 m. above sea level. Find the temperature of boiling water at Denver, Colo., altitude 1600 m. above sea level.

14. If water boils at 85.5° C. at the top of Mont Blanc, what is the altitude?

15. If 20 g. of steam at 100° C. are passed into 500 g. of water at 5° C., what will be the resulting temperature? *Ans.* 29.2° C.

16. How much heat will be required to convert 150 g. of ice at 0° C. into steam at 100° C.?

2. THE TRANSFERENCE OF HEAT

252. Three Modes of Heat Transmission. — Heat is transferred from one point to another in three different ways; viz. by **conduction, convection,** and **radiation.** By *conduction,* heat passes through the metal walls of a stove or along a metal rod from heated regions toward cold ones. By *convection*, the currents of air set up by a hot stove transfer heat to the distant parts of the room. By *radiation*, energy from the sun reaches the earth; and, by this process, the hands held before the fire in an open grate or near a hot stove become warm.

253. Conduction. — The *conduction* of heat is simply the transference of molecular motion from those portions of a body where such motion is greatest to those where the motion is less, *i.e.* from the warmest parts of a body to colder parts, without producing any sensible motion in the intervening parts. The facility with which the conduction of heat takes place varies widely with the nature of substances.

Join together three similar bars of copper, brass, and iron as shown in Fig. 182, and heat the junction in a flame for several minutes. By sliding the tip of a sulphur match from the cold end of each bar toward the flame, ascertain at what point it ignites. In this manner it may be shown that the copper has conducted heat the farthest from the source and is, consequently, the best conductor. The iron will prove to be the poorest conductor of the three metals.

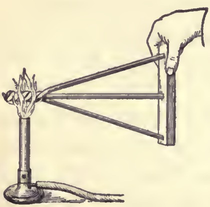


Fig. 182. — Testing the Conductivity of Metals.

Make tests by using a piece of glass tubing, the stem of a clay pipe, a piece of crayon, etc. These substances will be found to conduct very poorly.

254. Liquids and Gases Poor Conductors of Heat. — The conductivity of a liquid can be tested as follows:

Through a cork fitted in the neck of a glass funnel pass the tube of a simple air thermometer (Fig. 170), as shown in Fig. 183. Fill the funnel with water until the bulb is covered by about half an inch of the liquid. Pour about a spoonful of ether upon the water and ignite it. Although the flame is at all times separated from the bulb of the thermometer by only a thin layer of water, the liquid in the tube will remain stationary.

The experiment shows clearly that water is a very poor conductor of heat. Its conductivity is less than $\frac{1}{1000}$ that of copper. In general, all liquids, except mercury and other metals in a molten state, are to be classed as poor conductors.

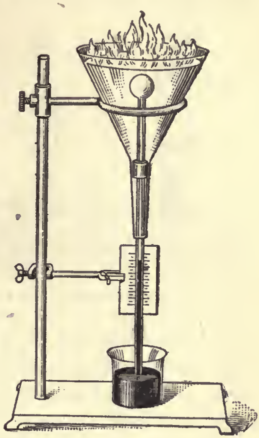


FIG. 183.—Testing the Conductivity of Water.

Gases have a lower conductivity than liquids. For this reason many substances, as wool, which inclose a large amount of air are poor conductors and are therefore used extensively in the manufacture of winter garments. Such articles of clothing owe their warmth to the fact that they prevent the loss of the natural heat of the body. Winter wheat and fruit trees are protected in a similar manner by a deep covering of snow. A flannel "holder" prevents the transference of heat from the flatiron to the hand, and a piece of ice wrapped in a woolen blanket is shielded from the heat of the atmosphere.

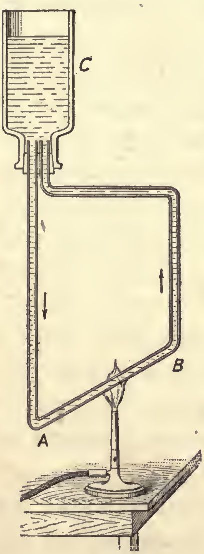


FIG. 184.—The Convection of Heat by Water.

255. Convection. — Heat is transferred in liquids and gases by the process of *convection*, *i.e.* by a general mass movement of the heated portions away from the source of heat.

1. Pass the ends of a glass tube, bent as shown in Fig. 184, through a rubber stopper fitted to the neck of a bottle from which the bottom has been removed. Fill the apparatus

with water, and place a small quantity of oak sawdust in the liquid to serve as an index of the motion. If a flame is moved back and forth between the points A and B, it will be seen that a current is set up in the direction from A toward B. In a short time the entire mass of water in the reservoir C will become hot.

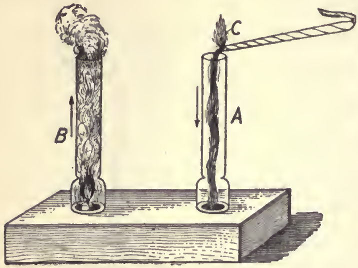


Fig. 185.—Convection Currents in Air.

2. Make two openings in the side of a crayon box, as shown in Fig. 185. Set a short candle over one hole, leaving an opening into the box. Set a lamp chimney over each hole and attach it to the box by means of melted candle wax. Light the candle and then hold some burning paper above chimney A. It will be observed that the flame and smoke of the burning paper will be drawn downward, thus showing the direction of the draft of air. At the same time heat is transferred upward from chimney B by the convection currents of the hot gases.

256. Convection Explained. — Convection is brought about by the expansion of fluids (*i.e.* liquids and gases) when heated. When a portion of the fluid is warmed, its volume increases, thus decreasing the density of the fluid. This portion of the body of the fluid is then forced upward in a manner similar to that in which a submerged piece of cork is forced up by water. (See § 121.) As the heated portions of the fluid rise, they carry their heat with them, and colder portions flow in to replace them. Convection currents are applied to the heating of buildings with hot air and hot water and to mine ventilation. The trade-winds and the Gulf Stream are convection currents of enormous proportions.

257. A Hot-air Heating System. — Figure 186 shows the method employed in heating a house by means of hot air. A furnace is placed in the cellar and supplied with fresh air through the duct A leading to the heating chamber *c, c.*

Here the air is heated and thence conducted through large pipes to the various rooms of the building. A large part of the air thus led into the rooms finds an outlet around the windows and doors. Sometimes provision is made for its escape into a cold-air flue leading through the roof. A cold-air duct *B* is often introduced for the purpose of reconducting the partially cooled air to the furnace, where it is again heated and sent into the rooms. For good ventilation,

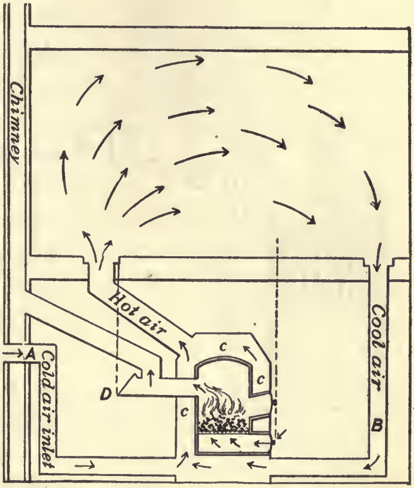


Fig. 186. — Hot-air Heating System.

however, an abundant supply of fresh out-door air should be admitted to the system at *A*. The circulation of air is indicated by the arrows. The fire in the fire-box is controlled by dampers. These are often regulated by chains extending to a room above, and shown here by the dotted lines.

258. A Hot-water Heating System. — A hot-water system of heating depends upon convection currents produced as shown in Experiment 1, § 255. Water is raised nearly to the boiling point in a heater *H*, Fig. 187, placed in the basement. From the heater it is conducted through pipes to iron radiators *R* placed in the various rooms of the building, while the cooler water from the radiators is led in return pipes back to the heater. Thus a continuous current of water is maintained until the pipes are closed by valves placed near the radiators. On account of the large exposed surface in each radiator, the heat emitted by the hot water is transferred to the surrounding air. In order to prevent the radiation of heat from the conducting pipes, a thick covering of asbestos, a very poor conductor of heat, is frequently provided. This plan of heating affords no means of ventilation.

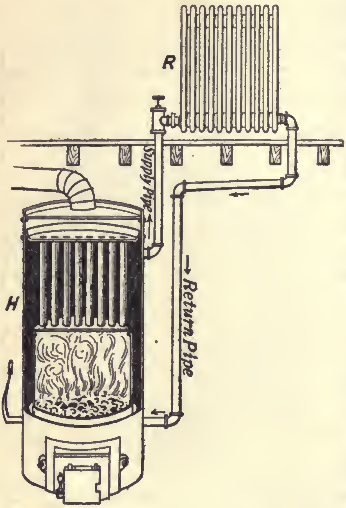


Fig. 187.—Heating a House by Means of Hot Water.

259. Radiation. — When we stand before a hot stove or a grate full of glowing coals, we readily perceive that the surface of the body nearest the fire is rapidly heated. However, the intervening air is not warmed as it would be if the heat passed to us by conduction or convection. Again, a room is frequently heated by the sun's rays, while the glass through which the rays pass remains cold. Plainly, the medium which transmits the heat energy in these instances is not the air nor the glass. In order that the phenomena just described and others of a similar nature may admit of explanation, it is assumed by physicists that all space is filled with an exceedingly light medium called the **ether.**[1] The properties of the ether are such that transverse wave motions are transmitted by it in a manner somewhat similar to that in which the waves produced by a falling pebble are carried along upon the surface of

[1] The term must not be confused with "ether," which is a well-known liquid and is used in several experiments.

water. Energy thus transmitted is called **radiant energy**, and the process is called *radiation.* If this energy affects the sense of sight, it is called **light.** When it falls upon the hands, it produces warmth. *Radiant energy becomes real heat only as it falls upon matter which is capable of absorbing it and converting it into the energy of molecular motion.*

260. Absorption of Radiant Energy. — The ability of a body to radiate energy depends both upon its temperature and the nature of its surface. Smooth and highly polished bodies radiate poorly, while rough, black bodies radiate well. On the other hand, bodies differ in their power to absorb radiant energy. Those that radiate well also absorb well.

Nail two pieces of tin A and B, Fig. 188, to a block of wood as shown. Coat the interior surface of B with lampblack and attach a match to the exterior surface of each with melted paraffin. Now place a hot iron ball midway between the plates. In a moment the wax on B will soften, and the match will fall.

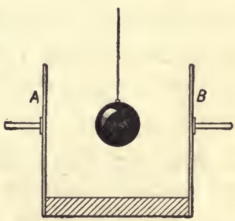


Fig. 188. — The Black Surface B Absorbs Heat Better than the Polished Surface A.

The experiment clearly shows that the blackened surface B absorbs the energy radiated by the ball faster than the bright one A. If pieces of black and white cloth are placed upon snow, the rapid absorption of the radiant energy of sunlight will cause the black body to melt its way into the snow. Since the white cloth reflects and transmits the greater portion of the energy, little remains to be converted into heat. This fact accounts for the general use of light-colored clothing in summer and, in part, for the stifling heat developed in attics under dark-colored roofing.

261. The Radiometer. — This interesting instrument, Fig. 189, was invented by Sir William Crookes,[1] of England, in 1873. It is used to detect radiant energy. The instrument consists of a glass bulb from which the air has been almost exhausted and within which four diamond-shaped mica vanes are delicately pivoted on light cross arms. One face of each vane is coated with lampblack. When radiant energy falls upon these vanes, a rotation is produced.

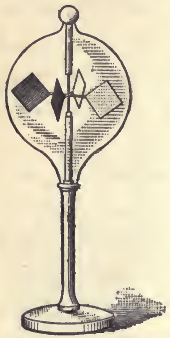


FIG. 189. — Crooke's Radiometer.

Since the blackened faces of the vanes absorb radiant energy, they are raised to a higher temperature than the bright faces. Thus the few remaining molecules of gas in the bulb have their speed greatly quickened as they come in contact with the black surfaces, and hence rebound from these faces with a strong reaction. It is this reaction that causes the vanes to move. The speed of rotation depends upon the intensity of the radiation falling upon the instrument.

262. Selective Absorption of Bodies. — Place a radiometer near a lighted lamp and between them set a glass beaker. After the rate of rotation of the vanes has become uniform, fill the beaker with water. The rate of rotation will be greatly diminished. Repeat the experiment, but fill the beaker with carbon disulphide. It will be found to have little effect on the motion of the vanes. Substitute a solution of iodine in carbon disulphide. Although nearly opaque to light, the solution will be found to transmit the radiations perfectly.

Water, which is transparent to the short, or visible, ether waves emitted by the lamp, transmits very poorly the longer waves of a slower rate of vibration. Likewise, glass transmits well the visible radiation (*i.e.* light) from the sun, but retards effectively the longer waves emitted

[1] See portrait facing page 466.

by the objects in a room. Substances like glass and water which absorb long waves are called **athermanous** substances. While the glass in a window admits light energy into a room, the energy of the longer waves sent out from the heated objects within is retained. The glass of a green-house or hot-bed transmits well the energy of short waves to the soil within, but the longer waves emitted by the heated soil cannot escape. Hence the temperature rapidly rises.

On the other hand, the carbon disulphide and the iodine solution transmit well the waves of the ether that are far too long to affect the sense of sight. Such substances are called **diathermanous** substances.

263. The Sun as a Source of Heat. — The process of radiation plays an important part in everyday life. The sun is continually sending out great quantities of radiant energy in all directions in space. A small fraction of this energy falls upon the earth's atmosphere, passes readily through it without producing any appreciable change, and reaches the earth's surface. Here a large part of the energy of the ether waves is transformed into heat, *i.e.* is absorbed. The earth also radiates heat; but being of a low temperature, the waves emitted by it are longer. Since the presence of water vapor in the atmosphere renders it athermanous, the radiation of energy away from the earth is greatly hindered.

It is the radiant energy from the sun converted into heat that evaporates water, resulting in the production of vapor and rain. Rains produce the flow of rivers and thus give rise to the energy derived from waterfalls. The wood we use and the food we consume owe their value to the energy which they have stored up within them. This they derive from the sunlight and warmth in which they grow. Coal received its energy from the plants that flourished under the solar radiation of past ages. It is this energy that we utilize in warming our houses, cooking our food, and that we convert into mechanical energy through the help of the steam engine for running factories and aiding transportation over both land and water.

EXERCISES

1. In the construction of brick and cement houses the walls are often made hollow. Why?

2. Why does a piece of iron feel colder than a piece of wood when both have the same temperature?

3. What is the normal temperature of the blood on the centigrade scale? of a living room?

4. Does woolen clothing supply the heat that maintains the temperature of the body?

5. Explain under what conditions a workman might be led to wear woolen garments to keep himself cool.

6. Why are coverings of sheets of paper often sufficient to prevent plants from freezing on frosty nights?

7. Why does dew seldom form on cloudy nights? Why are frosts almost entirely prevented by the presence of clouds?

8. In a fireless cooker a kettle containing vegetables that have been boiled a short time is surrounded by wool, felt, etc., and left a few hours to complete the process of cooking. Explain.

9. The poor conductivity of glass causes a tumbler to crack when hot water is poured into it. Explain. Why does not a thin glass beaker crack from the same cause?

3. RELATION BETWEEN HEAT AND WORK

264. Heat and Mechanical Energy.— The experiments made in section 212 show clearly that an intimate relation exists between heat and work. The production of heat at the expense of mechanical energy is one of the most common phenomena of nature. In fact, the energy expended in nearly all mechanical processes passes finally to the form of heat. An inquiry into the reverse transformation of

COUNT RUMFORD (SIR BENJAMIN THOMPSON)
(1753–1814)

The name of Rumford is prominent among the early physicists who engaged in confuting the theory that heat was a substance. His attainments as a soldier and public benefactor, however, are of no less interest.

Thompson was born on a farm near Woburn, Massachusetts, and as a young man was employed in teaching school. Having been made a major in the local militia by the governor of New Hampshire, he became the object of mistrust by the friends of American liberty. On this account, in 1776 he removed to London. Here his advance was rapid; within four years he became undersecretary of state. In 1779 he was elected a member of the Royal Society. In 1783 he planned to aid the Austrians against the Turks, but while on his way he met Prince Maximilian (afterwards elector of Bavaria), who induced him to enter the Bavarian military service. For eleven years he remained at Munich as minister of war, minister of police, and grand chamberlain to the elector. He reorganized the Bavarian army, suppressed begging, provided employment for the poor, and established schools for the industrial classes.

In 1791 Thompson was made a count of the Holy Roman Empire and took the name of Rumford. In 1799 he was instrumental in founding the Royal Institution of London and selected Sir Humphry Davy as the first lecturer. In remembrance of the help that he received in his early days from attending some lectures by Professor Winthrop at Harvard, Thompson later gave an endowment which founded the professorship that bears his name. His last years were passed near Paris, where he died in 1814. His tomb is at Auteuil.

energy, *i.e.* from heat into mechanical energy, is an interesting consideration.

Let a tube, bent as shown in Fig. 190, project through a rubber stopper into a flask *A* half filled with water. The tube should extend nearly to the bottom of the vessel. Heat the water over a burner, and the water will be elevated into a tumbler at *B*.

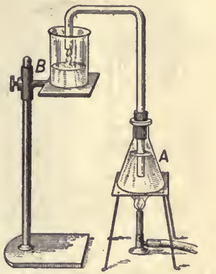


FIG. 190. — Water raised by Heat Energy.

In this experiment work is performed upon the water, and *heat is converted into potential energy*, which is stored in the elevated water.

265. Early Historical Experiments. — In the early development of the subject scientists looked upon heat as a kind of material substance called *caloric*. An important step toward ascertaining the true nature of heat is found in the experiments of Count Rumford.[1] He showed that one horse used as a source of power could develop sufficient heat by friction to raise 26.5 pounds of water from the freezing to the boiling point in $2\frac{1}{2}$ hours. About this time Sir Humphry Davy (1778–1829) showed that two pieces of ice kept below the freezing point could be melted by rubbing them together. The first, however, to establish the relation between heat and work by expressing one in terms of the other was James Prescott Joule[2] of Manchester, England.

266. Joule's Experiment. — The method employed by Joule to ascertain the exact relation between the calorie and the unit of mechanical energy consisted in measuring the heat produced by a definite quantity of work. The heat under consideration was produced by the rotation of paddles in a vessel of water *C*, Fig. 191. The work done upon the water produced heat enough to raise its

[1] See portrait facing page 256. [2] See portrait facing page 258.

temperature an appreciable amount. The rise in temperature being measured, the quantity of heat developed could be computed as the product of the mass of water and its change in temperature (§ 231). The paddles were turned by two weights *W*, *W*, attached to cords so arranged as to rotate the main shaft. The work performed by the paddles could be computed as the product of the weights and the distance through which they descended (§ 55). Thus the work corresponding to a calorie of heat could readily be determined.

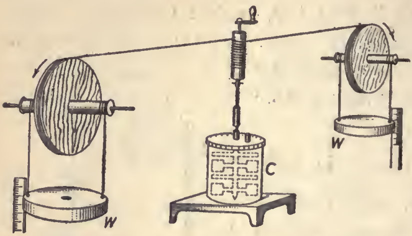


Fig. 191.—Illustrating Joule's Method for Determining the Mechanical Equivalent of Heat.

267. The Mechanical Equivalent of Heat. — Joule's experiments upon the relation of heat and work extended over more than one half his life. They have been carefully repeated (1879) by Professor Rowland of Johns Hopkins University, with apparatus of more refinement and precision. As a result of these experiments the following value of the calorie is generally accepted by physicists:

1 *calorie* = 427 *gram-meters, or* 41,900,000 *ergs.*

This result is known as the **mechanical equivalent of heat**, or simply **Joule's equivalent.**

268. Conversion of Energy. — The experiments performed by Rumford, Joule, Rowland, and others, relative to the conversion of mechanical energy into heat, serve to verify the principle of the Conservation of Energy first stated in § 64. Ignorance of this great law of nature has led men at all times, and even in this enlightened period, to undertake to construct devices whereby useful work

JAMES PRESCOTT JOULE (1818–1889)

The attention of Joule was turned at an early age in the direction of physics and chemistry by the influence of his teacher, John Dalton, the chemist. Under his tuition, Joule was initiated into mathematics and trained in the art of experimentation. His renown rests upon the thoroughness with which he established the doctrine of the Conservation of Energy (§64) upon an experimental basis. His epoch-making experiments were made to determine the mechanical equivalent of heat, which is recognized as one of the most important physical constants. Measurable quantities of energy were expended in revolving paddles in water, mercury, and oil, and cast iron disks were rotated against each other and the resulting quantity of heat ascertained. The results of these experiments left no doubt that the amount of heat produced by a definite quantity of mechanical work is fixed and invariable. For this great scientific achievement Joule received the Royal Medal of the Royal Society of England in 1852, and eight years later, when men of science more fully understood the value of the discovery, he was presented with the Copley Medal.

Joule was the son of a wealthy brewer of Manchester, England, at which place he carried on his experiments. Important laws relating to the heating of electrical conductors and valuable contributions to the subject of electro-magnetism must also be accredited to him. The *joule,* which is used as a unit of energy, has been so named in his honor.

can be obtained without the expenditure of an equivalent amount of energy of some form. Such devices, if possible, would supply enough energy to keep their parts in motion when once started, and are therefore called *perpetual-motion machines.* Any attempt, however, to secure useful energy from the wind, sunlight, ocean waves, tides, etc., is praiseworthy, and should be encouraged. To some extent, such efforts have not been fruitless. But since in the best machines that can be made some friction will exist, there will be a continual conversion of a part of the energy supplied to the machine into heat. If, therefore, energy is supplied only in starting the machine, no matter how it is constructed, it is sure to come to rest. Consequently, a machine at best can only transfer or transform the energy with which it is supplied.

The transformation of mechanical energy into heat is easily accomplished. For example, a bullet is warmed by its impact with a target, mercury can be warmed by being shaken vigorously in a bottle, a mass of shot will rise in temperature if allowed to fall several feet, a button grows hot when rubbed upon a piece of flannel, etc. The reverse transformation, *i.e.* from heat into mechanical energy, is not, however, so readily effected. The process, nevertheless, is accomplished in steam and gas engines by making use of certain properties of gases.

269. Gases Heated by Compression and Cooled by Expansion. — We have seen in § 212 that the energy used in compressing a gas is converted into heat. On the other hand, *when a gas is allowed to expand against pressure and perform work, heat is given up and the temperature of the gas falls.* In other words, molecular energy is expended by the gas when it does work. In order to utilize this process in converting heat into useful work, the gas is allowed to expand in a cylinder and thus move a piston *P*, Fig. 192.

It is clear that, in the case illustrated, the gas performs work in raising the weight W to a higher position. A modification of this process is employed in all steam and gas engines.

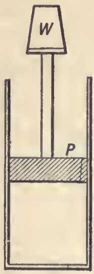


FIG. 192.—By Moving a Piston P a Gas Does Work on the Weight W.

270. The Reciprocating Steam Engine. — The essential parts of an ordinary steam engine are the cylinder, the piston, and the slide-valve mechanism, represented diagrammatically in Fig. 193. The gas employed is steam generated by the combustion of fuel. A to-and-fro motion is given to the piston P by the force exerted by the steam which is applied to its two faces alternately. The operation is as follows:

Steam under a pressure of several atmospheres (*i.e.* 100 to 250 pounds per square inch) enters the steam chest S from the boiler. From S the steam finds an entrance into the cylinder through the port N and drives the piston to the left, forcing any gas that may be contained in the space C' through the port M and out of the engine through the exhaust pipe E. The motion of the piston is communicated to the main shaft A through the connecting rod R and the crank D. As the piston approaches the left end of the cylinder, the sliding valve V is moved to the right by the eccentric F and the eccentric rod R', thus

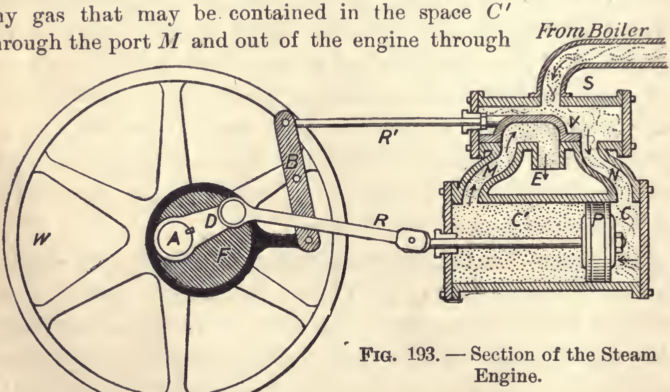


FIG. 193. — Section of the Steam Engine.

admitting "live" steam through M into the cylinder chamber C', and opening the port N to allow the expanded steam to escape. The piston is now driven back to the right, and the sliding valve V is forced back to its former position just before the piston reaches the end of its stroke as at first. The operations just described are then repeated. The shaft of the engine is provided with a heavy fly wheel E in order to maintain uniformity of speed.

In the so-called *non-condensing* or *high-pressure* engines the exhaust steam escapes through the exhaust pipe E into the open air. The piston of such an engine therefore moves continually in opposition to the pressure of the atmosphere. This disadvantage is partially removed in *condensing* engines. In engines of this type the exhaust pipe E conducts the exhaust steam to a condensing chamber in which a spray of cold water hastens its condensation. By the aid of a pump operated by the engine, the water together with the condensed steam is removed from the condensing chamber, leaving a back-pressure of only a few ounces per square inch instead of one atmosphere. Thus by decreasing the back-pressure against the piston, a larger quantity of useful work can be obtained from the steam, and the efficiency of the engine correspondingly increased. Condensers are not used on locomotives (1) because of the large supply of cold water necessary and (2) because of their inconvenient size.

271. The Gas Engine. — With the development of the automobile has come that of the gas engine as a source of power. To this class of machines belong all engines utilizing an explosive mixture of gases as the working agent, such as air and illuminating gas, or air and gasoline vapor. The operation of the so-called "four-cycle" gas engine in common use is shown in Fig. 194.

C is a cylinder within which moves the piston P. The piston is connected with the crank B by means of the rod A. Upon the crank shaft D is mounted a heavy fly wheel W, which is set in motion by the hand on starting the engine. When the piston moves downward to the position shown in *(1)*, an explosive mixture of gas and air is drawn into the cylinder through the inlet valve I. As the motion continues, the piston moves upward and compresses the mixture in the top of the cylinder, as shown in *(2)*. At about the instant the piston reaches the highest point, as in *(3)*, an electric spark at the spark-plug S ignites the gas; an explosion ensues, with the production of much heat, and the expanding gases exert an enormous pressure on the top of the piston. This forces the piston violently downward, giving motion, and hence kinetic energy, to the heavy fly wheel *W*. On the next upward stroke the products of the combustion of the gas are driven out of the cylinder through the exhaust valve, as shown in (*4*). This valve is opened automatically at the proper instant. The piston having traversed the length of the cylinder four times, the

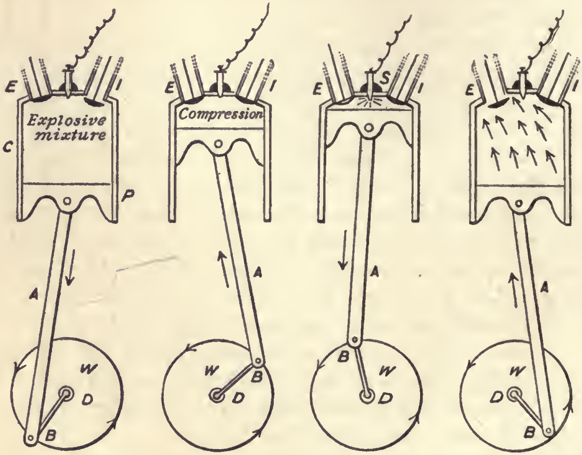


Fig. 194. — Operation of a Four-cycle Gas Engine.

initial conditions are restored, and the operations are repeated. It is obvious that the fly wheel must be made heavy, since the energy given it during the third stroke of the piston has to keep the engine and machinery in motion with almost constant speed during the three following strokes of the cycle.

When gasoline is used as fuel, the inlet pipe *I* leads from the "carburetor," into which the liquid enters as a spray, vaporizes, and is mixed with the proper amount of air. In order to prevent undue heating of the cylinder and piston, a current of water is kept in circulation through cavities cast in the walls of the cylinder. Some manufacturers supply so-called "air-cooled" engines, the cylinders of which are cooled by the circulation of air about their exterior surface. In this instance the cylinder is cast with numerous projections for the purpose of increasing as much as possible the amount of radiating surface.

On account of the lightness and compactness of the engine, and the small space occupied by the fuel, gasoline engines are extensively used to propel automobiles, in which motors of from one to six cylinders may be seen. Such engines are also widely used in launches, dirigible balloons, aeroplanes, pumping stations, machine shops, factories, etc., on account of the small attention required in their operation.

272. The Steam Turbine. — In the common reciprocating form of the steam engine a large amount of energy is lost in stopping and starting the piston and connecting rods at the end of each stroke. It

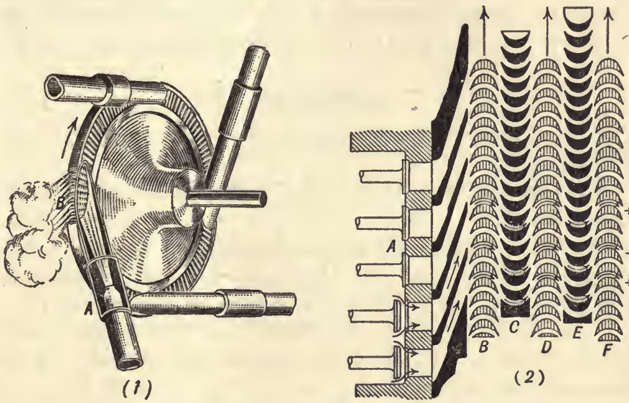


FIG. 195. — Principle of the Steam Turbine.

is only within the last few years that inventors have succeeded in designing efficient engines of the purely rotary type. The operation of a steam turbine is as follows:

Steam under high pressure is conducted through a series of stationary jets A, (*1*), Fig. 195, arranged in a circle, which directs it obliquely against a series of blades B which are attached to a rotating drum, called the *rotor*. The rotor is fastened to the main shaft of the engine. By the impact of the steam these movable blades are impelled in an upward direction and thus produce rotation. (*2*), Fig. 195, shows the arrangement of several series of movable and stationary blades used in the more powerful turbines. Each movable blade is a curved projection attached to the exterior surface of the rotor; each stationary blade is fastened to the interior surface of the metal case surrounding the rotor. The concave surfaces of the two sets of blades are turned opposite to each other as shown. The number of series used will vary largely in turbines of different power. After the steam has passed through the first series of movable blades B, a series of blades C, which are stationary, serves to direct it at the proper angle against the next series of movable blades D, and so on through the entire turbine.

The steam turbines will find extensive use in ocean-going steamships on account of the fact that they are free from the objectionable vibration that always accompanies engines of the reciprocating form. At the present time they are replacing the ordinary steam engine in the generation of electrical power, and in a few years, no doubt, will be found wherever energy is to be derived from steam.

EXERCISES

1. Explain how the energy contained in coal can be utilized in performing work.

2. What energy other than that of coal is often employed in running factories, etc.?

3. The heat developed by the combustion of a gram of coal of a certain grade is 5000 calories. How many kilogram-meters of work could be done if all the energy could be used for this purpose?

4. If all the potential energy stored in a 500-kilogram mass of rock at an elevation of 200 m. were converted into heat, how many calories would be produced?

5. The energy of a falling body is transformed into heat when it strikes. Compute the number of calories of heat produced when a 10-gram mass of iron falls 25 m.

Suggestion. — Compute in gram-meters the energy of the given mass at an elevation of 25 m., then reduce to calories.

6. A steam engine raises 8000 four-pound bricks to the top of a building 75 ft. high. How many calories of heat are thus expended? If the efficiency of the engine is 10 % (§ 100), how many calories must be developed by the combustion of the coal that is used?

7. How high could 100 g. of ice be elevated by the amount of heat required to melt the same amount, if all the heat could be utilized for that purpose?

8. Show that the energy required to vaporize 1 g. of water at 100° C. is equivalent to the work done by a force of 10 kg. in moving a body a distance of 22.89 m. in the direction of the force.

9. Show that it requires more energy to raise the temperature of

100 g. of iron from 0° C. to 100° C. than to elevate a weight of 400 kg. through a height of 1 m.

10. The average pressure of the steam in the cylinder of an engine is 125 lb. to the square inch and the area of the piston is 50 sq. in. Compute the work done by the steam during each 20-inch stroke of the piston.

11. If the mechanical equivalent of the calorie defined in § 231 is 3.1 foot-pounds, compute the heat units lost by the steam in Exer. 10 at each stroke of the piston.

12. At what rate does a 2-horse-power engine consume coal when working at its full capacity, if its efficiency is 10 % and the coal produces 5500 calories per gram?

SUMMARY

1. The state of a body depends largely on its temperature. The temperature at which a solid changes to a liquid is its *melting* or *fusing point*. Definite laws govern the fusion of all crystalline bodies (§§ 234 to 236).

2. The process of liquefaction is accompanied by an absorption of heat. Molecular kinetic energy (heat) is converted into molecular potential energy in the process. The *heat of fusion* of a substance is the number of calories per gram required to liquefy it without changing its temperature. The heat of fusion of ice is 80 calories. Noncrystalline substances when heated pass through a plastic state and have no definite melting point (§§ 237 and 238).

3. When a liquid solidifies, an amount of heat equal to the heat of fusion is given up for each gram (§ 239).

4. *Evaporation* may take place at any temperature. It is a process by which many of the more rapidly moving molecules of a body become detached and pass into the surrounding space. The gas composed of these detached molecules is called a *vapor* (§§ 240 and 241).

5. The *rate of evaporation* of a liquid varies with the amount of exposed surface and its temperature, and is decreased by the presence of the vapor of the liquid in the space around it (§ 242).

6. When the number of molecules leaving a liquid per second equals the number reëntering it, the vapor is *saturated.* Its pressure at this time is called the *maximum vapor pressure* at that temperature (§ 243).

7. The quantity of a given vapor required to produce saturation in a given space is the same whether the space is occupied by other vapors or not. This quantity depends, however, on the temperature (§§ 243 and 244).

8. On account of the abundance of water, the atmosphere always contains more or less water vapor. The temperature to which air would have to be reduced to cause moisture to form is called its *dew-point* (§ 245).

9. The *relative humidity* of the air is the ratio of the amount of water vapor present in a given volume to the amount required to produce saturation at that temperature (§ 245).

10. Every liquid has its own boiling point, which is invariable under the same conditions. This point rises or falls according as the pressure upon the liquid is increased or decreased (§§ 246 and 247).

11. The heat of vaporization of a liquid is the number of calories required to convert 1 g. of it at its boiling point into vapor at the same temperature. The heat of vaporization of water is 536 calories. This amount of heat is given up by the vapor when it condenses (§§ 249 and 250).

12. Heat is transferred by *conduction*, *convection*, and *radiation.* Substances are classed as good and poor conductors of heat. In general, liquids (except liquid metals) and gases are poor conductors (§§ 252 to 254).

13. Liquids and gases transfer heat by a general mass movement of the heated portions away from the source of heat, *i.e.* by *convection.* Convection is the result of the expansion which accompanies a rise in temperature. Heating systems depend on the convection of heat by air, steam, or hot water (§§ 255 to 258).

14. *Radiation* is the process in which energy is transferred by ether waves. The energy of these waves is transformed into heat when absorbed by bodies. The best radiators and absorbers are rough, black bodies. The sun is our great source of energy. This energy we receive through the process of radiation (§§ 259 to 263).

15. Heat may be made to perform work. One calorie is equivalent to 427 gram-meters, or 41,990,000 ergs. This result is known as *Joule's equivalent*, or the *mechanical equivalent of heat* (§§ 264 to 267).

16. Heat energy is transformed into mechanical energy in steam and gas engines. A gas does work in expanding against a movable piston. As a result of the work done, the temperature of the working gas is lowered (§§ 268 to 272).

CHAPTER XIII

LIGHT: ITS CHARACTERISTICS AND MEASUREMENT

1. NATURE AND PROPAGATION OF LIGHT

273. Meaning of the Term "Light." — Just as *sound* is defined as undulations in the air, or some other medium, that produce the sensation which we call "sound," so *light*, in the same sense, consists of undulations or waves in the ether that produce the sensation which we often call by the name "light." (See § 259.) Not all ether waves can be regarded as light waves, since not all affect the organ of sight; but all ether waves, from the longest to the shortest, transfer energy, and therefore may properly be classed as carriers of *radiant energy*.

274. The Ether and Ether Waves. — The theory that light is wave motion in the ether was advocated by the Dutch physicist Huyghens (1629–1695) in 1678. The theory, however, was not well established until the beginning of the nineteenth century, when the experiments by Thomas Young of England and Fresnel of France placed it on a firm basis. Ether fills all interstellar space as well as the spaces between the molecules in bodies of matter. The ether is also of extreme rareness, or tenuity, since planets passing through it suffer no appreciable retardation in their orbits.

Ether waves possess several well-known characteristics. They are transverse waves and are propagated with a definite speed, and this speed becomes less when they pass through matter such as glass, air, water, etc. Ether waves may be reflected, transmitted, bent from their courses, or their energy may be transformed into other forms than radiant energy.

Ether waves that produce an effect upon the sense of vision vary in length between about 0.00004 and 0.00008 centimeter. Hence our sense of sight, with its narrow limitations, does not enable us to perceive directly ether waves which are shorter than the former of these two numbers or longer than the latter.

275. Speed of Light. — An achievement of great scientific importance was the discovery that *light travels with a definite speed.* Previous to the year 1676 it was supposed that light moved infinitely fast, because no one had found a way to measure so great a velocity. In that year the Danish astronomer Roemer (1644–1710), as the result of several months' work with the instruments at the Observatory of Paris, correctly inferred that the time required for light to traverse the diameter of the earth's orbit (about 186,000,000 miles) was almost 1000 seconds. Roemer was led to this conclusion after making a series of observations upon the eclipses of one of the satellites of the planet Jupiter. At each revolution of the satellite *s*, Fig. 196, in its orbit around Jupiter *J*, it passes behind that planet into its shadow and becomes invisible from the earth at *E*. By measuring the interval between two successive eclipses of the satellite it was apparently possible to predict the precise time of each eclipse for many months in advance. But when the earth was at *E′*, on the opposite

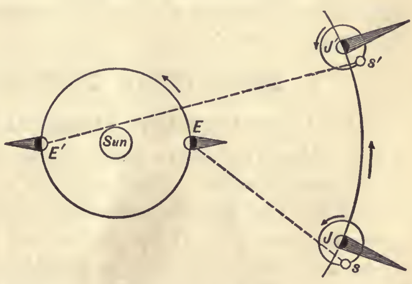


FIG. 196. — Roemer's Method for Determining the Speed of Light.

side of the sun, the time of each eclipse of the satellite was found to be about 1000 seconds later than predicted. In order to account for this difference, Roemer advanced the idea that this interval was precisely the time that light requires to pass over the diameter of the earth's orbit. *On this assumption the speed of light is* 186,000,000 ÷ 1000, *or* 186,000 *miles per second.*

Recent measurements of the speed of light by different methods continue to show that it is about 186,000 miles, or 300,000 kilometers, per second.

EXERCISES

1. The circumference of the earth is about 25,000 mi. How many times could this distance be traversed by light in a second?

2. The distance of the north star from the earth is so great that it requires about 43 yr. for its light to reach the earth. Express the distance in miles.

3. How many minutes are required for light to reach the earth from the sun?

4. Since the sun moves (apparently) through 360° in 24 hr., over what arc will it move while a light wave is on its way from the sun to the earth?

2. RECTILINEAR PROPAGATION OF LIGHT

276. Light Travels in Straight Lines.— Set up a small screen about midway between a candle flame and the wall so that it casts a well-defined shadow. Mark the edge of the shadow, and extinguish the candle. Stretch a cord between the candle wick and the line marking the edge of the shadow, and it will be found to graze the edge of the screen. Since the cord is straight, the course taken by the light from the candle to the wall is a straight line.

Further evidence regarding the fact that light follows straight lines may be obtained by observing the path taken by a beam of light as it enters a partially darkened room where the air contains dust. We unconsciously utilize this important fact in many ways. In order that we may see an object, light must come from that object to the eye;

and *we always assume that the object sending the light to us is located in the straight line which marks the direction of the light as it enters the eye.* The marksman trains his gun along the line of direction of the light which comes from the object he wishes to hit, and the carpenter selects a straight piece of lumber by "sighting" along its edge.

We shall see later, however, that light deviates from a straight line under certain conditions, but that the deviation is ordinarily inappreciable.

277. Shadows. — *A* **shadow** *is a space from which the light from a luminous body is wholly or partially excluded by an opaque body.* The nature of a shadow depends both upon the form of the opaque body and upon the form of the source of light.

1. Hold an opaque body, as a book, between a very small source of light, as an electric arc light, and a white wall or screen. A very sharply outlined shadow will be produced upon the screen for all positions of the opaque body.

2. Place two electric arc lights about 15 centimeters apart in a line parallel to a screen or wall, and produce a shadow, as in Experiment 1. Two portions of the shadow are now easily distinguished, viz. a dark central part and a partially illuminated area just outside. The experiment may be performed with a single gas flame or by using two oil lamps placed a few centimeters apart.

When the source of light *L*, Fig. 197, is small, and an opaque body *AB* intercepts the light, a region of darkness

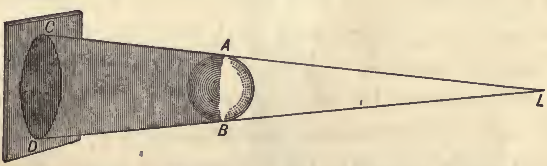


Fig. 197. — Illustrating the Shadow Cast by a Sphere.

ABCD is produced behind it as shown by the shading. This *space* is the shadow of *AB*. It is obvious that the form of the shadow may be found by drawing straight lines from L just touching the edge of the object AB. If AB is a sphere, it is clear that the form of the shadow will be that of a truncated cone whose top rests against the sphere.

When two sources of light L and L', Fig. 198, are used, no light from the source L enters the region $CABD'$,

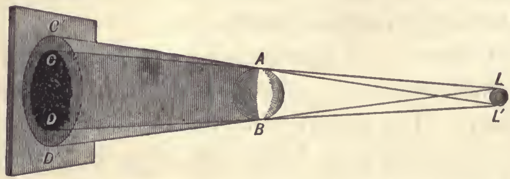


Fig. 198. — Showing the Production of Umbra and Penumbra.

and none from L' enters the space $C'ABD$. Now the space $CABD$ lies within both of these spaces and hence *receives no light from either source.* The remaining shaded portions *receive light from one or the other of the sources.* Furthermore, if other luminous points exist between L and L', the space $CABD$ will receive no light from any of them. *The portion of a shadow that is wholly dark is called the* **umbra**, *and the portions that are only partially illuminated are called the* **penumbra**.

278. Eclipses Produced by Shadows. — In the preceding section it was seen that the section of a shadow that falls upon a wall or the ground will have a distinct outline only when the source of light is small. If, therefore, the source of light is the sun, we find no sharply defined shadows, *i.e.* every shadow is surrounded by an indistinct region which is partially illuminated.

Let the sun, Fig. 199, be the source of light, and the earth the opaque body. By drawing lines tangent to both sun and earth, as shown, we find that the earth casts a shadow of which the umbra is cone-shaped and has its apex at *A*. Surrounding this is the penumbra which varies from total darkness near the umbra to practically full illumination near its outer limits. If the moon *M* in its monthly revolution about the earth, in the orbit shown by the dotted line, passes *completely* into the earth's umbra, it receives no light from the sun and is thus eclipsed. Since we see the moon only by the light which it reflects from the sun to the eye, this phenomenon constitutes *a total eclipse of the moon.* But if only a portion of the moon enters the earth's umbra, it suffers only a partial *eclipse*.

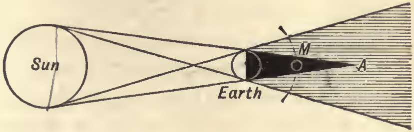


FIG. 199. — Showing the Moon Eclipsed by Entering the Earth's Umbra.

279. Pin-hole Images. — Images are readily produced by means of small apertures. If a hole 2 or 3 millimeters in diameter is made in the window shade of a darkened room, images of trees, clouds, and other outdoor objects will be produced on a screen held a short distance from the opening. Each dimension of an image is proportional to the distance from the aperture to the screen, but the larger the image, the less distinct it is in every detail, since the light is distributed over a larger area.

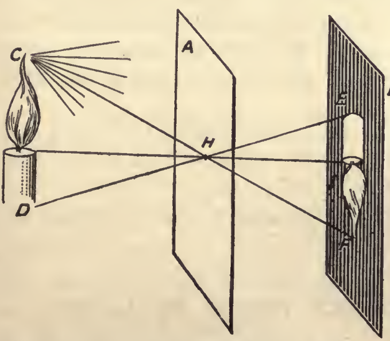


FIG. 200. — An Image Produced by a Small Aperture.

The reason for the formation of images in this manner is made clear in Fig. 200. *CD* is a candle, *A* an opaque piece of wood or cardboard having a small aperture at *H*, and *B* is a white screen. Light from the tip of the candle *C*, for example, falls at all points on *A*, but only that falling at H is transmitted. This portion follows the straight line CH to F. Likewise, only light from D can fall at E on the screen. Thus the portions of light from the several points of the object CD build up the inverted image EF.

Numerous images of the sun may often be observed upon the sidewalk when the light passes through the small openings between the leaves of a tree. These images assume interesting, crescent-shaped figures during a partial eclipse of the sun.

EXERCISES

1. Hold a book in direct sunlight, and from the section of the shadow that is outlined upon the floor, infer whether we should treat the sun as a point source of light. Describe the shadow.

2. Hold a ball in direct sunlight about 5 ft. from the floor or wall, and ascertain whether or not it casts a distinct shadow. Do the same beneath an uncovered electric arc light. Draw figures to illustrate the difference in the two shadows.

3. Describe the shadow cast by the moon. In what direction does its umbra point? Does its umbra ever reach the earth?

4. When the moon enters the earth's umbra, is its darkening gradual or sudden? Explain.

5. If the earth should pass into the moon's umbra, what phenomenon would be observed by a person standing in the shadow? Would any of the sun be visible? Would any of the sun be visible to a person standing in the moon's penumbra?

Suggestion. — Draw a figure representing sun, moon, and earth in such a position that the umbra of the moon just touches the earth.

6. If the sun's rays make an angle of 45° with the horizontal plane, how long is the shadow cast on level ground by a vertical pole 50 ft. high?

7. A vertical rod 10 ft. in height casts a shadow 12 ft. long on a level sidewalk. How tall is a tree whose shadow at the same time is 72 ft. in length?

8. How could one find the height of a building by employing the method suggested by Exer. 7?

3. INTENSITY AND CANDLE POWER OF LIGHTS

280. Intensity of Illumination. — If one realizes that the waves of light that are sent out from any given source spread out in all directions, it is readily inferred that the intensity of illumination will decrease as one recedes from the luminous body. We are also led to the same conclusion by the fact that we *decrease* the distance from a lamp to a printed page when we wish to *increase* the amount of illumination. The exact law is readily shown by experiment.

Cut in a large cardboard screen A, Fig. 201, an aperture just 2 inches square. Place the screen 1 meter from a point source of

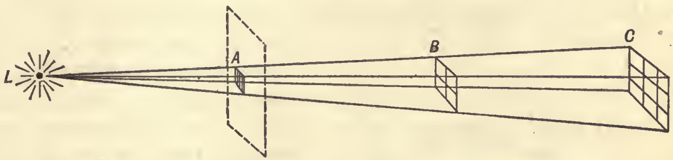


FIG. 201. — The Intensity of Light Varies Inversely as the Square of the Distance.

light, preferably an electric arc. Now place a second screen B, upon which is drawn a square precisely four times as large as the aperture in A, *i.e.* 4 inches square, 2 meters from the light. The light, which at a distance of 1 meter falls upon an area A, at a distance of 2 meters is found to cover precisely 4 equal areas. Hence each area at B receives only one fourth as much light as a similar area at A. When the second screen is carried to C, a distance of 3 meters from L, the light which passes through A illuminates 9 equal areas at C. Hence each area at C receives only one ninth as much light as an equal area at A.

It is now plain that when the distance from a source of light is doubled, the intensity of illumination is divided by 4; and when the distance is made three times as great, the intensity of illumination is $\frac{1}{9}$. Hence the experiment leads us to the conclusion that *the intensity of illumination is inversely proportional to the square of the distance from the source of light.*

281. Candle Power of Lights. — The law of intensity shown in the preceding section is used to compare the illuminating powers of two sources of light. If the intensity of one of the lights is known, that of the other can be found.

Place a lighted candle A, Fig. 202, 1 meter from a paper screen S and four similar candles at a point B, the same distance on the opposite side of S. It is now clear that the side of the screen facing the 4 candles receives 4 times as much illumination as the other. But the two illuminations may be equalized by moving the 4 candles to a greater distance. If, now, a drop of oil or candle wax is placed on the paper screen, it becomes possible to ascertain when the illuminations are equal, since the spot will look alike on the two sides when viewed at the same angle. To produce equal illumination on the two sides of S (*i.e.* to divide the illumination produced by the stronger light by 4), it will be found necessary to move the 4 candles to a distance of 2 meters from the screen. Hence the light-producing powers of the two lights are *directly* proportional to the squares of their respective distances from the screen.

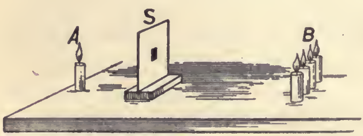


FIG. 202. — Showing a Method of Measuring the Candle Power of Lights.

It is clear that this method may be employed in the comparison of the light-emitting powers of two sources. The process is to set the lights so that they illuminate the two sides of a screen equally; then *the ratio of the squares of their respective distances from the screen expresses the ratio of the intensities of the two lights.* A screen upon which is an oiled spot is used in the *Bunsen photometer* for the measurement of the power of lights.

The unit used in the measurement of the power of lights is called a **candle power** and is approximately the power of a sperm candle of the size known as "sixes" (meaning six to the pound), burning 120 grains per hour.

The candle power of a Welsbach gas lamp consuming about 3 cubic feet of gas per hour is from 50 to 100, and that of ordinary open gas flames is from 15 to 25, while the consumption of gas is from 5 cubic feet per hour upward. The incandescent electric lamps containing a carbon filament in most common use are of 16 candle power, but those of greater power can be procured.

EXERCISES

1. A 2-candle-power light is placed 1.5 m. from a screen. Where must an 8-candle-power light be placed to produce the same illumination on the screen?

2. In measuring the candle power of an electric light it was found that a 4-candle-power light placed 2 m. from a disk produced the same illumination as the electric light at 10 m. Compute the power of the electric light.

3. If a book receives ample illumination when placed 10 ft. from a 50-candle-power lamp, how far must it be placed from a light of 5-candle-power to be equally well illuminated?

SUMMARY

1. Light, physically speaking, consists of ether waves which produce the sensation called *light*, *i.e.* which excite the optic nerve (§§ 273 and 274).

2. The speed of light is about 186,000 miles (300,000 km.) per second (§ 275).

3. Light is propagated in straight lines in a uniform medium. This fact gives rise to shadows, eclipses, pinhole images, etc. (§§ 276 to 279).

4. When a luminous body is of appreciable size, the shadows of opaque bodies consist of two parts, the *umbra*, or region of no illumination, and the *penumbra*, or partial shadow.

5. The intensity of illumination is inversely proportional to the square of the distance from a source of light (§ 280).

6. The illuminating power of a source of light is measured in terms of the *candle power*. This unit is about equal to the power of the ordinary household candle (§ 281).

CHAPTER XIV

LIGHT: REFLECTION AND REFRACTION

1. REFLECTION OF LIGHT

282. Reflection and Transmission. — It is a familiar fact that a piece of glass both reflects and transmits light; for we frequently see the bright sunlight reflected by the glass of a window when we are outside, although, as we know, a large portion of the light is transmitted to the interior of the house.

By means of a mounted mirror M, Fig. 203, reflect a bright beam of sunlight upon a pane of glass AB, held obliquely. If a sheet of paper be placed behind the glass at C, the transmitted light will fall upon it; if, again, it be placed in the position D, it will be brightly illuminated by reflected light.

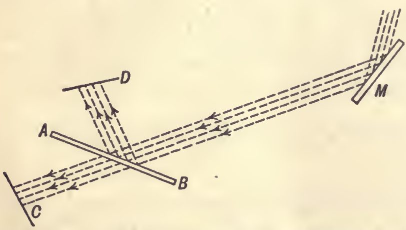


FIG. 203.—Reflection and Transmission of Light by a Pane of Glass.

The ordinary mirror makes use of the reflection of light from the surface of the opaque film of mercury that covers its back. Polished metals are often excellent reflectors.

283. The Law of Reflection. — Every one is accustomed to the manner in which light is reflected by a mirror on account of the many purposes which it serves in everyday life; but the following experiments may be performed in order to establish the law which ordinary observation does not reveal:

1. By means of a mirror held in the hand, reflect a beam of sunlight in various directions, and observe the position of the mirror in each case. Attach a cardboard index so that it shall be perpendicular to the mirror, and observe how it points in relation to the beam of light before and after its reflection. It will be found that the index always points in a direction midway between the direct and reflected beams of light, as shown in Fig. 204.

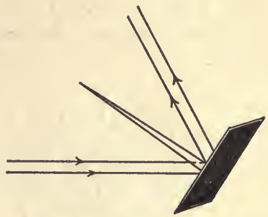


FIG. 204. — Illustrating the Reflection of Light by a Mirror.

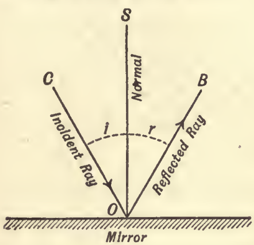


FIG. 205. — The Angle of Reflection r Equals the Angle of Incidence i.

2. Attach a block of wood to a plane mirror, and set it upon a line ruled across a large sheet of paper. Place a small candle about a foot from the mirror at C, Fig. 205. Now place the eye near the plane of the paper, and set two pins in line with the image of the candle wick seen in the mirror. Draw a line, as BO, through these pins to the mirror. Draw also a line, as CO, from the center of the candle to the point where the first line intersects the mirror. Draw the line OS perpendicular to the mirror at this point. Angles COS and BOS will be found to be equal.

Now part of the light from the candle C follows line CO to the mirror and line OB after being reflected. Angle COS is called the **angle of incidence**, and angle BOS the **angle of reflection.** In every case it will be found that these two angles are equal. Hence, *the angle of reflection equals the angle of incidence.* It is to be observed that these two angles are in the same plane, which in Experiment 2 is represented by the plane of the paper.

284. Diffused or Scattered Light. — Objects are visible to us either by the light which they emit, as in the case of the sun, a candle, or a live coal, or by the light which, after falling upon them from some luminous body, they scatter, or *diffuse.* Most objects, unlike a smooth piece of glass, reflect light in many directions. Thus the sunlight which falls upon the snow is diffused; but when it falls upon smooth ice, it is reflected as from a mirror. This is because the tiny reflecting surfaces of snow lie in all con-

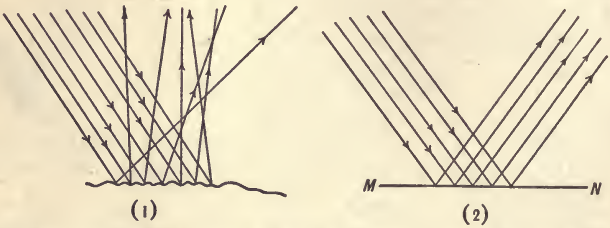


FIG. 206. — Diffusion Compared with Reflection from a Smooth Surface.

ceivable positions, as shown in (1), Fig. 206, while those of ice all lie in one smooth plane, as in (2).

By the help of the light which objects send to our eyes, we judge of their distance, form, size, color, and brilliancy. Leaves, grass, flowers, etc., diffuse in every direction the sunlight that falls upon them. The moon also is visible because of the sunlight diffused from its illuminated surface; and we are often able to trace the dim outline of the new moon, although it is in shadow, because of the sunlight which the earth diffuses back upon the moon's dark area.

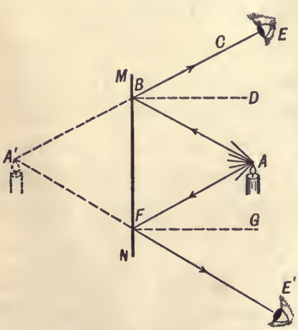


FIG. 207. — Production of an Image by a Plane Mirror.

285. Image of a Point in a Plane Mirror. — It was found in § 283 that light is reflected by a plane mirror so that the angle of reflection is equal to the angle of incidence. Hence the light which starts from the point A, Fig. 207, and takes the direction AB is reflected by the mirror MN in the direction BC, so that angle CBD equals angle ABD. All other rays that may be drawn from A to the mirror are reflected in the same manner; and when the eye is placed at E or E', the reflected rays appear to come from a point A' behind the mirror.

286. Waves and "Rays." — It is easy to conceive of a train of waves moving outward from the point A, Fig. 208, and striking against a plane mirror MN. The waves are sent back from the mirror as though they emanated from the point A' behind the mirror. Hence, to an eye placed at E the effect is just the same as though A' were the light-emitting point. It is obviously more convenient to locate the image of a point by the help of "rays," as in Fig. 207, rather than by the use of waves, as in Fig. 208. However, *it should always be remembered that a so-called "ray" of light is simply a symbol used to represent the direction taken by a portion of a wave.* Thus that part of a wave of light which starts from A toward B in Fig. 207 follows the course ABC.

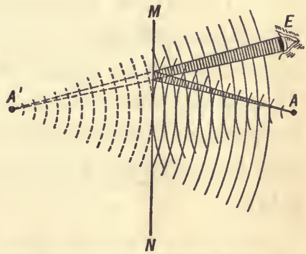


FIG. 208. — Reflected Waves of Light from A Seem to Come to the Eye from A'.

287. Image of an Object in a Plane Mirror. — By applying the law of reflection it is easy to locate the image produced by a plane mirror. Let AB, Fig. 209, be an object and MN the mirror. Let AC be an incident ray from A drawn perpendicular to the mirror. The reflected ray will take the direction CA. (Why?) Let AD

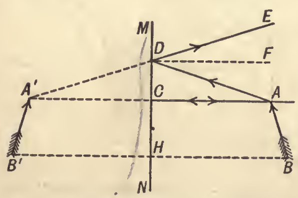


FIG. 209. — Manner of Locating an Image Illustrated.

be another incident ray from A, whose direction DE, after being reflected, is found by making angle EDF equal angle ADF. The image of A lies at the intersection A' of the reflected rays CA and DE produced backward behind the mirror. It is plain from the equality of the triangles ACD and $A'CD$ that AC equals $A'C$; *i.e. the image of a point is as far behind a plane mirror as the point itself is in front.* We may now employ this fact in locating the image of the point B at B'.

288. Seeing the Image. — Imagine an eye to be placed at E, Fig. 209. Light enters the eye from the mirror MN as though it came from A', although it actually comes from A. Similarly, the eye receives light by other rays as if its origin were at B', whereas it is really at B. There is nothing behind the mirror that concerns our vision, and the light is not propagated by the medium except in front of the mirror. The image is called a **virtual** image to distinguish it from the **real** images formed by small apertures (§ 279) and in other cases, to be studied later.

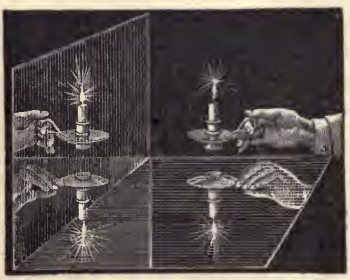


Fig. 210. — A Result of Double Reflection.

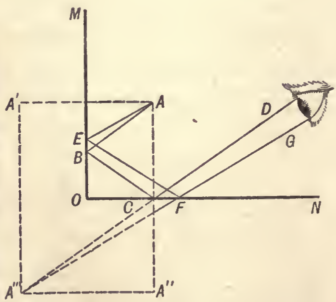


Fig. 211. — Diagram. Illustrating Double Reflection.

289. Double Reflection. — If two plane mirrors are placed at right angles to each other, as shown in Figs. 210 and

211, it is clear that a large portion of the light emanating from a point A will be reflected twice, once at the surface of each mirror. Thus the ray AB is reflected from B to C by the vertical mirror OM, and from C toward D by the horizontal mirror ON. Likewise, the ray AE takes the course $AEFG$, being reflected at E and F. Now FG and CD, and all other rays that have undergone double reflection, diverge as though they emanated from the point A''', which is at one corner of the rectangle $A'AA''A'''$. It is therefore evident that three images may be seen by placing the eye in such a position as to receive light that has suffered two reflections as well as that which has been reflected but once.

EXERCISES

1. A pole is inclined at an angle of 45° to the surface of water in a quiet pond. Construct the image of the pole seen in the water.

2. Look at your image in a mirror, and lift your right hand. Which hand of the image appears to be lifted? Is the image direct or reversed?

3. Set a candle or a tumbler on a horizontal mirror, and observe the position of the image.

4. What would be the result of covering the mirror OM in Fig. 211? How could the formation of image A''' be prevented without interfering with image A' or A''?

Suggestion. — Place an opaque screen so that no light can be reflected twice. Show how this can be done.

5. A man approaches a plane mirror with a velocity of 3 m. per second. How rapidly is he approaching his image?

6. Try to read a printed page by looking at its image in a mirror. Write your name backward on a sheet of paper, and then look at the image of the writing in a mirror. What effect is produced by the mirror in each case?

7. Find by construction the shortest vertical mirror in which a man 6 ft. tall can see his entire image when standing erect.

Suggestion. — Diagram the case, and then draw lines from the man's eye to the highest and lowest points of the image. Consider the length of the mirror employed between these two limits.

8. Two lines AB and BC make an angle of 60° with each other. Show how to place a mirror so that AB may represent an incident ray of which BC is the reflected ray.

2. REFLECTION BY CURVED MIRRORS

290. Spherical Mirrors. — A spherical mirror is a polished or silvered portion of the surface of a sphere.

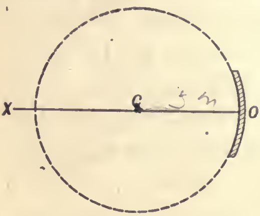


FIG. 212. — Section of a Spherical Mirror.

If the side of the surface toward the center of the sphere is used to reflect light, see Fig. 212, the mirror is *concave;* if the outer surface is used, the mirror is *convex.* The center of the sphere C is the **center of curvature**, and the radius OC of the sphere is the **radius of curvature.** The line of symmetry XO is called the **principal axis.**

Let direct sunlight fall upon a concave mirror parallel to the principal axis, and hold a small card in front of the mirror to receive the reflected light. Move the card back and forth until the illuminated spot is as small as possible. Measure the distance from this spot to the mirror. If a piece of tissue paper be held at the spot where the reflected light is concentrated, it will probably take fire. If the air in front of the mirror be filled with crayon dust, the convergence of the reflected light is easily made visible.

Figure 213 shows the effect produced when parallel rays of sunlight fall upon a concave mirror. At every point on the mirror light is reflected according to the law of reflection (§ 283); but on account of the curvature of the mirror, each reflected ray from the beam of parallel rays is sent through the point F, which is therefore called the **principal focus.** When the energy thus concentrated at the principal focus

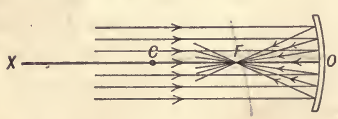


FIG. 213. — A Beam of Parallel Rays is Focused at the Point F.

falls upon paper, enough of the energy is transformed into heat to ignite it. The principal focus is located midway between the center of curvature and the mirror; *i.e.* OF equals one half OC. The distance OF is called the **focal length** of the mirror or the **principal focal distance.**

291. Convex Mirrors. — Try to concentrate direct sunlight by employing a convex mirror in the same manner as the concave mirror. While sunlight is falling upon the mirror, look toward its convex surface through a piece of black glass. An exceedingly bright point will be seen located apparently behind the mirror.

Figure 214 illustrates the manner in which a convex mirror reflects the parallel rays of sunlight. The light at every point follows the law of reflection; but on account of the form of the surface, it diverges as though it came from the point F behind the mirror. Of course no heat will be produced at the point F, inasmuch as the light does not actually pass through it.

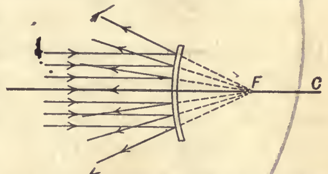


Fig. 214.—Parallel Rays are Made Divergent by a Convex Mirror.

Since F is only an *apparent* meeting point or focus of the reflected rays, it is called a **virtual** or **unreal** focus. The principal focus of a *concave* mirror, however, is a **real** focus. See § 288.

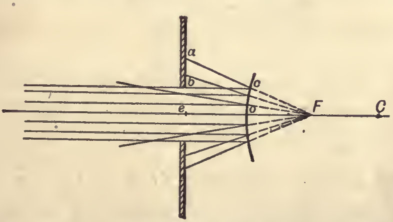


Fig. 215.—Determining the Focal Length of a Convex Mirror.

To locate the principal focus of a convex mirror, let a beam of sunlight pass through a round hole in a piece of cardboard, as shown in Fig. 215. Around this aperture draw a circle whose radius is just twice that of the aperture. Let the beam fall upon a convex mirror, and then move the mirror back and forth until the reflected light just covers the larger circle. Triangles *abc* and *coF* are practically equal. (Why?) Hence the distance from the cardboard to the mirror oe is equal to oF, the focal length of the mirror.

292. Images Formed by a Convex Mirror. — The student is probably familiar with the small image that is seen as one looks at the polished surface of a glass or metal ball or the convex side of the bowl of a spoon. The experiment may be made with a lighted candle, as shown in Fig. 216. *The image will appear to be behind the mirror and is always smaller than the object*, which in this case is the candle. Compare with Fig. 208.

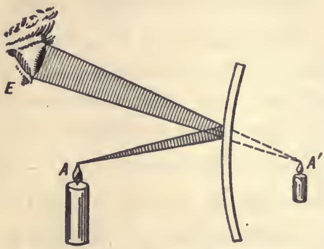


Fig. 216. — Image Formed by a Convex Mirror.

293. Constructing the Image. — To show diagrammatically how an image is produced by a convex mirror, let the arc MN, Fig. 217, whose center of curvature is at C, represent a convex mirror. Let the arrow AB be the object, and draw the ray AG parallel to the principal axis OX. The reflected ray GD will apparently come from F (§ 291), which is midway between O and C. Let a second ray AH be drawn from A along a radius of the mirror. Since this ray falls perpendicularly upon the mirror, it will be reflected back along the same line HA. Now let the reflected rays GD and HA be produced until they intersect at some point, as a behind the mirror. This locates the image of the point A. The image of the point

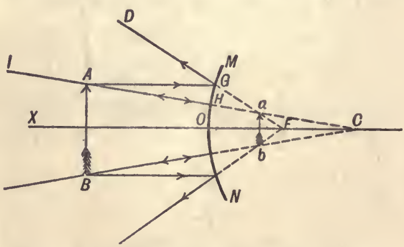


Fig. 217. — Locating the Image of AB by Construction.

B may be located in the same manner at *b*. A line drawn from *a* to *b* represents, therefore, the image of the object *AB*. Is the light from *A actually* focused at *a*? Is the image therefore *real* or *virtual?* Is it *erect* or *inverted?* Is it *larger* or *smaller* than the object? Could any rays other than those selected be employed? How would you find the direction of any other reflected ray? (See § 283.)

294. Images Formed by a Concave Mirror. — The nature of the images produced when light from some object, as a candle, falls upon a concave mirror is readily shown by a series of experiments. Excellent results can be obtained by using a mirror whose radius of curvature is 20 inches or more.

1. Let a lighted candle be placed before a concave mirror at a distance somewhat *greater than the radius of curvature.* Place a small

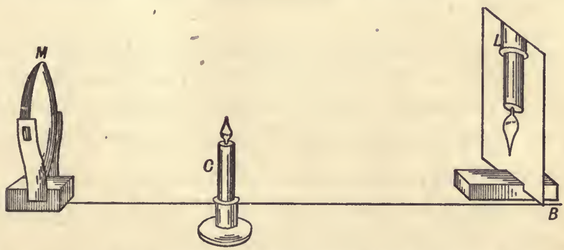


FIG. 218. — Production of a Real Image by a Concave Mirror.

cardboard screen between the candle and the mirror, and move it back and forth until a good image of the candle appears upon it. The image will be found between the principal focus and center of curvature.

This image differs greatly from that produced by a convex mirror in that it can be caught upon a screen. We are not obliged to look into the mirror to see the image, because the light that emanates from a point in the candle is *actually* reflected to a corresponding point on the screen. Such an image is a *real* image. The experiment plainly shows that *when the object is beyond the center of curvature, the image is between the center and the principal focus, is real, inverted, and smaller than the object.*

2. Let the candle be placed at any point *between the center of curvature and the principal focus* and the image caught upon a screen. In this case the screen has to be placed beyond the center. (See Fig. 218.)

Here we shall readily find that *when the object is placed between the center and the principal focus, the image is beyond the center, is real, inverted, and larger than the object.*

3. Let the candle be placed *between the principal focus and the mirror.* In this case, in order to locate the image, direct the eye toward the mirror in such a manner as to receive some of the reflected light. An erect image will be seen.

When the object is between the principal focus and the mirror, the image is behind the mirror, is virtual, erect, and larger than the object.

4. Project upon a screen the images of some *distant* object, — clouds, trees, buildings, etc. In all cases it will be found that the images are small and lie practically midway between the mirror and its center of curvature. Parallel rays of sunlight are also focused at this point.

The experiment shows that *the image of an object at a great distance lies near the principal focus, is real, inverted, and smaller than the object itself.*

295. Construction of Images Formed by Concave Mirrors. — We have seen in § 293 how an image can be located by geometrical construction. The same method may be applied to the cases arising from the use of a concave mirror.

Case I. — When the object is beyond the center of curvature.

Let MN, Fig. 219, be a concave mirror whose center is C. Let the object be AB. Locate first the principal focus F (§ 290). Now let a ray AG be drawn from the point A of the object parallel to the principal axis. This ray will be reflected through the point F.

(Why?) Let a second ray AD be drawn through the center of curvature C. Since this ray is perpendicular to the surface of the mirror at

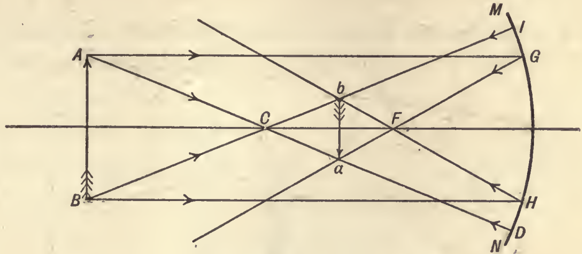


FIG. 219. — A Real Image of AB is Located at ab.

D, the reflected ray takes the direction DC. The two reflected rays GF and DC obviously meet at the point a. Could other incident rays be drawn from A? How could their direction be found after reflection? Where would they meet the reflected rays already drawn? Hence the image of A is at the point a. In a similar manner the image of the point B is located at b. Thus ab is the image of the object AB.

When two points are so related that the image of one falls at the other, as A and a, or B and b, they are called **conjugate foci** (pronounced *fō′ sī*).

Case II. — When the object is between the center and the principal focus.

The conjugate foci of the points A and B are located by a method similar to that used in the preceding case. (See Fig. 220.) The two

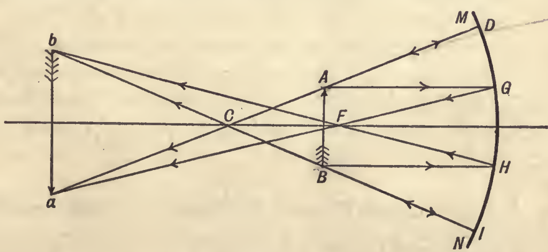


FIG. 220. — A Real Image of AB is Located at ab.

cases should be compared, and the constructions actually made. A real, inverted, and magnified image is found at ab.

Case III. — When the object is between the principal focus and the mirror.

As we undertake here to carry out the method of construction used in the preceding cases, we find that the reflected rays emanating from

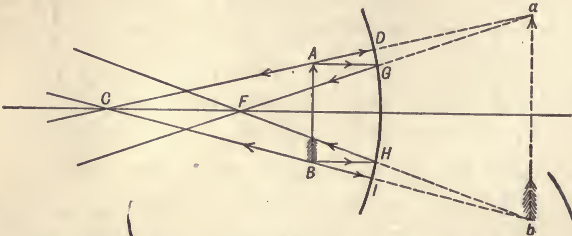


FIG. 221. — A Virtual Image of AB is Formed at ab.

the point A diverge after leaving the mirror. (See Fig. 221.) This fact shows at once that A can have no real focus. The image will be seen only by looking into the mirror. In such cases the reflected rays DC and GF are to be produced until they meet at some point as a behind the mirror. Similarly the image of B is found at b. Thus a magnified, erect, and *virtual* image is found at ab.

Case IV. — When the object is at the center of curvature or at the principal focus.

When light from a point at the center of curvature falls upon a concave mirror, it strikes at an angle of 90° with the reflecting surface and is, consequently, reflected back along the same path. Hence all such rays will be focused at the center of curvature. But, when light from a point placed at the principal focus is reflected, it follows lines parallel to the principal axis; *e.g.* FGA and FHB, Fig. 219. Since such rays never meet, no image of the point F could be produced.

296. A Real Image Viewed Without a Screen. — We have already seen (§ 294) that a real image of a bright object can be projected by a concave mirror upon a suitable screen. Now if the eye be placed about 10 inches beyond the image and turned toward the mirror, the screen may be removed, and the image will be visible. Imagine an eye at E, Fig. 222. Light waves emanating from A, a point in the object, advance toward the mirror with convex wave fronts, as those of water waves which are started by a falling pebble. These are here represented by arcs, having their common center at A. But on account of the curvature of the mirror MN, the waves are reflected with concave fronts having as their common center the point a, which is called the *conjugate focus* of A. As the waves are not obstructed by a screen, they leave a with convex fronts and enter the eye at E.

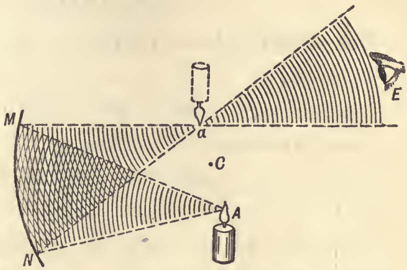


FIG. 222.—The Eye Can Observe a Real Image without a Screen.

EXERCISES

1. An object is placed 6 in. in front of a convex mirror whose radius of curvature is 12 in. Find by construction the position of the image.

SUGGESTION.—Make a drawing, using for blackboard work the dimensions and distances given, but divide each by 4 for pencil drawings.

2. An object is placed 44 cm. from a concave mirror whose radius of curvature is 50 cm. Find by construction the location of the image.

3. Place a small object slightly above the center of curvature of a concave mirror whose principal axis is horizontal, and find its image by construction. Does this exercise suggest a method for finding the radius of curvature by experiment?

4. An object 8 in. in height is placed 30 in. in front of a concave mirror whose radius of curvature is 15 in. Find the distance from the mirror to the image.

5. Make an accurate construction of the case described in Exer. 4, and carefully measure the size of the image. Measure also the distances of the object and the image from the center of curvature. Do you find any relation between the distances and the sizes of image and object? If so, express it in a single sentence. Can you prove the same relation by similar triangles?

3. REFRACTION OF LIGHT

297. Refraction of Light. — Numerous examples of the refraction or bending of the course taken by light come before our attention daily, although we seldom give the phenomenon much thought. A simple case is the apparent bending of a spoon standing in a tumbler of water, or an oar at the point where it enters the water. Again, if a coin be placed in a tumbler of water and viewed obliquely, two coins become visible, — a small one seen through the horizontal surface of the water and a magnified one seen through the side of the vessel. If, now, a pencil be placed obliquely in the water contained in the tumbler, a bent section may be seen below the upper surface of the liquid, while a magnified portion is visible through the side of the vessel. In every case the illusion is due to the bending of light rays as they pass from one medium into another. No principles of optics are of more value to us than those relating to the phenomenon of refraction, for upon this effect are based not only our most important optical instruments, including the microscope, telescope, and camera, but also the structure of the eye.

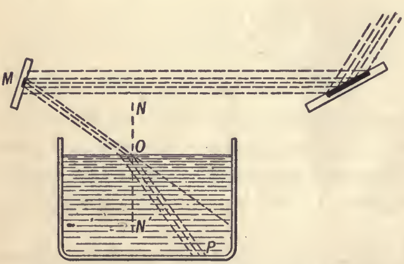


Fig. 223. — Refraction of Light as it Enters Water.

298. Refraction Illustrated. — **1.** By means of the mirror M, Fig. 223, about one half an inch in width, let a beam of sunlight be reflected obliquely upon the surface of water in a tank. By scattering crayon dust in the air above the water, the course of the beam before and after entering the water becomes visible. Another excellent way to make the path of the light easy to trace is to hold a piece of white cardboard or tin partly under water so that it receives the beam of light both above and below the liquid surface. The result will show that at the surface of the liquid O the beam of light turns toward the perpendicular, or normal, NN' which is drawn at O. If the obliquity of the incident light is increased, the bending of the beam is made more pronounced.

This experiment shows that *when light passes obliquely from air into water, it undergoes a refraction or bending toward the perpendicular to the surface at the point where the beam enters the water.*

2. Place a plane mirror in the bottom of the tank used in the preceding experiment so as to reflect a beam of light from water into the air, as shown in Fig. 224. The course of the beam MOP may be traced before and after entering the air by employing the means used in the preceding experiment. In fact, the path of the light MOP is precisely the reverse of that which the beam would take if it were passing into water along the line PO.

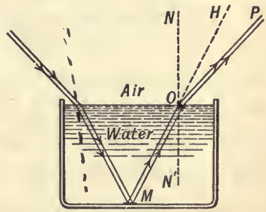


FIG. 224. — Refraction of Light as it Emerges from Water.

It will readily be observed in this experiment that *when light passes obliquely from water into air it undergoes refraction away from the perpendicular to the surface at the point where the beam emerges from the water.*

The angle MON, Fig. 223, is called the **angle of incidence,** and angle PON', the **angle of refraction.** The perpendicular ON is usually called the normal at the point O.

299. Cause of Refraction. — It has been found by direct experimentation that light waves travel with less speed in water than in air; in fact, the speed of light in water is almost exactly three fourths of that in air. When a beam of light AB, (1) Fig. 225, strikes at right angles to the surface of water CD, all parts of a given wave front strike the medium at the same time. Within the water the waves travel with less speed and are shorter. Likewise, if all parts of the wave front emerge at the same time, they resume their original speed without being refracted. But when the waves fall obliquely upon the surface, the case is quite different. See (2), Fig. 225. The parts of a given wave front *abcd* do not enter the medium at the same instant; but *a* enters first and continues with reduced speed, while the other parts *bcd* are still in air.

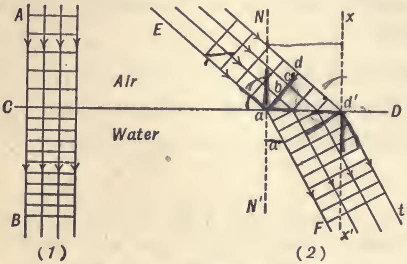


FIG. 225.—Illustrating the Cause of Refraction.

Similarly, *b* enters the medium before *c* and *d*, then *c* before *d*, and finally *d*. Thus the portion *a* travels the distance *aa'*, while *d* is traveling the larger distance *dd'*. The result is that the wave is "faced" in a different direction, namely *aF*, having suffered a bending *toward the normal* to the surface, *NN'*. From this explanation of refraction it is clear that the ratio of the distance *dd'* to the distance *aa'* is the same as the *ratio of the speeds of light in the two media.*

300. Index of Refraction. — It is obvious from Fig. 223 that the amount which the course of light is changed when it enters a medium where its speed is less than it is in air depends upon the relation that the distance *aa'* bears to the distance *dd'*; or, in other words, upon the relation between the speeds of light in the two media. Although it is not easy to measure the speed of light in a medium, it is comparatively a simple matter to measure the amount of refraction and from this to compute the *relative* speed of light. *The number which expresses the ratio of the speed* *of light in air to its speed in another medium is called the* **index of refraction** *of that medium.* Hence

$$\textbf{Index of refraction}^1 = \frac{\textbf{speed in air}}{\textbf{speed in other medium}} = \frac{dd'}{aa'}. \quad (1)$$

301. Index of Refraction Measured. — By referring to Fig. 225 in which xx' is the normal at the point d', we observe that angle dad' = angle $dd'x$, which is the angle of incidence. (Why?) Angle $ad'a'$ = angle $x'd't$, the angle of refraction. (Why?) Now the ratio $\frac{dd'}{ad'}$ is defined in mathematics as the *sine of angle dad'*, and is written *sin dad'*. Similarly, the ratio $\frac{aa'}{ad'}$ is the *sine of angle ad'a'*, and is written *sin ad'a'*. Hence

$$\frac{\sin dd'x}{\sin x'd't} = \frac{\sin dad'}{\sin ad'a'} = \frac{\frac{dd'}{ad'}}{\frac{aa'}{ad'}} = \frac{dd'}{aa'} = \textbf{index of refraction.} \quad (2)$$

Therefore the index of refraction of a substance is equal to the quotient obtained by dividing the sine of the angle of incidence by the sine of the angle of refraction.

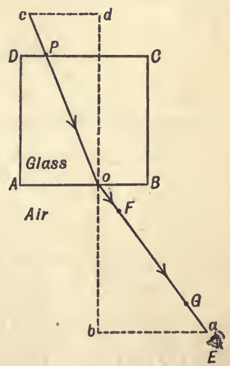


Fig. 226. — Measuring the Index of Refraction of Glass.

302. Index of Refraction by Experiment. — Let a glass cube *ABCD*, Fig. 226, be placed against a pin *P* set upright in a horizontal board or table. Place the eye at some point as *E*, and set two other pins *F* and *G* in line with *P* as seen through the glass. Draw line *AB* and remove the cube. Next draw lines *GFo* and *oP*. The line *PoG* is the

[1] The term "absolute" index is used to refer to the ratio of the speed of light in a vacuum to its speed in a medium. The index of refraction defined above is often called the "relative" index.

course taken by the light that enters the eye from the pin P. Draw the normal bd at the point o, and then draw the lines ab and cd perpendicular to the normal after making $oa = oc$. Now $ab \div ao$ is the sine of angle aob, and $cd \div co$ is the sine of angle cod. Hence, by equation (**2**), and substituting ao for its equal, co, we have

$$\text{index of refraction} = \frac{\frac{ab}{ao}}{\frac{cd}{ao}} = \frac{ab}{cd}.$$

Therefore we can readily find the value of the index of refraction by dividing the length ab by the length cd.

RELATIVE SPEED OF LIGHT OR ABSOLUTE INDEXES OF REFRACTION FOR SOME COMMON TRANSPARENT SUBSTANCES

Air	1.00029	Flint glass	1.54 to 1.71
Water	1.333	Carbon disulphide	1.64
Crown glass	1.51	Diamond	2.47

303. Total Reflection. — Since the rays of light which pass from water or glass into air are bent away from the normal, as POA and $PO'B$, Fig. 227, it is readily observed that when the angle of incidence below the surface is great enough, the refracted ray will follow close to the surface, as ray $PO''S$. Hence the ray PO'' is the last one that can emerge from the surface. If the angle of incidence is still increased, the light is reflected wholly beneath the surface, as $PO'''C$. This phenomenon is called *total reflection*.

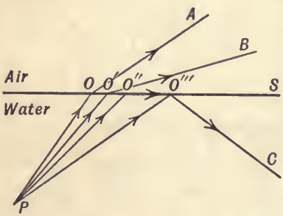


FIG. 227. — The Total Reflection of Light.

1. Examine the glass cube used in § 302 by looking through the face AB, Fig. 226, toward face BC, which has the appearance of a mirror. Place the finger upon face BC. It cannot be seen through AB. Transfer the finger to face CD. It can now be seen in face BC by reflected light.

2. Look obliquely upward against the surface of water in a tumbler. It will be seen to have the appearance of a plane mirror. If the point of a pencil is held in the water, only that part of it is visible that projects below the surface, and this portion can also be seen by reflected light.

3. Reflect a narrow beam of sunlight obliquely upward through the side of a glass tank containing water. (See Fig. 228.) By varying the angle of incidence below the surface until it is greater than 48.5°, the totally reflected beam can be traced back into the water. Both the incident and reflected beams may be made visible in the manner described in § 298.

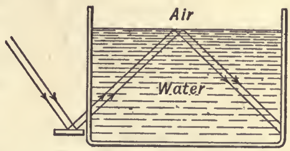


FIG. 228. — An Illustration of Total Reflection.

4. Hold a test-tube obliquely about 5 centimeters under water, and look vertically downward upon it. The portion of the tube below the liquid surface has the appearance of a mirror.

These experiments serve to illustrate the fact that when the angle of incidence with which light undertakes to emerge from glass or water exceeds a certain value, *the light is totally reflected at the point of incidence back into the medium. The angle of incidence at which the effect changes from refraction to total reflection is called the* **critical angle.**

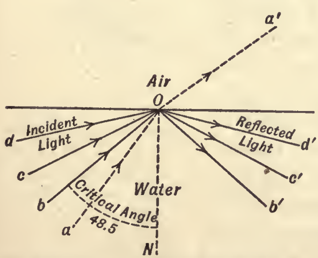


FIG. 229. — Illustrating the Critical Angle.

The phenomenon of total reflection occurs only when light is proceeding in one medium toward another in which the speed is greater. (See Fig. 229.) The critical angle for water is 48.5°; for crown glass, 41°; for diamond, 24°.

304. Critical Angle Constructed. — If the index of refraction of a medium is known, the critical angle can be found readily from the following construction:

Let the line AB, Fig. 230, be the boundary between air and water, and let the index of refraction be $\frac{4}{3}$. With O, the point of incidence, as the center draw two concentric circles whose radii have the ratio of 4 : 3. Since the ray that emerges for the critical angle follows the surface, erect the normal at the point C, where the surface intersects the inner circle. This cuts the larger circle at E. The line EO produced below the surface gives the angle DOF as the critical angle.

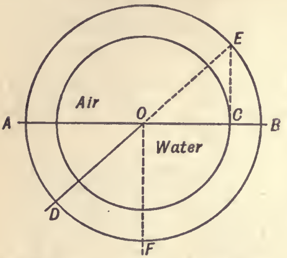


FIG. 230. — Method of Constructing the Critical Angle.

The proof is as follows: When light passes from water into air, the index of refraction is $\frac{3}{4}$; and for the critical angle of incidence below the surface, the angle of refraction is 90°. It is to be shown that $\frac{\sin DOF}{\sin 90°}=\frac{3}{4}$. Now, angles DOF and OEC are equal. (Why?) Further, $\sin OEC=\frac{OC}{OE}=\frac{3}{4}$ by construction (§ 301), and $\sin 90° = 1$. Therefore

$$\frac{\sin DOF}{\sin 90°}=\frac{\sin OEC}{\sin 90°}=\frac{3}{4}\div 1=\frac{3}{4}.$$

305. Total Reflecting Prism. — The most important use that is made of the phenomenon of total reflection is accomplished by employing a right-angled prism of glass whose cross section is an isosceles triangle, as shown in Fig. 231. If incident light enters the prism perpendicular to either of the faces forming the right angle, it will not suffer refraction and will strike the oblique face at an angle of incidence of 45°. Since the critical angle for glass is less than 45°, the light will be totally reflected in the direction perpendicular to the third face, from which it will emerge without undergoing refraction. Such prisms are used when it is desired to turn the course of light through an angle of 90° without excessive loss.

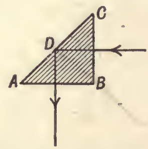


FIG. 231. — Turning the Course of Light through 90° by Total Reflection.

306. Path of Light through Plates and Prisms. — The effect of a parallel-sided plate of glass upon a ray of light is readily determined by the following experiment:

Look obliquely through a glass cube or a parallel-sided tank of water and set four pins, two on each side of the cube or tank, so that they will form apparently a straight line. Remove the refracting object, and draw the lines connecting the two pins of each pair. If these two lines are produced, it will be found that they are parallel.

Hence, when light passes obliquely through a medium with parallel faces, it does not suffer a permanent change in direction. In other words, the refraction toward the normal at the first surface *DC*, Fig. 232, is canceled by the refraction from the normal at the second surface *AB*. As the experiment shows, *the ray suffers a lateral displacement only.*

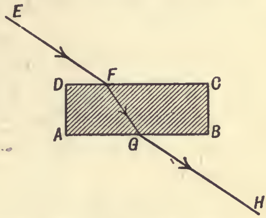


FIG. 232. — Light Suffers no Permanent Change in Direction in Passing through Parallel Surfaces.

The course taken by a ray of light which falls obliquely upon one of the faces of a glass prism may be traced in a manner similar to that just described:

Set four pins, two upon each of the opposite sides of a glass prism, Fig. 233, whose angle at *A* is about 60°, so that all four pins lie apparently in a straight line. Draw a line around the prism and remove it. Join the two pins in each pair by a straight line, and produce these lines until they meet the sides of the prism *AB* and *AC* at *O* and *O′*. Then *OO′* is the path of the light through the prism.

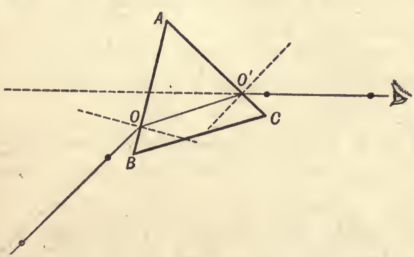


FIG. 233. — Refraction of Light by Means of a Prism.

Since the ray which enters the prism is bent toward the normal at *O* and away from the normal at *O′* where it emerges, it is clear that the effect of the prism is to turn the ray always away from the refracting angle *A* of the prism.

EXERCISES

1. What is the speed of light in water, the index of refraction being $\frac{4}{3}$? The speed of light in air is 186,000 mi. per sec.

2. Compute the speed of light in crown glass, assuming that the index of refraction is $\frac{3}{2}$.

3. Compare the speed of light in water with that in crown glass. What simple fraction will represent the relative speed? Show how to get this same fraction from the indexes of refraction given in Exercises 1 and 2.

4. The answer obtained in Exer. 3 is called the relative index of refraction for light on passing from water into crown glass. In a similar manner find the relative index of refraction for light which passes from flint glass into carbon disulphide, using the indexes given in the table.

5. For a given angle of incidence will light be refracted more in passing from air into crown glass or from water into crown glass? From water into crown glass or from carbon disulphide into flint glass?

6. Construct the critical angle for air and water. On which side of the boundary surface does the critical angle lie? Does the critical angle lie within the medium where the speed of light is the greater or the less?

7. Upon which side of the boundary surface separating water and crown glass does the critical angle lie?

8. The angle of incidence at which a ray of light enters a medium from air is 45°, and the angle of refraction 38°. Find by construction the index of refraction of the medium.

Suggestion. — By the help of a protractor construct accurately a figure. Measure the lines *ab* and *cd*, as in § 302, and compute the index of refraction.

9. Draw the figures, and ascertain the indexes of refraction for the following angles: angle of incidence 30°, angle of refraction 22°; angle of incidence 50°, angle of refraction 31°; angle of incidence 60°, angle of refraction 40°.

4. LENSES AND IMAGES

307. Lenses. — The student will call to mind many instruments with which he is acquainted that make use of of lenses, — the camera, microscope, spyglass, spectacles, opera glass, etc. The value of these instruments can hardly be too highly estimated. The study of lenses and their application is therefore of great interest and utility.

A lens is usually made of glass and has either two curved boundary surfaces or one curved and one plane surface. The curved surfaces are usually (not necessarily) spherical, and the lens thus formed is called a spherical lens. The center of the sphere C, Fig. 234, of which the lens surface is a part, is called the *center of curvature*, and the radius of the sphere, the *radius of curvature*.

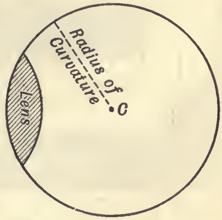


FIG. 234. — A Spherical Lens.

Lenses are of two general classes, — *convex* lenses, which are thicker at the middle than at the edge, and *concave* lenses, which are thickest at the edge. Figure 235 shows the three lenses belonging to each of the two general classes.

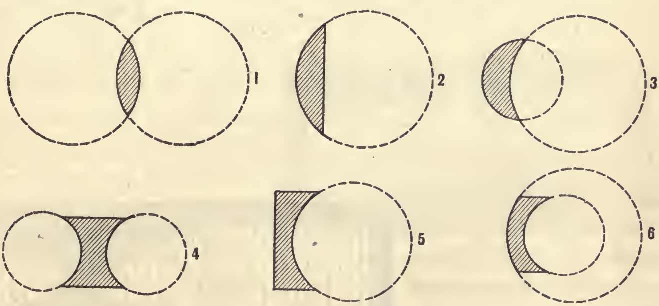


FIG. 235. — Forms of Lenses.

Convex, or Converging Lenses	1. Double Convex 2. Plano-Convex 3. Concavo-Convex, or a Meniscus	Concave, or Diverging Lenses	4. Double Concave 5. Plano-Concave 6. Convexo-Concave

308. Effect of a Convex Lens on Light. — The most important feature of any lens is not its form, but the manner in which it acts upon light. The effect of a lens that is most familiar to all is that employed in the so-called "burning glass," which was in general use for producing fire before the introduction of cheap matches. The following experiment shows this effect:

Allow a beam of sunlight to fall upon a large convex lens in a darkened room. If the air be made dusty, the light will be seen to form a cone-shaped figure as *AA*, Fig. 236. If a piece of tissue paper be placed at the vertex of the cone, it is readily ignited. Beyond this point the light diverges precisely as from the principal focus of a concave mirror (§ 290). If a convex lens of another form be substituted for the first, the action is practically the same.

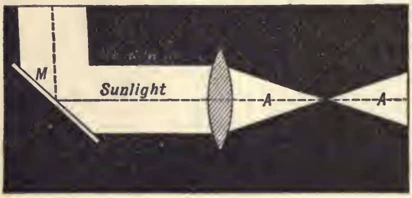


Fig. 236.—Effect of a Convex Lens on Parallel Rays.

The vertex of the cone formed by the sunlight transmitted by a convex lens is called the *principal focus* of the lens. The distance from the principal focus to the center of the lens is the *focal length*, or *principal focal distance*, of the lens. Since all convex lenses cause rays of sunlight to converge to a point, they are often called *converging* lenses. (See Fig. 235.)

Figure 237 shows the change in form that waves of light undergo while passing through a double convex lens. The plane waves of sunlight, whose direction of motion is represented by arrows drawn in the figure, are retarded by the lens in proportion to the thickness of

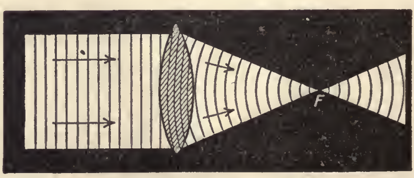


Fig. 237.—Plane Waves are Converged to the Principal Focus *F*.

the glass through which they pass. As a result of this retardation, the emerging light has concave wave fronts whose centers are at F, the principal focus. On leaving F, however, the waves have convex fronts; or, in other words, the light diverges from the point F.

Ordinary glass lenses, whose index of refraction is very nearly $\frac{3}{2}$, have a focal length equal to the radius of curvature, if they are double-convex and the two surfaces have the same curvature. If one surface is a plane, the focal length is double the radius of curvature.

309. Concave Lenses. — The effect of a concave lens on sunlight is very different from that of a convex lens, as the following experiment will show:

Let a beam of sunlight fall upon a concave lens mounted in a wide board. By making the air dusty, or holding a piece of cardboard obliquely in the transmitted light, it will be readily observed that the light diverges as it leaves the lens.

The part of a plane wave that passes through the center of a concave lens, Fig. 238, is retarded least on account of the fact that the lens is thin at that point, while that passing through near the edge suffers the greatest retardation. The result is that the front of the emerging waves is convex, as though the light had emanated from the point F, which in this case is a *virtual* focus and located very near the center of curvature. Since the general effect of all concave lenses is to cause rays of sunlight to diverge, they are classed together as *diverging* lenses. (See Fig. 235.)

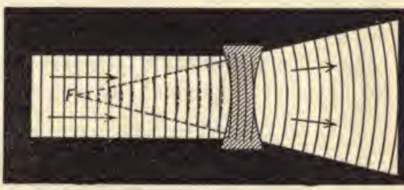


FIG. 238. — Parallel Rays of Light are Scattered by Concave Lenses.

310. Conjugate Foci. — Place a candle flame close to a small hole P in a piece of tin S, Fig. 239. Now set a convex lens about twice its focal length away from the hole P, and place a screen S' on the opposite side of the lens upon which will be produced a sharp

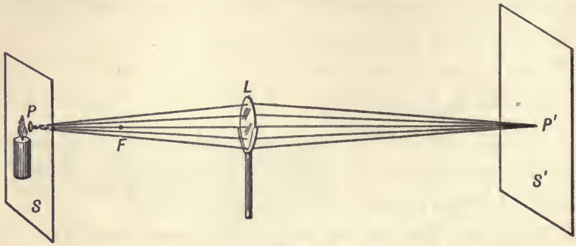


Fig. 239. — The Conjugate Focus of P is at P'.

image of P at P'. Cover up one half the surface of the lens, and the image will remain at P'. Cover the lens almost entirely, and the image will be weakened but not destroyed.

It is to be observed that all the light emanating from the point P and passing through the lens is collected at P'. Likewise, any other point beyond the principal focus F has a corresponding point on the opposite side of the lens where its image would appear. *Two points so related that the image of one of them, as P, falls at the other, as P', are called conjugate foci.* See also § 295.

311. Virtual Focus. — Let the candle and screen S that were used in the experiment of the preceding section be placed between a convex lens and its principal focus, and the result shown in Fig. 240 will be secured. The rays diverging from the point P will not be brought to a focus by the lens, but the divergence will be greatly reduced. Of course no image of P can be produced upon a screen; but upon looking through the lens the eye E locates (apparently) the image of P at the point P'. If the screen be removed a virtual image of the candle may be seen.

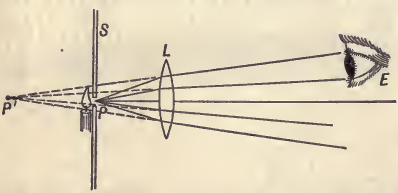


Fig. 240. — The Conjugate Focus of P is Virtual and at Point P'.

In the location of an image the eye is always governed by the divergence of rays which enter it (§ 276). The divergence is in this case as if the rays had come from P'

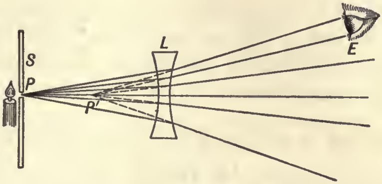


FIG. 241. — A Concave Lens always Forms a Virtual Focus.

instead of P. Since the point P' is only the apparent meeting place of the rays that enter the eye, this point is called the *virtual* focus of P. If the experiment be repeated with a diverging (concave) lens, as shown in Fig. 241, the conjugate focus of P is *virtual no matter what the position of P may be*, because the effect of such a lens is always to scatter the transmitted light.

312. Images Formed by Convex Lenses. — Every one who has viewed the pictures projected by a stereopticon or a moving-picture machine has seen the real images that convex lenses are able to produce. In the process of forming images the convex lens presents precisely as

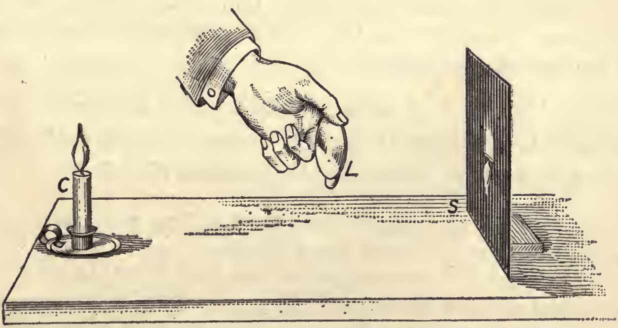


FIG. 242. — Focusing a Real Image on a Screen by Means of a Convex Lens.

many cases as the concave mirror (§ 295). These cases are easily illustrated by experiments.

1. Let a lighted candle C, Fig. 242, be placed at a little more than twice the focal length from a convex lens L. By moving the screen S back and forth an image will be found distinctly focused upon it. The distance from the lens to the image should be measured and compared with the focal distance.

When the object is situated at more than twice the focal length from a convex lens, the image is at less than twice and more than once the focal length from the lens on the opposite side, is real, inverted, and smaller than the object.

2. Let the candle be placed at less than twice but more than once the focal length from the convex lens. The image may be found by moving the screen away from the lens. The distance to the image should be measured and its position and size noted.

When the object is situated at more than once and less than twice the focal length from a convex lens, the image is at more than twice the focal length on the other side, is real, inverted, and larger than the object.

3. Let the candle be placed at less than the focal length from the convex lens. Of course no image can be produced upon the screen since the conjugate foci of all points in the object are virtual. (See § 311.) But by allowing some of the transmitted light to enter the eye (*i.e.* by looking through the lens), an apparent magnified image may be seen behind the lens.

When the object is situated at a point between a convex lens and its principal focus, the image is apparently behind the lens, is virtual, erect, and larger than the object.

4. Focus upon a screen the images of objects which are situated at a great distance, — clouds, trees, buildings, etc. If, now, the distance from the lens to the images be measured, it will be found practically equal to the focal length of the lens. Repeat the experiment with different lenses. Let the size and position of these images be noted.

When an object is at a great distance from a convex lens, its image is at the focal distance from the lens, is real, inverted, and smaller than the object.

It will be observed that this follows from the fact that the rays which come from a given point on the object to the lens are practically parallel to each other like rays of sunlight. This case affords a good method for determining the focal length of a lens.

Since rays of light which diverge from the principal focus of a convex lens become parallel to the principal axis after passing through, it follows that there can be no image of a small object placed at the principal focus.

313. Images Formed by Concave Lenses. — It is obvious from § 311 that a concave lens cannot produce a real image, since it always tends to scatter the light. This fact, however, is of value in the construction of certain optical instruments, as will appear later.

Hold a concave lens between the eye and a candle flame. No matter how far the flame is from the lens, the only image produced is a small erect one behind the lens.

Hence the image produced by a concave lens is always apparently on the same side of the lens as the object, is virtual, erect, and smaller than the object.

314. Construction of Images Formed by Lenses. — The construction of the images produced by lenses will serve to bring out clearly the nature of the image in each of the cases illustrated by experiment in § 312.

The different cases of lenses now to be studied will be found to correspond closely to those of curved mirrors which were treated in section 295. In each case the complete construction should be accurately made on a scale somewhat larger than that used in the illustrations as here given.

Case I. — When the object is placed at more than twice the focal distance from a convex lens.

Let ED, Fig. 243, be a convex lens whose centers of curvature are at C and C'. Let AB represent an object so placed that the distance

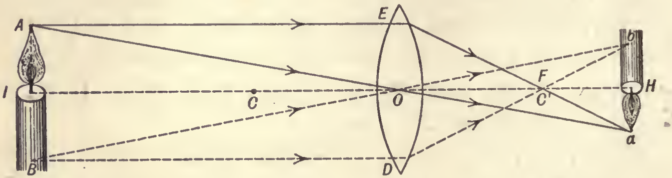


Fig. 243. — Method of Locating the Image Produced by a Convex Lens.

OI is more than twice the focal length OC. Now if rays AE and BD be drawn parallel to the principal axis HI, the refracted rays will pass through the principal focus F, which is at C' (§ 308). Again, the rays that pass through the *optical center* of the lens O enter the lens and emerge from it *at points where the surfaces are parallel* and therefore suffer no permanent change in direction. (See § 306.) Let the line AO be produced through the lens until it meets the line EF at a. Thus a is the conjugate focus of A. Similarly b is found to be the conjugate focus of B. Therefore ab is the image of the object AB.

Case II. — When the object is at more than once and less than twice the focal distance from a convex lens.

In this case the method of construction is precisely the same as in the preceding one; but on account of the fact that the object has been brought nearer the lens, the divergence of the rays before refrac-

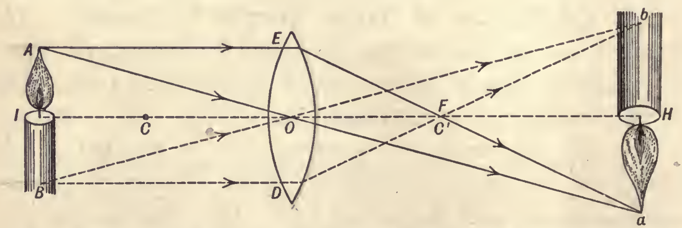


Fig. 244. — Illustrating the Production of a Real and Magnified Image.

tion (which is represented by angle OAE) is greater than before, and hence they are brought to a focus a at a greater distance from the lens.

Figure 244 shows clearly the construction. Does the description given in § 312 apply to the image that is found by construction?

It is to be observed that Case I changes to Case II when the object is placed at twice the focal length from the lens. In this instance the image and object are of equal size and are equidistant from the lens.

Case III. — When the object is at less than the focal distance from a convex lens.

Two rays are drawn from each of the points A and B, Fig. 245, precisely as in the two preceding cases. But since the angle of divergence EAO is so large, the lens is not able to cause the rays to converge to a real focus; *i.e.* the refracted rays EF and AO *never meet after leaving the lens.* If, however, the refracted rays enter an eye, an apparent image of A will be seen at a, which is the intersection of the

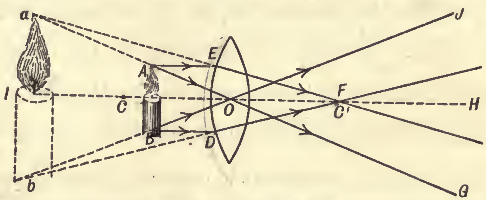


FIG. 245. — The Production of a Virtual Image by a Convex Lens.

refracted rays when produced behind the lens. Since the effect is an illusion, the image is a virtual one. Does the description given in § 312 apply to this image?

Case IV. — When an object is viewed through a concave lens.

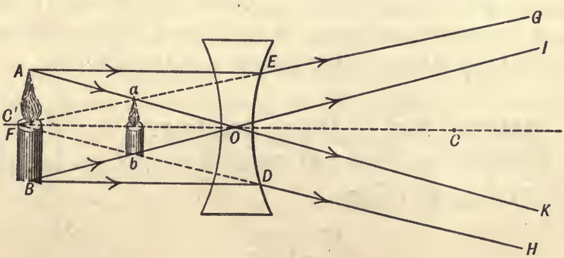


FIG. 246. — Constructing the Image Produced by a Concave Lens.

Rays AO and BO, Fig. 246, are drawn as in the preceding cases. But the parallel rays AE and BD are turned away from the principal axis CC' as if their origin were at the center of curvature C' (§ 309). Thus the angle of divergence EAO is increased by the lens to the value EaO. Consequently, when the refracted light is received by an eye, the image which is virtual and erect appears to be behind the lens, but nearer and always smaller than the object.

315. The Lens Equation. — The experiments of the preceding sections have shown that the position of an image formed by a lens is determined by the focal length of the lens and the distance from the lens to the object. If p represents the distance from the lens to the object, q the distance from the lens to the image, and f the focal length, then the following relation between these distances will be found to exist:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}. \tag{3}$$

Referring to Fig. 243, we observe that the triangles AOI and aOH are similar. Hence $\frac{AI}{aH}=\frac{OI}{OH}$. If the lens be thin and a line be drawn from E to D, it may be assumed to pass through the point O. Then the triangles EFO and aHF are similar. Therefore, $\frac{EO}{aH}=\frac{OF}{HF}$. Since $AI = EO$, the first member of these two proportions are equal. Hence, by substituting p for OI, q for OH, and f for OF, we obtain

$$\frac{p}{q}=\frac{f}{q-f}.$$

Clearing of fractions, transposing, and dividing by pqf gives

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}.$$

Equation (3) is useful in determining the focal length of a lens. For example, let the image of an object which is 60 cm. from a lens be focused 40 cm. from the lens. By substituting these values for p and q, we find the value of f, the focal length, to be 24 cm.

316. Relative Size of Object and Image. — By referring to Fig. 243 and the above discussion the following proportion is found to be true:

$$\frac{AI}{aH}=\frac{OI}{OH}. \tag{4}$$

But aH is the image of AI. Therefore, *the ratio of the size of the object to the size of the image is equal to the ratio of their respective distances from the lens.* This relation may be easily verified by experiment.

EXERCISES

1. What kind of mirrors and lenses always produce virtual images?

2. Under what conditions do convex lenses produce real images? When is the image produced by a convex lens virtual?

3. If one half of a convex lens be covered with an opaque card, what will be the effect upon the real images produced by it? Test your answer by experiment.

4. How can you test a spectacle lens to ascertain whether it is convex or concave?

5. By the help of equation **(3)** compute the focal length of a lens when the image of a candle flame 120 cm. away is focused at a distance of 60 cm.

6. The focal length of a lens is 50 cm. If an object is situated at a distance of 75 cm. from the lens, how far from the lens will its image be focused?

7. An object is placed 30 cm. from a lens whose focal length is 45 cm. Locate the image by employing equation **(3)**.

Suggestion. — When the minus sign precedes the result obtained by using the equation, the inference is that the image is virtual.

8. What is the height of a tree 350 ft. away when its image on a screen 10 in. from a convex lens is 2 in. in height? *Ans.* 70 ft.

5. OPTICAL INSTRUMENTS

317. The Simple Magnifier or Reading Glass. — It is a common occurrence to see a botanist examining the details of a flower or a jeweler adjusting the minute parts of a watch by the help of a convex lens. Large convex lenses are frequently used as an aid in reading, and are therefore often called *reading glasses.* In these cases the object to be examined is placed a little nearer the lens than the principal focus, while the image is viewed by placing the eye on the opposite side of the lens, as shown in Fig. 240. The instrument owes its importance to the fact that a magnified image is visible behind the lens, as shown in Fig. 245.

318. The Photographic Camera. — Two important principles are employed in the photographic camera: (1) a convex lens produces a real image of objects placed beyond its principal focus, and (2) light has the property of producing chemical changes in certain compounds of silver.

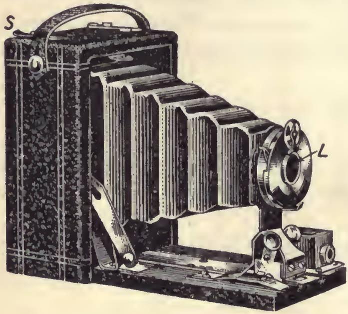


FIG. 247. — The Camera.

The camera is a light-proof box, or chamber, Fig. 247, provided at the front with a convex lens L and at the back with a ground-glass screen which can be replaced by a "sensitized" plate or film for receiving the image. The image is first focused on the screen by varying its distance from the lens, after which the sensitive plate is introduced in a light-proof holder. The shutter of the lens is now closed, and the cover of the plate-holder removed. When all is in readiness, the lens is uncovered for a sufficient time to enable the transmitted light to produce the desired effect upon the plate. The plate is now covered and taken to a dark room, where a "developing" process brings out a visible and permanent image. The plate thus treated is called a "negative" because of the reversal of light and shade in the picture upon it, and may be used in the reproduction of any number of positive photographs on prepared paper.

319. The Eye. — Although the eye is a very complicated structure, its action depends on one of the simplest cases of refraction that we have studied. The eye is essentially a small camera at the front of which the cornea C, Fig. 248, the aqueous humor A, and the crystalline lens O take the place of the convex lens of that instrument. When the eye is directed toward an object, a small, real, and inverted image is produced on the retina, which is an expansion of the optic nerve N and covers the inner surface of the eyeball at the back. This does not mean, however, that we see things upside down. The relative position which we ascribe to objects is the result of experience aided by the sense of touch, etc., and the fact that images are inverted on the retina has little effect on the ideas which the impression gives us. It will be observed that the impressions remain the same even when the eye is tilted or inverted.

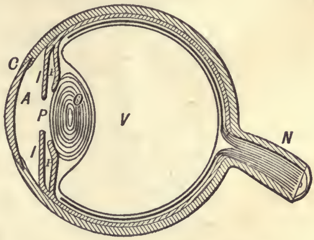


Fig. 248. — Sectional View of the Eye.

The eye adjusts itself to objects near and far by changing the focal length of the crystalline lens. When a normal eye is completely relaxed, the lens has the proper curvature for focusing light from distant objects (*i.e.* parallel rays) upon the retina. When, however, we wish to view an object near at hand, as in reading, small muscles within the eye cause the curvature of the crystalline lens to increase until the image is again focused upon the sensitive retina.

320. Spectacles and Eyeglasses. — Although a normal eye with complete relaxation focuses parallel rays upon the retina, as in Fig. 249, it should also be able to focus with ease light that comes from an object at a distance of 10 inches or more. The eye is defective when it cannot accomplish the performance of these functions without unnatural effort.

If the retina is too far from the crystalline lens, parallel rays will not be brought to a focus upon it, but in front of it. This defect is called *myopia*, or *near-sightedness.* (*1*) Fig. 250 illustrates this condition. It is at once obvious that the crystalline lens produces too great a convergence of the light. This can be corrected by using a concave lens of suitable curvature, as shown in (*2*) Fig. 250. Hence *concave* spectacles are used to assist myopic eyes.

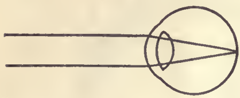


FIG. 249.—A Normal Eye when Relaxed.

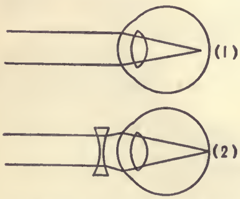


FIG. 250.—The Myopic Eye and Its Correction.

Again, the eyeball may be too short from front to back, in which case the focus of parallel rays will be behind the retina, as shown in (*1*), Fig. 251. It is clear in this instance that the crystalline lens does not converge the rays sufficiently for distinct vision. This defect is called *hypermetropia*, or *far-sightedness.* In order to correct the fault, a convex lens of suitable curvature is needed, which assists the crystalline lens to produce an adequate convergence of the light, as shown in (*2*).

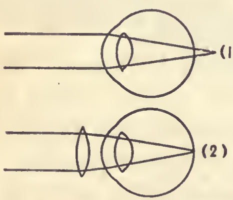


FIG. 251.—The Hypermetropic Eye and Its Correction.

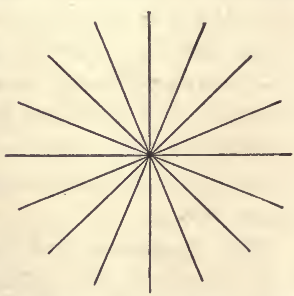


FIG. 252.—An Astigmatic Eye Sees these Lines with Unequal Distinctness.

The most prevalent defect in the eyes of young people is that known as *astigmatism.* Astigmatism is due to the fact that the crystalline lens is not symmetrical about its axis. In other words, a vertical section through the lens differs in form from a horizontal section.

Such an eye sees the lines of Fig. 252 with unequal distinctness. This defect is corrected by the use of a lens whose vertical and horizontal sections possess suitable curvatures to make up for the deficiencies in the crystalline lens.

321. The Compound Microscope. — The compound microscope consists of a convex lens O, Fig. 253, of short (say $\frac{1}{4}$ in.) focal length, which is called the *objective*, and a larger convex lens E, called the *eye-piece*. When the object to be viewed, AB, is placed a little beyond the principal focus of O, a real, inverted, and magnified image is produced at ab (§ 312). When this *real image* is viewed through the eye-piece, a magnified virtual image is seen at $a'b'$.

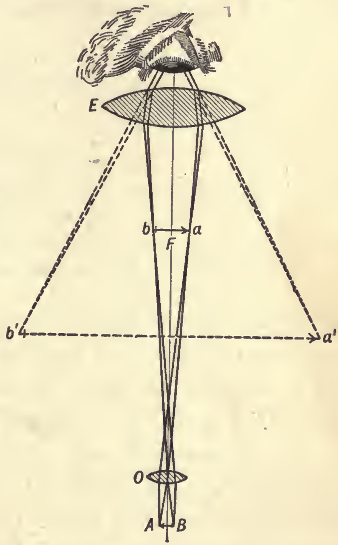


Fig. 253. — The Compound Microscope.

322. The Astronomical Telescope. — The principal part of a modern astronomical telescope is the large convex lens O, Fig. 254, called the object glass. This is designed to collect a large amount of light in order that the real inverted image ab that is formed may be sufficiently brilliant. This image

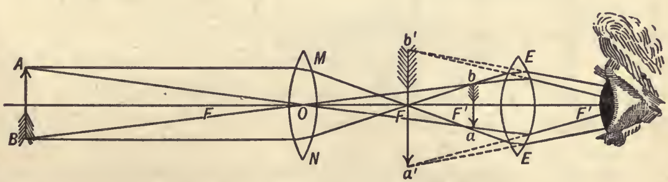


Fig. 254. — The Astronomical Telescope.

falls close to the eye-piece E whose function is precisely the same as in the compound microscope which is described in the preceding section. The inversion of the image is of little consequence in astronomical telescopes, but for viewing terrestrial objects this feature would be a defect.

323. The Opera Glass, or Galileo's Telescope. — The honor of having invented the original form of the tele-

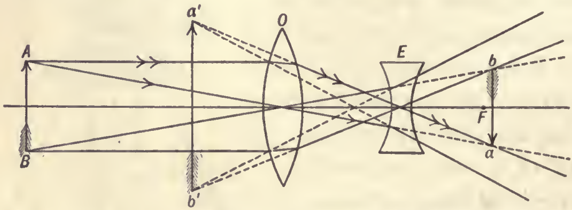


FIG. 255. — Illustrating the Principle of the Opera Glass.

scope belongs to Galileo,[1] who constructed the first instrument about 1610. Two Galilean telescopes arranged side by side form an opera glass or a field glass. An object glass O, Fig. 255, converges the light to form a real image at ab. Before the rays reach the focus, however, they are intercepted by a concave lens which gives them a slight divergence as they enter the eye. As a consequence, an erect and magnified virtual image is seen at $a'b'$.

324. The Projecting Lantern. — The projecting lantern, Fig. 256, consists of a powerful source of light A whose

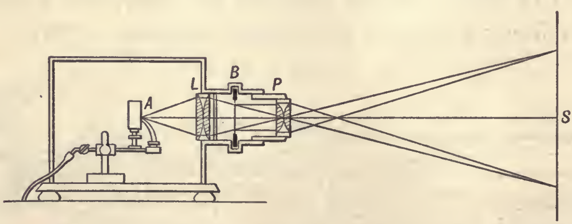


FIG. 256. — Illustrating the Principle of the Projecting Lantern.

[1] See portrait facing page 70.

rays are concentrated upon a transparent picture B by the convex *condensing lenses* L. At the front of the instrument is placed the *projecting lens* P which forms a real, inverted, and enlarged image of B upon the screen S. The source of light is usually an electric arc lamp or a calcium light. The latter is produced by directing an exceedingly hot flame, produced by burning a mixture of hydrogen and oxygen, against a piece of lime. When raised to a high temperature, the lime becomes intensely luminous.

325. Binocular Vision. — The Stereoscope. — On account of the fact that the two eyes are separated by a distance of 6 or 7 centimeters, the images produced upon the two retinas are not precisely alike. This has the effect of giving to an object the appearance of solidity or depth. Advantage has been taken of this fact in the *stereoscope*, an instrument which is so constructed as to present to each eye a similar image to that which it would receive if the object itself were present. A double photograph is first made by means of a camera having two objectives separated by a distance about equal to that between the two eyes. The two pictures thus taken differ just as much as would the corresponding images upon the retinas of the eyes. These pictures are now mounted on the stereoscope at A and B, Fig. 257, so as to be viewed by the two eyes through the half-lenses m and n. These lenses are so adjusted that the images produced upon the retinas are related in position precisely as in ordinary vision. On this account a perfectly natural blending of the two impressions is brought about as though the object itself were in the direction of C. The observer is therefore conscious of the presence of only one photograph, which gives the effect of extension in a degree that is remarkably true to nature.

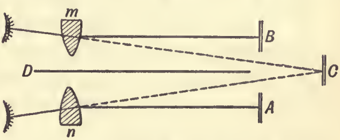


FIG. 257. — Illustrating the Principle of the Stereoscope.

326. The Kinetoscope. — The kinetoscope, kinematograph, or moving-picture machine, is a common object in most cities and villages. A life-like motion is given to pictures projected on a screen by means of a series of transparent photographs taken as follows:

A camera is provided with a shutter that opens and closes automatically about 12 times a second. The instrument also contains a long, narrow, sensitive film *which moves along about 2 centimeters while the shutter is closed, and remains stationary while the shutter is open.* With this a series of pictures is taken, each of which differs slightly from the preceding, *provided any moving object is in the field of the camera.*

These pictures are thrown upon a screen with a projection lantern in precisely the same order and with the same rapidity as they were taken. On account of the fact that the sensation produced by one picture remains until the next picture appears, the observer is unconscious of any interruption in the illumination of the screen upon which the pictures are produced.

EXERCISES

1. While changing the attention from a distant object to a near one, does the crystalline lens flatten or thicken?

2. A photographer finds that the desired image of a building more than covers the area of the plate to be used. How can the size of the image be reduced to fit the plate?

3. When the spectacle lens used to correct a myopic eye is placed in front of a normal eye, is the image of a distant object behind or in front of the retina?

4. How can you test your eyes for astigmatism?

5. The crystalline lens becomes less elastic with age. Account for the spectacles with double lenses which are frequently worn.

6. The picture projected on a screen by a projecting lantern is found to be too large. Which way must the instrument be moved in order to reduce its size?

7. Why is it necessary to "focus" a microscope or a telescope upon the object to be viewed?

SUMMARY

1. Light is reflected from polished surfaces in such a manner that *the angle of reflection equals the angle of incidence* (§§ 282 and 283).

2. Light is *diffused* from unpolished surfaces. It is the power of diffusing light that renders objects visible from different points of observation when light falls upon them (§ 284).

3. The image produced by a plane mirror is as far behind the mirror as the object is in front of it (§§ 285 to 289).

4. The tendency of a *concave* mirror is to *collect* rays of light. Thus parallel rays are reflected through a common point called the *principal focus* (§ 290).

5. The tendency of *convex* mirrors is to *scatter* light. Hence the principal focus is unreal, or virtual (§ 291).

6. The images formed by a convex mirror are always virtual, erect, smaller than the object, and behind the mirror (§ 292).

7. The images formed by a concave mirror depend on the position of the object relative to the center of curvature (§ 294).

8. Light is refracted or bent toward the perpendicular to the surface where it enters a medium — as glass or water — from the air, and away from the perpendicular when it emerges from the medium into the air. Refraction is due to the fact that the speed of light is less in water, glass, etc., than in air or a vacuum (§§ 297 to 299).

9. The index of refraction of a medium expresses the ratio of the speed of light in air to its speed in that medium (§§ 300 to 302).

10. Total reflection always takes place when light travels in one medium toward another in which its speed is greater, provided the angle of incidence upon the boundary surface is greater than the critical angle (§§ 303 to 305).

11. Lenses are classed as *convex* or *concave* according to form, and as *converging* or *diverging* according to their effect on light (§ 307).

12. *Convex*, or converging, lenses tend to *collect* light. Rays which are parallel before reflection are caused to pass through a common point, called the *principal focus* (§ 308).

13. *Concave*, or diverging, lenses tend to *scatter* light, and therefore the principal focus is unreal or virtual (§ 309).

14. The images formed by convex lenses depend upon the relative position of the object and the principal focus (§ 312).

15. The images formed by a concave lens are always virtual, erect, smaller than the object, and on the same side of the lens as the object (§ 313).

16. Conjugate focal distances p and q are related mathematically to the focal distance f as shown by the following equation:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}. \qquad (\S\ 315.)$$

17. The sizes of image and object are in proportion to their respective distances from the lens (§ 316).

18. The convex lens is employed in the reading glass, camera, spectacles, and the eye. Two or more convex lenses are used in the compound microscope, telescope, projecting lantern, etc. Concave lenses are used in some spectacles, and in the eye-piece of the opera glass (§§ 317 to 326).

CHAPTER XV

LIGHT: COLOR AND SPECTRA

1. DISPERSION OF LIGHT: COLOR

327. Decomposition of White Light. — Let sunlight pass through a narrow slit into a well-darkened room and fall obliquely on a glass prism, as shown in Fig. 258. As previously shown (§ 306), the beam will be refracted; but when the refracted light is allowed to fall on a white screen, a beautiful band of different colors will be seen. Among these will be recognized red, orange, yellow, green, blue, indigo, and violet, although there is no sharp line of demarcation between them.

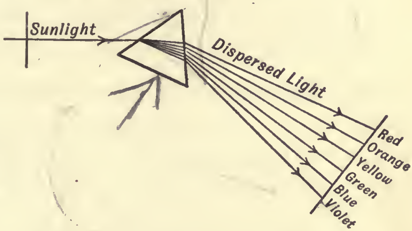


FIG. 258. — The Separation of White Light into its Components.

From this experiment we can only infer that white light is a mixture of the several colors seen on the screen. In fact, if a similar prism is available, the colors may be turned together again to form white light by placing the two prisms as shown in Fig. 259.

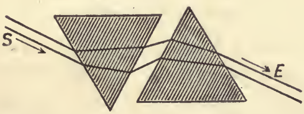


FIG. 259. — Colors of the Spectrum Added to Produce White.

The band of color produced by the separation of light of any kind is called a **spectrum,** and the process of separation in which the light of different colors is refracted in varying degree is called **dispersion.**

328. Cause of Dispersion. — Direct measurements of the waves of light of different colors show that they are of very different lengths, those of red light being longest and of violet shortest. It is therefore clear that waves of different lengths are refracted unequally by the prism, — the red or longest waves being bent least, and the violet or shortest waves most. The following table shows the approximate wave lengths of the various colors:

WAVE LENGTHS OF LIGHT

Red . . .	0.000068 cm.	Green . . .	0.000052 cm.
Orange . .	0.000065 cm.	Blue . . .	0.000046 cm.
Yellow . .	0.000058 cm.	Violet . . .	0.000040 cm.

329. Achromatic Lenses. — When white light passes through a simple lens, it is dispersed as well as refracted, *i.e.* the violet light is brought to a focus somewhat nearer the lens than the red, since it suffers the greater refraction (§ 328). On this account the images formed by simple lenses are always fringed with color, which is a serious fault in optical instruments. It is, however, possible to remedy this defect by combining two lenses, — one being a double convex lens of crown glass, the other a plano-concave lens (plane on one side, concave on the other) of flint glass, as shown in Fig. 260. In this lens system the dispersion produced in one part is just neutralized by the other, while the refraction is reduced only about one half. Such a system of lenses is called an *achromatic lens*, since all color is eliminated.

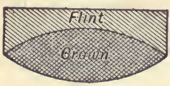


FIG. 260. — Achromatic Lens.

330. Color of Objects. — Let the spectrum of sunlight be projected on a white screen. Hold small pieces of paper or cloth of different colors in the various parts of the spectrum, and observe the effect. A red object will be a brilliant red when held in the red of the spectrum, and black in other parts. Blue and violet objects show a coloration only when placed near the violet end of the spectrum. A black object will appear black everywhere, and a white one reflects that color of the spectrum in which it is placed.

The experiment shows that the color of an object depends (1) on the light which falls upon it and (2) on the light it reflects to the eye. A red object is red because it reflects mainly red light, all other incident light being absorbed by the material, *i.e. transformed into heat* (§ 259). If, therefore, no red light falls upon it, it can reflect no light and is black. A black object appears black because it absorbs practically all colors alike; and a white body is white because it reflects all colors to the same extent.

331. Color of Transparent Bodies. — Project the spectrum of sunlight, and hold pieces of glass of different colors in the beam at any point. It will be observed that a red glass, for example, transmits mainly red light and absorbs the rest, that green glass transmits mainly green light, etc. Placing two pieces at once in the path of the light results in the transmission of that light only which neither can absorb.

It is therefore clear that the color of a transparent body depends on the color of the light which it transmits. *The color that we see in any body is the combination of all the light that is not absorbed by that body.* A colorless body is one that transmits all colors equally.

332. Complementary Colors. — Project the spectrum of sunlight or the light from an electric arc, and reunite the colors by means of a similar prism, as shown in Fig. 261. The result is, of course, white. Now place a card C between the prisms, and let it intercept the red light only. The spot of light on the screen S turns to bluish green. If the card is now caused to cut out the violet and blue, a greenish yellow results.

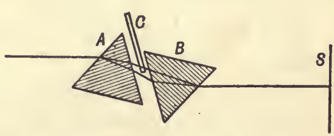


Fig. 261. — Production of Complementary Colors.

Two colors, as red and bluish green, into which white light can be resolved are called *complementary* colors. The union of complementary colors results in the production of white. When any color, or combination of colors, is removed from the spectrum of white light, the color produced by combining the remaining colors is the complementary of the part removed. The two complementary colors most easily obtainable are yellow and blue. If one half of a circular disk be colored blue and the other half yellow, the color effects may be combined by rotating the disk on a whirling machine. If the whirling disk be strongly illuminated, it appears white.

TABLE OF COMPLEMENTARY COLORS

Red and bluish green.	Green and purple.
Orange and greenish blue.	Violet and greenish yellow.
Yellow and blue.	

333. Color of Pigments. — The color of pigments used in paints and coloring materials is due to their power of absorbing incident light, similar to that shown in § 330 in the case of colored cloth and paper. The mixing of pigments is a very different thing from mixing lights or colors. A striking example is furnished by pigments that reflect the complementary colors blue and yellow.

Pulverize pieces of yellow and blue crayon, but keep the powders separate. If now about equal portions of the two powders be thoroughly mixed together, a bright green appears.

The cause of the phenomenon is the imperfect absorption of the pigments. The yellow powder subtracts from white light all *except yellow and green*, and the blue powder subtracts all *except blue and green*. Hence the only color not absorbed by one powder or the other is *green*.

2. SPECTRA

334. The Solar Spectrum. — The spectrum of sunlight, or the **solar spectrum**, frequently presents itself in nature in the rainbow. In the production of the rainbow the sunlight is dispersed by spherical raindrops. Light from the sun strikes a drop at *A*, Fig. 262, and is refracted to *B*. At *B* a portion of the light is reflected to *C*, where it emerges and enters the eye *E*. At the points *A* and *C* dispersion accompanies refraction, and the red light takes the direction *rrr*, while the violet follows the path *vvv*. A study of Fig. 263 will show that the eye receives the red from a greater angle of elevation than the violet; hence red lies at the outside of the rainbow.

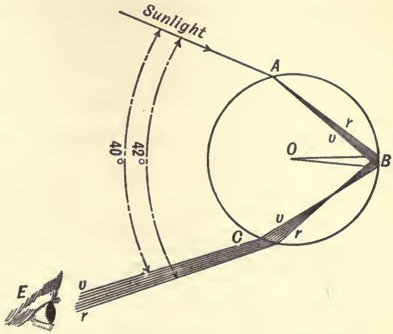


Fig. 262. — Dispersion of Sunlight by a Raindrop.

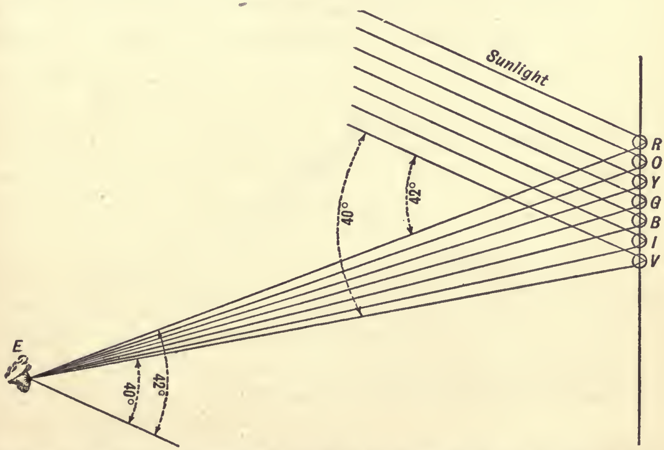


Fig. 263. — Showing the Spectrum Colors in their Relative Positions in the Rainbow.

The *secondary* rainbow often accompanies the primary one. In this case, as Fig. 264 shows, light suffers two reflections within the drop and emerges with the violet at the greater angle of elevation; hence here in the secondary bow the red band is the inner circle of color.

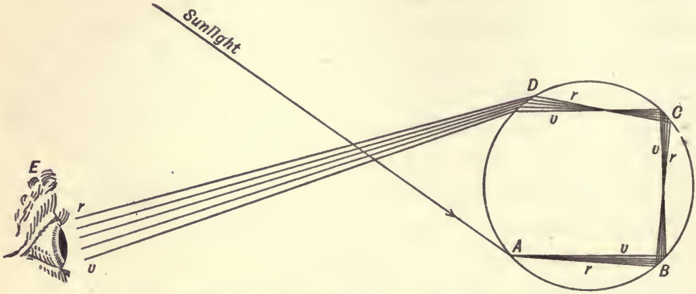


FIG. 264. — Showing the Formation of a Secondary Bow.

335. Absorption of Light by the Medium. — Let a narrow, horizontal beam of sunlight come into a darkened room through a vertical slit *S*, Fig. 265, and pass through a convex lens *L* to the carbon disulphide prism *P*. (The dispersion produced by a glass

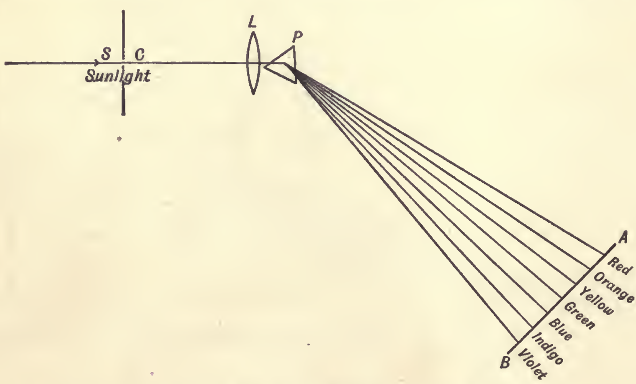


FIG. 265. — Method of Projecting the Solar Spectrum

prism is too small to give satisfactory results.) Reflect some of the light from the back of the prism to the screen *AB* where the spectrum is to be formed, and adjust the lens until it focuses a sharp image of the slit on the screen. A good spectrum can now be produced by setting the prism in the position shown in the figure.

While the spectrum is on the screen, place before the slit at C a flat tank or bottle containing a dilute solution of chlorophyl, made by soaking a quantity of grass in warm alcohol. It will be observed that a broad dark band is formed in the red portion of the spectrum and a less marked band appears in the green.

It appears from this experiment that a certain portion of the solar spectrum may be removed by the absorption of certain wave lengths in the medium through which the light passes. The resulting spectrum is called an **absorption spectrum** to distinguish it from the **continuous spectrum** that is obtained when none of the light is absorbed. Continuous spectra are, in general, given off by all luminous solids and liquids. It will appear presently that the solar spectrum is an absorption spectrum.

336. The Solar Spectrum in Detail. — The most interesting and remarkable features of the solar spectrum are not revealed in the rainbow, nor in the spectrum ordinarily projected by a prism, on account of the overlapping color bands. In order to produce a *pure spectrum* in which the colors are more distinctly separated than before, it is necessary to work with a narrow slit which is very sharply focused by a lens on a white screen.

Let the solar spectrum be projected as before, but make the slit about one half a millimeter in width and use a long focus (about 50 centimeters) lens. Place the screen about 1 meter from the lens and prism. By making all the adjustments with care, it will easily be seen that the spectrum is crossed vertically by many dark lines. By moving the screen back and forth the best position for showing the lines can readily be found.

These dark lines across the solar spectrum are known as **Fraunhofer lines** in honor of the German astronomer who first mapped them out about 1814. When the spectrum is magnified sufficiently, hundreds of these lines become visible. The presence of the Fraunhofer lines shows that certain wave lengths are either absent entirely from sunlight or that they are so weak as to appear dark by contrast. The solar spectrum is thus observed to be an absorption spectrum.

337. Fraunhofer Lines Produced by Absorption. — The explanation of the dark lines of the solar spectrum is due to Stokes and Kirchhoff, the former an English, the latter a German, physicist. The theory is based on the following experiment:

Project a well-defined solar spectrum as in the preceding experiment. Now sprinkle common salt (sodium chloride) on the wick of an alcohol lamp, place the lamp just below the slit at *C*, Fig. 265, and ignite the alcohol. A bright yellow light will be emitted by the volatilized salt, but on the screen will appear a darkening of a Fraunhofer line in the yellow part of the spectrum.

It thus appears that when light passes through the so-called sodium flame, the absorption of a certain wave length takes place. In the same manner the light from the white-hot central mass of the sun, which alone would give a spectrum without dark lines, passes through surrounding vapors, each of which, like the sodium vapor, has the power of removing light of certain wave lengths. *The wave lengths absorbed by a heated gas or vapor are precisely those which the vapor itself is capable of emitting.*

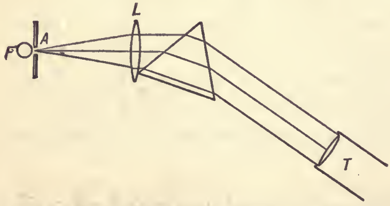


FIG. 266. — Apparatus Arranged for Bright-line Spectra.

338. Bright-line Spectra. — Prepare an alcohol lamp for producing yellow light as in the preceding experiment. Place the lens *L*, Fig. 266, so that the slit *A* is at its principal focus, and set the prism as shown in the figure. Focus a small telescope *T* on a distant object, and then place it for receiving light from the prism. Place the lamp at F, and ignite the alcohol. If all is properly adjusted, a bright double yellow line will be visible in the telescope. This is the spectrum of incandescent sodium vapor. Repeat the experiment by using strontium chloride in another lamp. Red and blue lines should appear. By using calcium chloride, red and green lines become visible. The yellow sodium spectrum will always appear, since sodium exists as an impurity in the lamp wick and in practically all salts.

These experiments show that *incandescent vapors emit light in which only certain wave lengths are present.* The spectra of such bodies are therefore **bright-line spectra.** Since these spectra are characteristic of the chemical elements, many substances can be identified by the wave lengths of the light which their incandescent vapors emit. The spectra of the chemical elements are as well known as their densities, specific heats, and other physical and chemical properties. Several new elements have been discovered by the observation of bright spectral lines which could not have been produced by any known substances. An incandescent vapor also affords a convenient means of obtaining light of one wave length or color, *i.e. monochromatic light.*

339. Solar Elements. — Since the spectra of many incandescent vapors have been examined and compared with the Fraunhofer lines of the solar spectrum, most of these lines have been accounted for just as the existence of incandescent sodium vapor surrounding the sun accounts for the dark sodium line. (See § 337.) If the correspondence between lines of the solar spectrum, shown in the middle in Fig. 267, and the spectrum of iron vapor, be noted, there can be no doubt as to the existence of iron

FIG. 267. — Showing the Coincidence of the Bright Lines of the Spectrum of Iron with Some of the Dark Lines of the Solar Spectrum.

in the sun. Other solar elements are calcium, hydrogen, sodium, nickel, magnesium, cobalt, silicon, aluminium, titanium, chromium, manganese, carbon, barium, silver, zinc, and many others. It is an interesting fact that the element helium was first discovered in the sun by Sir Norman Lockyer, of England, by means of its dark lines in the solar spectrum. Later, in 1895, Sir William Ramsay discovered an element that gave the same spectrum, and which was therefore given the same name. Helium is at present a comparatively well-known gas that can be produced in all chemical laboratories.

The instrument by which the spectra of celestial and terrestrial objects are produced is called the **spectroscope.** By combining the spectroscope with the telescope, astronomers have succeeded in ascertaining to some extent the composition of many stars and in collecting other information of great scientific value.

EXERCISES

1. What kind of a spectrum should moonlight give?

2. What kind of a spectrum would you expect to obtain by dispersing the light from a live coal by means of a prism?

3. The luminosity of an oil or gas flame is due to heated particles of carbon. What should be the nature of the spectrum of a kerosene lamp flame?

4. What is the position of the sun relative to falling rain and an observer when a rainbow is seen?

5. Why do colored fabrics often appear different when viewed by artificial light?

6. If the waves producing the sensation of red were all absorbed from sunlight, what color would remain? What color would objects that were formerly red appear to have?

3. INTERFERENCE OF LIGHT

340. Color Bands by Interference. — We have seen how composite light can be separated into its component colors by means of a prism, but there still remain a great many cases of color formation which are not at all due to dispersion or absorption.

Let two strips of plate glass, A and B, Fig. 268, about 1 inch wide and 5 inches long, be separated at one end by a piece of tissue paper C (exaggerated in cut) and the other ends clamped tightly together. (The plate glass covers accompanying sets of small weights may be used.) Produce a yellow sodium flame by bringing in contact with a Bunsen or alcohol flame a piece of asbestos D, wet with a solution of common salt. Now hold the glass strips behind the flame, and observe the images produced. A series of dark and yellow bands will be seen extending transversely across the glass.

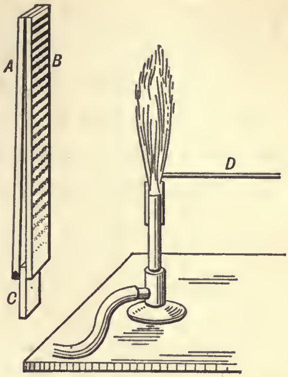


FIG. 268. — Alternate Dark and Bright Bands Produced by Interference of Light Waves.

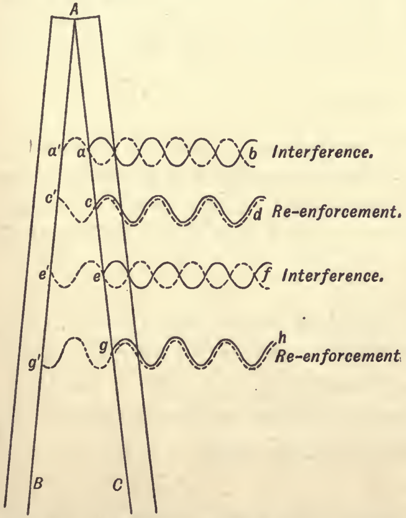


FIG. 269. — Diagram Showing Points of Interference and Reënforcement.

The explanation of this interesting phenomenon depends on the wave theory of light. The flame, as we know, sends out only yellow light (§ 338). This is in part transmitted and in part reflected at each glass surface (§ 282). Let AB and AC, Fig. 269, represent the interior surfaces of the plates much enlarged, and let the wave line ab represent the light reflected at the point *a* on the surface *AC*. Also let the dotted line *a′b* represent the waves reflected at *a′* on the surface *AB*. These two trains of waves obviously interfere and destroy each other just as do two trains of sound waves when they coincide in this manner (§ 190). Therefore, since at this place on the glass plates the light waves cancel each other, we see a dark band. But at the point *c* the case is different. Since the thickness of the air film is here a quarter of a wave length greater than at *a*, the train of waves reflected from *c′* coincides with that reflected at *c* in such a manner as to produce reënforcement. Hence at this point we see the bright yellow band. Similarly the wave trains reflected at *e* and *e′* interfere and cancel, while at *g* and *g′* we again find reënforcement. Thus the dark and bright bands appear alternately in the plates.

341. Color Bands in Soap Films. — The following experiment can easily be performed by letting a soap film take the place of the wedge-shaped air film of the preceding experiment.

Let a film of soapy water be produced on a wire loop as described in § 130. Hold the film behind a yellow sodium flame, and observe the image in the film by reflected light. Many narrow bands of yellow will appear which continue to grow broader and farther apart until the film breaks.

When the wire loop is held in a vertical position, the liquid runs slowly down from the top, forming a wedge-shaped film that is thinnest at the upper edge. One portion of the yellow incident light is reflected from the front surface, and another portion from the back, after traversing the thickness of the film. Hence the portion reflected from the back passes through the film twice. Now if the two reflected trains interfere, a dark band is produced; but if they reënforce, a bright band of yellow appears, precisely as described in the preceding section.

342. White Light Decomposed by Interference. — The result of the experiment described in § 341 leads to the explanation of the beautiful coloration produced when the sunlight falls on soap bubbles, oil films on water, etc.

Let sunlight fall upon the plates of glass arranged as in § 340. Variegated bands occurring alternately will appear instead of the yellow bands obtained when sodium light is used.

On account of the fact that sunlight, or white light, is a composite of different wave lengths, the interference of the reflected waves of red, for example, takes place at a different point from that of the yellow. When the waves of red are canceled by interference, the complementary color, or bluish green, is left. At the point where waves of yellow interfere, its complement, or blue, appears. Thus bands consisting of the complements of all the spectral colors are produced.

SUMMARY

1. White light is decomposed into the spectral colors by passing through a triangular prism of glass, water, ice, etc. The cause of dispersion lies in the fact that different wave lengths are retarded different amounts, and hence are refracted in differing degree by the prism. Long waves (red) suffer the greatest refraction, while the shorter waves (violet) suffer least (§§ 327 to 329).

2. The color of an object depends both on the light which falls upon it and that which it reflects to the eye. Various wave lengths are absorbed by all except white bodies (§ 330).

3. The color of a transparent body depends on the color of the light which it transmits (§ 331).

4. Complementary colors are those which when added in proper proportions produce white (§ 332).

5. The color of pigments used in the manufacture of paints, etc., is due to the absorbing power of the material (§ 333).

6. The solar spectrum is the spectrum of sunlight. The rainbow is an example. The pure spectrum is crossed by numerous dark lines produced by the absorption of certain wave lengths by heated gases in the solar atmosphere. These lines reveal the chemical elements in the sun's composition (§§ 334 to 339).

7. Interference of light is brought about by the coincidence of waves in such a manner as to weaken or cancel each other. The alternate interference and reënforcement of light waves in thin wedge-shaped films gives rise to the colors seen in oil films, soap bubbles, etc. (§§ 340 to 342).

CHAPTER XVI

ELECTROSTATICS

1. ELECTRIFICATION AND ELECTRICAL CHARGES

343. An Electric Charge Produced. — It is a matter of common observation that a hard rubber comb acquires the power of attracting bits of tissue paper and other very light bodies simply by being drawn through dry hair. The comb may also be put into this condition by being rubbed with silk or flannel. Knowledge of this phenomenon dates from the Greeks of 600 B.C., when it was known that amber would attract light objects after having been rubbed with silk. No advance was made in the science of electricity until the time of Sir William Gilbert of England, about 1600 A.D. At this time Gilbert found that a great many substances, as glass, ebonite, sealing wax, etc., could be given this power of attraction by rubbing with fur, wool, or silk.

Test rods of glass, ebonite, sealing wax, etc., for the power of attracting small pieces of dry pith before and after being rubbed with silk, fur, and flannel. See Fig. 270.

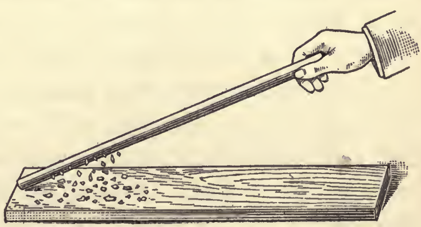


Fig. 270. — Action of an Electrified Glass Rod.

When a body possesses the property of attracting light objects, as hair, pith balls, and bits of paper, it is said to be **electrified**. The change is brought about by a **charge of electricity**. The process by which a body is electrified is called **electrifica-**

tion. These terms are all derived from the Greek word *electron*, meaning amber.

344. Electrical Charges of Two Kinds. — Procure two large sticks of sealing wax and a glass rod. Electrify a stick of sealing wax by rubbing it with a flannel or fur and suspend it in a wire stirrup as shown in Fig. 271. Rub the other stick of sealing wax and bring it near the suspended one. A decided repulsion will be observed. Now rub the glass rod with silk and bring it near the suspended wax. Attraction takes place.

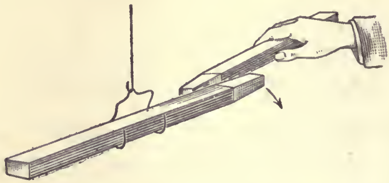


Fig. 271. — Repulsion between Two Similarly Charged Bodies.

The experiment shows that the electrified glass and sealing wax cannot be in precisely the same condition, since they act differently toward the suspended body. This difference in the behavior of electrified bodies is said to be due to the kind of charge developed when they are rubbed. Thus glass is said to be charged *positively* when rubbed with silk; and sealing wax, *negatively* when rubbed with flannel or fur. Some specimens of glass are electrified negatively when rubbed with cat's fur or flannel.

345. A Law of Electric Action. — The preceding experiment shows that two sticks of sealing wax repel each other after being charged negatively. In the same manner two positively charged glass rods will show a repulsion. But any negative charge will attract a positive charge, just as the glass attracts the suspended sealing wax when both are electrified. Hence, we may infer that *electrical charges of a similar kind repel each other, and those that are dissimilar attract.*

346. The Electroscope. — In order to detect the presence of a charge upon a body, an instrument called the *electro-*

scope is employed. The *gold-leaf* electroscope is shown in Fig. 272. This instrument consists of a metal rod which penetrates the rubber stopper of a flask. At the top the rod terminates in a plate or ball, and at the lower end it is provided with two strips of gold foil. Under ordinary conditions the leaves hang parallel; but when an electrical charge is brought near, they diverge and thus show the presence of electrification.

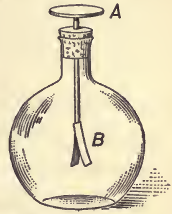


FIG. 272. — The Gold-leaf Electroscope.

347. Determining the Kind of Charge. — The electroscope is frequently used to ascertain the kind of charge on an electrified body. The following experiment will show how this can be done.

Make a *proof plane* by sealing a small coin or tin disk to the end of a small glass rod to be used as a handle. Touch the disk of the proof plane to a positively charged glass rod, and then to the plate of the electroscope. The divergence of the leaves will show that the instrument is charged. Now, if the charged glass rod is carefully brought near the electroscope, the divergence will increase; but if the negatively charged sealing wax is brought near, the divergence at once decreases. While the electroscope is thus charged, bring up an electrified rod of ebonite or a charged rubber comb, and note whether the divergence *increases* or *decreases*. In this case the charge on the ebonite will be found to act like that on the sealing wax and hence produce a decreased divergence of the leaves.

In a word, the nature of an unknown charge is determined by observing whether its effect on a *charged* electroscope is like that of a positively charged glass rod or the negatively charged sealing wax.


FIG. 273. — Opposite Charges Developed at the Same Time.

348. Positive and Negative Charges Developed Simultaneously. — Let a flannel cap about 6 inches long be made that will fit closely over the end of an ebonite rod as in Fig. 273. *A* represents a silk thread by which the cap may be handled. Now turn the rod in the cap to electrify it and, without removing the cap, hold the rod near the plate of the electroscope. Little or no charge will be detected. Remove the cap by means of the thread *A* and present it to the electroscope. When tested as in § 347, the cap will be found to be charged positively. If the rod be now tested, a negative charge will be found.

On account of the fact that the rod and cap together produce no effect on the electroscope, we may infer that their charges exactly cancel. The conclusion is, therefore, that *when a certain quantity of one kind of electrification is produced by rubbing a rod, an equal amount of the opposite kind appears on the object with which it is rubbed.*

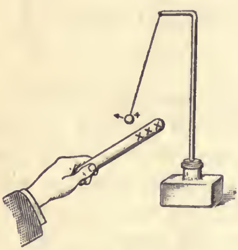


FIG. 274. — Experiment with a Suspended Pith Ball.

349. Charging by Contact. — By means of a silk thread or silk fiber suspend a small ball of dry elder pith as shown in Fig. 274. Hold near the ball a positively charged glass rod. The ball is at first attracted to the rod and then violently repelled. The repulsion shows that the ball by contact with the rod has been charged *with a kind of electrification similar to that in the rod.* In this case the charge is *positive.* Likewise the ball will be charged *negatively* by contact with a negatively charged body.

The experiment shows clearly that when an insulated body comes in contact with a charged body, it becomes charged with electricity of the same kind as that on the charged body. In this way the proof plane used in § 347 carried from the glass rod to the electroscope an electrical charge of the same kind as that on the glass.

350. Conductors and Insulators. — Select a number of pieces of wood, metal, glass, ebonite, cardboard, leather, etc. Let one end of a metal rod be supported on the electroscope and the other end on an ebonite rod *A*, Fig. 275. Bring a charged body *R* against the end of the rod opposite the electroscope. A sudden divergence of the leaves shows the transfer of a charge from R to the instrument. Experiment with a rod of wood in the same manner. The divergence, if any is produced, takes place more slowly than before if the wood is dry. Glass and ebonite should be found not to transfer any appreciable charge unless their surfaces are moist.

FIG. 275. — Testing the Conductivity of Rods.

Some substances have the power to transfer charges of electricity, while others do not. Metals are shown to be good **conductors**; dry wood is a poor conductor, while glass and ebonite are practically non-conductors. Non-conductors are called **insulators**. Among the best insulators may be mentioned dry air, glass, mica, shellac, silk, rubber, porcelain, paraffin, and oils. Substances of all degrees of conductivity exist, varying from the best conductors down to the best insulators.

EXERCISES

1. Place a piece of paper against the wall and stroke it with cat's fur. It will be found to adhere to the wall for several minutes. Explain.

2. Why is it more difficult to brush lint from clothing in cold dry weather than at other times?

3. Draw a rubber comb through dry hair and test the charge developed on the comb. Now present a positively charged rod to the charged hair and observe whether it is repelled or attracted. Compare this experiment with that of § 348.

4. After walking on a silk or woolen rug or over a glass floor, one often finds that sparks will jump between the finger and a gas fixture near which it is held. The gas may even be lighted in this manner. Explain.

5. Support a dry pane of glass about 1 in. above a quantity of small pieces of dry pith. Rub the glass with silk and explain the agitation of the pith observed.

6. Balance a meter stick on the smooth bottom of a flask and see if it is affected by a charged glass rod. Are only very light bodies attracted?

2. ELECTRIC FIELDS AND ELECTROSTATIC INDUCTION

351. Lines of Force. — We have already observed that an electrified body has an effect upon another body even when placed at a distance of several inches. Light objects will rise from the table toward a charged glass rod, and the leaves of an electroscope will often show a divergence at a distance of four or five feet from the charge. The space in which an electric charge affects surrounding objects is called the **electric field** due to that charge. If lines are drawn in an electric field to represent at every point the direction in which a charge, as A, Fig. 276, tends to move a very small, positively charged body B, they are called **lines of force**. Thus every electric field is assumed to be filled with lines of force.

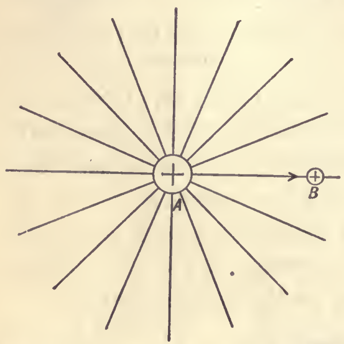


Fig. 276. — Lines of Force Emanating from a Charge.

Lines of force extend from positive charges to negative charges, as shown in Fig. 277. In other words, a positive charge always exists at one end of a line of force and a negative one at the other. Therefore, since each line of force must have two extremities, for every positive charge there must be somewhere a corresponding negative charge. This fact gives rise to the phenomena shown in the following sections.

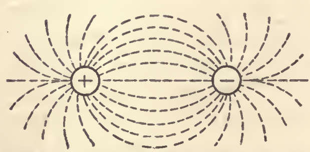


Fig. 277. — The Electric Field between Two Unlike Charges.

352. Electrification by Induction. — Let a charged glass rod be brought near a gold-leaf electroscope. The leaves begin to diverge when the rod is at a distance and spread more and more as the rod approaches. On removing the rod the leaves fall together again.

The temporary effect produced in the electroscope, due to the presence of a neighboring charge, is the result of *electrostatic induction.* The leaves of the instrument are obviously affected only while they are in the electrical field around the charged rod, since they collapse as soon as the rod is removed. In this case there is no change in the amount of electrification on the glass rod, and the effect can easily be shown to take place through an intervening plate of glass or other insulating material. Hence, in the process of electrostatic induction a transfer of electricity from one body to the other does not take place.

353. Electrical Separation by Induction. — The condition of an electroscope which is placed near a charged body, as in the preceding section, is readily understood after the following experiment:

Place two metal vessels A and B, Fig. 278, in contact on an elevated block of paraffin, or some other good insulating material. Each vessel should be provided with a small pith ball attached to the top by means of a piece of cotton thread. Now bring a positively charged glass rod R near one of the vessels, as B, and, by means of the silk thread C, separate the vessels before removing the rod. When the rod is taken away, the pith balls will show that each vessel has acquired a charge, but the charge on the rod R has not been diminished. With the help of the proof plane and electroscope test each charge; that of B will be negative, that of A positive.

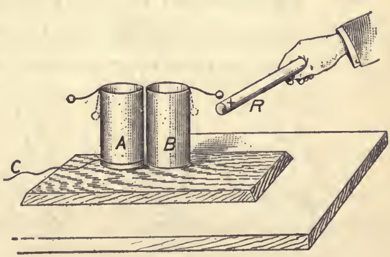


FIG. 278. — Separation of Positive and Negative Charges.

From this experiment it is clear that *whenever an electrified body is brought near an unelectrified insulated conductor,* *the opposite kind of electrification is induced on the nearer side and the same kind on the remote side.*

354. Charging by Induction. — An insulated metallic body may be charged by making use of the separation of positive and negative electricity, as shown in the preceding section.

Hold the positively charged glass rod about 6 in. from the plate of the gold-leaf electroscope. The leaves diverge because the positive charge on the rod induces a negative charge in the plate and a positive charge in the leaves, as shown in Fig. 279. While the rod is in this position, touch the electroscope with the finger. The leaves collapse. Now remove the finger and then the rod. The leaves diverge again, thus showing that a charge is left on the instrument. If this charge be tested by bringing a negative charge near the electroscope, an increased divergence will show that it is negative.

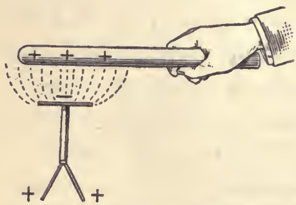


Fig. 279. — Charged Rod Acting by Induction on an Electroscope.

355. Explanation of the Process. — The electrical conditions that exist during the process of charging a conductor by induction are represented in Fig. 280. The presence

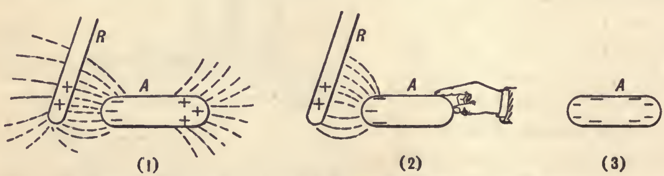


Fig. 280. – Illustrating the Process of Charging by Induction.

of the positively charged rod R produces a separation of positive and negative electricity in the conductor A (§ 353), just as if an uncharged body possessed equal amounts of these two kinds, as shown in (1). Lines of force extend from the positive on the rod to the induced negative on the conductor, while other lines extend away from the positive at the remote end to the walls of the room, near-by objects, etc. When the conductor is touched with the finger, as in (2), the repelled or "free" charge, which in this case is positive, is permitted to escape through the body to the earth. The negative or "bound" charge remains on the conductor on account of the influence of the charged rod *R*. On removing first the finger and then the rod, the negative charge distributes itself over the conductor, as shown in (3). If *R* is a negatively charged body, the signs of all the charges will simply be reversed.

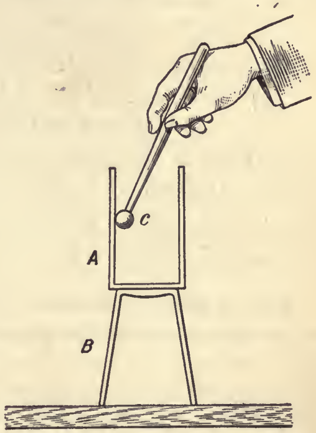


FIG. 281. — No Charge can be Taken from the Interior Surface of a Charged Body.

356. Distribution of Electricity on a Conductor. — Insulate a metal vessel *A*, Fig. 281, by placing it on a dry tumbler *B*, or a plate of paraffin or beeswax. Charge the vessel as highly as possible and then try to take charges from its interior surface to the electroscope by means of a proof plane *c*. It will be found that no charge can be obtained in this way. Now try to take a charge from the exterior surface of the vessel. The electroscope will show that this attempt is successful.

This experiment shows very conclusively that an electrical charge distributes itself over the exterior surface of an insulated conductor. This is just the result that is to be expected if we consider that the various parts of any charge are mutually repellent. On this account they will separate to the greatest extent possible, which is the case when the charge is distributed over the exterior surface of the body charged.

357. Distribution of a Charge not Uniform. — Charge a large insulated egg-shaped conductor, Fig. 282. By means of a proof plane transfer a charge from the large end of the body to the electroscope and observe the divergence of the leaves. Now discharge the electroscope and test the small end of the conductor in a similar manner. A greater divergence of the leaves will be obtained.

FIG. 282. — Electrical Density is Greatest at the Pointed End of this Conductor.

It thus appears that an electrical charge distributes itself over a conducting surface in accordance with the shape of the body. The quantity per square centimeter is greatest at the small end. In other words, the **electrical density** is greatest where the conductor is the most sharply pointed. In fact, if a sharp point be attached to the charged conductor, the density at the point may be so great that the charge will be spontaneously discharged into the surrounding air.

358. Effect of Points. — By the help of sealing wax or shellac attach the center of a sharp needle to a glass handle. Bring a charged glass rod over the electroscope, and, while the rod remains in this position, bring the eye end of the needle against the plate of the electroscope and the point toward the charged rod. Now remove both needle and rod, and the electroscope will be positively charged. Again, hold a charged glass rod near an insulated conductor, as a metal vessel, and then place the eye end of the needle against the opposite side of the conductor. Remove the needle and then the rod and test the conductor for a charge. A negative charge will be found.

The results of these experiments depend upon the electrical discharge from pointed conductors. On account of the great electrical density at a point, an intense field of force exists in the immediate neighborhood. Air particles are forcibly drawn against the point and charged by contact with the same kind of electricity, after which they are violently repelled. Thus a so-called **electrical wind** is set up which conveys away the charge at a rapid rate.

359. Lightning Rods. — Positive evidence regarding the identity of lightning and electrical discharges was secured by the classical experiments of Benjamin Franklin[1] in 1752, when he succeeded in drawing an electrical charge from a thunder cloud along the string of a kite. Through his suggestion lightning rods were first used as a protection for buildings against damage by lightning. The principle has been demonstrated in the preceding section. A strongly charged cloud passes over a building, and between the cloud and the building there is set up an intense field of force (§ 351). If this field becomes of sufficient intensity, the air, being no longer able to insulate, breaks down as an insulator. The sudden discharge tears the roof of the building, and the intense heat often produces a conflagration. The presence of a pointed conductor, however, leading from above the roof to the earth, permits a gentle and harmless discharge of electricity to take place between the cloud and the earth. The effectiveness of lightning rods is often impaired by the use of dull points and poor ground connections.

The thunder that accompanies a lightning flash is caused by the impact of the air as it is forced in to fill the partial vacuum which is developed along the line of electrical discharge. At a distance from the discharge, the direct report is followed by echoes from the clouds and woods, causing the rumblings so common in most localities.

EXERCISES

1. In testing a charge, why is it necessary to work with a charged electroscope?

2. When a gold-leaf electroscope is charged with negative electricity, for example, why will the approach of a positively charged body produce first a decrease and then an increase of the divergence of the leaves?

[1] See portrait facing page 346.

3. If several insulated metal vessels are placed in a row, but not in contact with each other, what will be the electrical condition of each when a positively charged rod is held near one end of the row? Illustrate by means of a diagram showing four such vessels.

4. If the vessel at the end of the row near the charged rod is touched with the finger and then both finger and rod are removed, what will be the electrical condition of each vessel? Illustrate this case by a diagram.

5. Represent by diagrams the fields of force existing under the conditions described in Exer. 3.

6. Represent the fields of force as they exist under the conditions given in Exer. 4 and compare each field with the corresponding one of the preceding exercise.

3. POTENTIAL DIFFERENCE AND CAPACITY

360. Electrical Flow. — It has already been observed in § 349 that a charge of electricity can be transferred by a conductor from an electrified body to the electroscope. The effects that accompany such a transmission of electricity, as will be seen later, enable it to be employed as one of the most important agents under the control of man. The following experiment will bring out more clearly the conditions under which a transfer of electricity takes place.

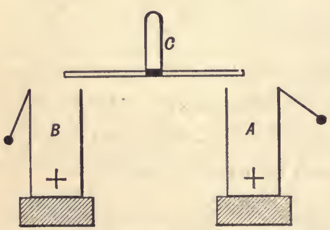


Fig. 283. — Conductor A has a Higher Potential than B.

Provide two similar metal vessels A and B, Fig. 283, with pith balls and insulate them on blocks of paraffin. Let each vessel be positively charged, but A more highly than B, as measured by the repulsion of the pith balls. Now connect the vessels by means of a wire attached to a sealing wax handle C. The pith ball on A will fall slightly while that on B will rise. Thus some of the charge on A moves along the wire to B.

Although both insulated vessels were originally charged with the same kind of electricity, in the language of

BENJAMIN FRANKLIN (1706–1790)

The achievements of Franklin in the field of electricity are no less brilliant than his successes as a statesman and diplomat. His attention was first turned to the study of electrical phenomena by witnessing some experiments which at that time (1746) were regarded as no less than marvelous. His experiment to prove the electrical nature of lightning has become classic. In order to ascertain whether electricity could be obtained from clouds during a storm, Franklin constructed a kite which was provided with a pointed metal rod for "drawing off" the electrical charge, if there should prove to be any. At the approach of a storm, Franklin and his son raised the kite, which was held by a hempen cord. The lower end of the cord was tied to a metal key through which was passed a ribbon of silk to protect the body from severe and dangerous shocks. As soon as the cord became moistened by the rain, electric sparks were readily drawn from the key. In regard to this experiment, Franklin writes: "At the key the Leyden jar may be charged; and, from the electric fire thus obtained, spirits may be kindled and all other electrical experiments performed which are usually done by the help of a rubbed glass globe or tube, and thereby the sameness of the electrical matter with that of lightning completely demonstrated."

"Antiquity would have erected altars to this great and powerful genius who, to promote the welfare of mankind, comprehending both the heavens and the earth in the range of his thought, could at once snatch the bolt from the cloud and the scepter from tyrants."—Mirabeau

Physics, *A was charged to a higher* **potential** *than B. Thus a charge moves from a point of higher to one of lower potential,* just as heat flows along a heat conductor from a point of higher to one of lower temperature.

361. Potential Difference. — It has just been observed that an electrical charge will flow along a conductor as long as there exists a *potential difference.* It is precisely this difference of potential that determines whether electricity will flow and the direction it will take. Figure 284 will show to some extent the differences that may exist between electrical charges. *AB* represents the level ground, and *a*, *b*, *c*, and *d* four tanks containing water. Tank *a* is analogous to an insulated body charged positively to a high potential, as vessel a', while tank *b* represents one of a lower potential than a', as b'. It is assumed that all positive charges are of a higher potential than the earth and will discharge to the earth unless insulated from it. On the other hand, negative charges, as c' and d', are assumed to have lower potentials than the earth. The potential of the earth is thus on the dividing line between positive and negative charges; hence, *the potential of the earth is regarded as zero.*

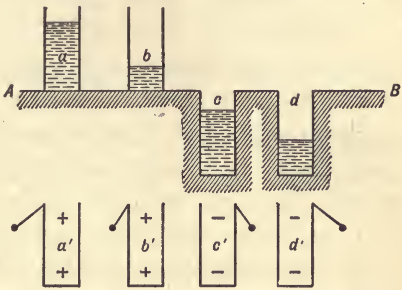


Fig. 284. — Analogy between Electrical Potential and Water Level.

Connecting any two of the tanks shown in the figure would evidently result in a transfer of water from left to right; thus joining any two of the charged vessels by means of a conductor would bring about a transfer of electricity in the same direction.

362. Mixing Positive and Negative Charges. — Let the two insulated metal vessels used in § 360 be charged with equal amounts of positive and negative electricity as shown by the divergence of the pith balls. Now connect them by means of a wire provided with an insulating handle as before. Both balls fall against the sides of the vessels, showing that the two charges have neutralized each other. Repeat the operations just described after charging A positively until its pith ball diverges considerably more than that of B, which is charged negatively. After the connection has been made between the vessels, a positive charge will be found on each vessel.

Not only does electricity tend to flow from a positively to a negatively charged body, but a mixture of unlike charges tends to produce a cancellation of both. If, however, one of the charges exceeds the other in amount, the excess remains distributed over both surfaces. In the final condition both bodies are of the same potential.

363. The Electrostatic Unit of Quantity. — *Unit charges of electricity are such equal quantities as exert upon each other a force of one dyne* (§ 35) *when separated by one centimeter of air.* The force, as we have seen, is repellent when the charges are of the same kind and attractive when they are different. If two equal quantities of positive and negative electricity are mixed, they cancel each other; but if both of the two mixed charges are of the same kind, the resulting charge is their sum.

Example. — If 10 units of positive electricity are mixed with 12 units of negative, what will be the result?

Solution. — The 10 positive units will cancel 10 negative units, leaving 2 units of negative electricity, which will be distributed over the surface of the conductors.

364. Electrostatic Capacity. — By means of a proof plane transfer a charge from an electrified glass rod to the electroscope and note the approximate divergence of the leaves. Now place one of the metal vessels used in § 353 upon the plate of the electroscope and again transfer a charge with the proof plane as before. In this case the divergence will be found to be much smaller than before. Continue to transfer charges until the divergence is the same as at first.

By placing the metal vessel upon the electroscope, the surface over which a charge will distribute itself is materially increased. Hence a given quantity of electricity will produce a smaller electric density, and the mutual repulsion of the gold leaves will be lessened. On this account, *a larger quantity will be required to produce the same divergence of leaves; or, in other words, to produce the same potential as at first.* The change produced in the condition of the conductor is expressed by saying that the **electrostatic capacity** is increased by the increased area presented by the metal vessel.

365. Condensers. — In many of the practical applications of electricity, it is necessary to make use of some device that has many times the capacity of anything we have used in the preceding experiments. The manner in which the desired result is accomplished is made clear by the following experiments:

1. Place a flat metal plate *C*, Fig. 285, having well-rounded corners, upon the plate of the gold-leaf electroscope and charge the instrument with the glass rod and proof plane. Now bring a similar metal plate *B*, which is provided with a handle of sealing wax *A*, near *C*, but not touching it. The divergence of the gold leaves will decrease, showing that the potential of the leaves has been lowered. Withdraw *B* and the potential of the leaves will rise to its original value. Repeat these operations while the fingers are allowed to remain in contact with *B*, thus connecting it with the earth. The divergence of the leaves becomes very small when *B* and *C* are near together, but returns to its former value when the plates are again separated.

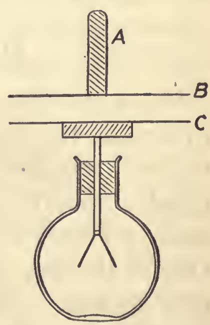


Fig. 285. — Showing the Effect of a Neighboring Conductor on Capacity.

2. Mount a plate *B* in a clamp so that it is about ½ cm. from *C* and insulated. Count the number of charges that must be carried to *C* by a proof plane to produce a given divergence, *i.e.* to produce a given potential. Now discharge C and connect B with the earth by means of a wire, and see how many charges must be transferred to C to produce approximately the same divergence as before. A large number will be required.

These experiments show (1) that the potential that a given charge produces when placed on an insulated conductor depends upon the proximity of other conductors and whether they are "earthed" or not, and (2) that the capacity of a conductor is enormously increased by the presence of an earthed conductor placed very near, but insulated from it. *A combination of plates separated by an insulator constitutes an* **electrostatic condenser.** The capacity of a condenser is proportional to the size of the plates and becomes greater as the distance between them is reduced.

366. Influence of the Insulating Material. — Arrange the electroscope and plates as in Experiment 2 of the preceding section. Connect B with the earth and charge C until a moderate divergence of the leaves is produced. Now introduce between B and C a pane of glass and note the effect upon the divergence. The potential of C will fall when the glass is introduced, and will rise again when it is removed. Let the experiment be made by using beeswax or paraffin instead of glass. The effect is more marked than before.

Since the introduction of another insulator, or **dielectric,** to replace the air between the plates of a condenser, reduces the potential, it is obvious that it will require a greater charge to bring the potential back to its original value. Hence the capacity of the condenser is increased. One of the best insulators used in condensers is mica, not only because it can easily be obtained in thin sheets, but because of its advantageous influence as a dielectric upon the electrical capacity of the condenser.

367. Forms of Condensers. — One of the earliest forms of condensers is known as the *Leyden jar*, from Leyden in Holland, the place of its origin. It was first used in 1745. This condenser consists of a glass jar, Fig. 286, which is coated with tin foil to about two thirds of its height on its interior and exterior surfaces. Through a cover of insulating material passes a metal rod terminating at the top in a ball and at the lower end in a chain which makes contact with the inner coating of the jar.

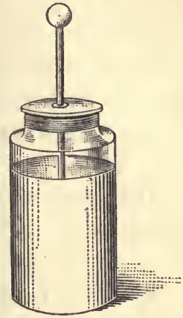


Fig. 286. — The Leyden Jar.

Another form of condenser that is widely used is represented diagrammatically in Fig. 287. This condenser consists of a large number of sheets of tin foil of which alternate sheets are connected at *A* and the intervening sheets at *B*. In the best condensers of this type the insulating material separating the sheets of foil is mica; but in cheaper forms, paraffined paper is used. The capacity of such condensers is large on account of the large area of tin foil and the extremely small distance between the conducting surfaces of the foil (§ 365).

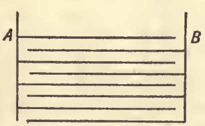


Fig. 287. — Diagram of a Mica, or Paper, Condenser.

368. Charging and Discharging a Leyden Jar. — In order to charge a Leyden jar, the outer coating is connected with the earth by a metallic conductor, or the jar is simply held in the hand. A very imperfect earth connection is produced by setting the jar upon a table of dry wood. If now the knob of the jar be connected with some source of electricity, a charge is communicated to the inner surface. If this is positive, an equal amount of negative will be induced on the inner surface of the outer coating, and a similar quantity of positive repelled to the earth. In this manner a strain is set up in the glass which in some instances is sufficient to break the jar.

To discharge a Leyden jar, it is necessary to make an electrical connection between the two tin-foil coatings. If the charge is not large, this can be done safely by simultaneously touching the knob and the outer coating with the hands. The best method, however, is to bend a metal conductor in such a form that one end can be kept in contact with the outer coating while the other is brought near the knob. At the instant of discharge a bright spark will be seen to jump between the knob and the end of the conductor.

EXERCISES

1. Three Leyden jars are charged with − 4, − 7, and + 10 units respectively. How many units will remain in the jars after the knobs have been connected?

2. A Leyden jar that is placed on a plate of glass has a small capacity. Why?

3. If a Leyden jar be highly charged and then placed on a plate of glass or paraffin, the knob can be safely touched with the hand. In this condition only a small portion of the entire charge will be taken from the jar. Explain.

4. Having a metal globe positively electrified, how could you electrify any number of other globes with negative electricity?

5. With a positively charged globe, how could you positively charge one of the other globes without reducing the charge on the first?

4. ELECTRICAL GENERATORS

369. The Electrophorus. — In order to produce larger electrical charges than those used in any of the preceding experiments, the principle of induction (§ 352) is advantageously employed. The simplest form of generator for this purpose is the *electrophorus* (pronounced *e lĕk trŏf′o rus*), an apparatus invented by Volta,[1] an Italian physicist, in 1777. This instrument consists of a plate of

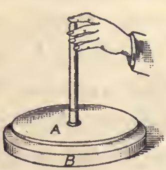


Fig. 288. — The Electrophorus.

[1] See portrait facing page 352.

COUNT ALESSANDRO VOLTA (1745–1827)

The age of practical electricity began with the invention of the voltaic cell by Volta, an Italian, professor of physics at the University of Pavia. Before his invention it was not known that a continuous current of electricity could be produced. In 1793, Volta announced to the Royal Society of London a discovery made by Galvani (1737–1798), professor of anatomy at Bologna. Galvani had observed that by joining together two different metals and touching one to the muscle of a frog's leg and the other to the nerve, violent contractions were produced even after the animal's death. Most extravagant hopes were founded on this discovery, and many believed that a cure had been found for all diseases. In reality, the discovery paved the way to other and greater ones which have become a vital part of the history of the nineteenth century.

The use of the two metals in Galvani's experiment suggested to Volta the basic principle of the modern cell consisting of two plates of different metals immersed in a liquid. This, as we know, becomes a continuous source of electricity. The invention was received with great enthusiasm. In a short time batteries made by joining several of the so-called "voltaic piles" were used in experimental work in many of the laboratories of Europe. With the advent of this means of generating electric currents began the series of discoveries that have led up to the phenomenal use of electricity at the present day.

To Volta is ascribed the invention of the electrophorus, electroscope, and the condenser. The practical unit of potential difference is called the *volt* in his honor.

ebonite or a shallow metal dish *B*, Fig. 288, about 25 cm. in diameter, into which has been poured melted resin or shellac, and a flat circular metal disk *A*, somewhat smaller than the plate and having well-rounded edges. The metal disk is provided with an insulating handle of ebonite.

Let the plate of an electrophorus be stroked with cat's fur and its electrification tested. Place the metal disk upon the plate and test the kind of electricity that can be taken from its upper surface. Touch the disk with the finger in order to "ground" or "earth" it, lift it by the insulating handle, and test its charge. Repeat the experiment, without "grounding" the disk. Explain the result.

The action of the electrophorus is as follows: When the non-conducting plate *B* is stroked with fur, it is given a negative charge. If, now, the metal disk *A* be placed upon the plate, the negative on the plate induces a positive charge on the lower surface of the disk and repels a negative charge to the upper side, as shown in Fig. 289. When the disk is touched with the finger, the negative charge escapes, leaving the positive charge, which is distributed over the disk when it is lifted. Compare § 355. Any number of charges may be obtained from the electrophorus without producing any appreciable change in the charge on the plate.

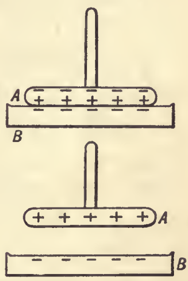


Fig. 289. — Representing the Theory of the Electrophorus.

370. Source of the Energy Derived from the Electrophorus. — Although any number of charges can be produced by the electrophorus, we know that the energy obtained in this manner cannot be brought into existence without the expenditure of a like quantity on the part of some working agent (§ 64). The source of the energy is obvious from the following:

After the metal disk of an electrophorus has been placed on the electrified plate and touched on its upper surface to draw away the negative charge (§ 369), connect a gold-leaf electroscope with the disk by means of a slender wire. No divergence in the leaves is produced, although we know that the disk has a positive charge induced by the negative charge on the plate. Slowly lift the disk by the insulating handle and the leaves will diverge widely.

When the disk with its induced positive charge rests upon the charged plate, it is at zero potential because it has been in connection with the earth. The disk, like a weight lying on the ground, manifests no energy until its positive charge is separated from the negative charge on the plate in opposition to their attractive force. The agent, therefore, that lifts the disk from the plate furnishes the energy which appears in the disk as electrical energy. When the disk is discharged, the electrical energy is transformed into heat which appears in the spark that is produced. The heat of the spark can be utilized to light illuminating gas or explode powder.

371. The Toeppler-Holtz Influence Machine. — The discovery of X-rays and their necessity for generators capa-

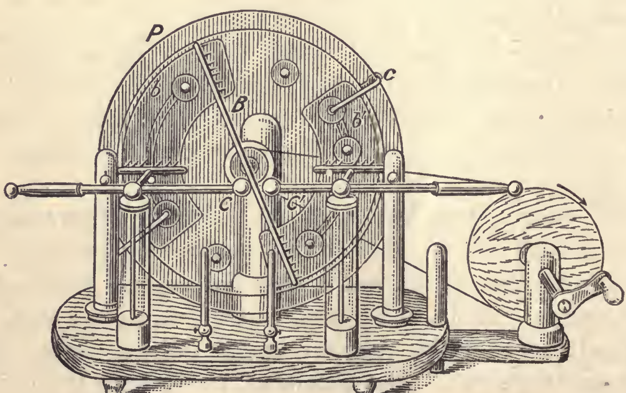


FIG. 290. — The Toeppler-Holtz Electrical Machine.

ble of producing a continuous supply of electricity has brought such generators into extensive use. The simplest form of induction, or influence, machines is the *Toeppler-Holtz*, shown in Fig. 290. This machine makes use of a glass plate about 50 cm. in diameter, which is provided with six or eight metallic disks, as shown. This plate revolves in front of a second stationary plate of glass *P*, which is furnished with two strips of tin foil *b*, *b′*, covered with paper sectors called armatures, or inductors. In front of the revolving plate is the stationary neutralizing bar *B*, provided with points and tinsel brushes at the ends. *C*, *C′* are two conductors having at their other extremes points that come close to the revolving plate. The action of the machine is best understood from a study of the diagram shown in Fig. 291.

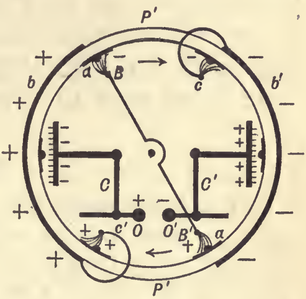


FIG. 291. — Diagram Showing Action of the Toeppler-Holtz Machine.

Imagine a small positive charge placed on the sector *b*. This charge acts inductively through the glass on the rod *BB′*, attracting a negative charge to *B* and repelling a positive one to *B′*. These induced charges rapidly escape from the points (§ 358) and electrify the glass plate as well as each metallic disk as it passes. As the plate rotates in the direction shown by the arrow, the disks at the top carry negative charges to the right, while those at the bottom carry positive charges to the left. These disks touch the small brushes *c* and *c′*, through which negative charges are given to sector *b′* and positive to *b*. The charges on the sectors are thus continually increased, an effect which continually increases the first inductive action through the glass on *BB′*. Again, when the negatively charged glass plate reaches C', positive electricity is drawn from the points and negative left on the ball O'. Similarly, on the opposite side of the machine, the positively electrified plate neutralizes the negative charge that it induces at the points C, and thus leaves a positive charge on ball O. The unlike charges on O and O' continue to increase until the difference of potential is sufficient to force a discharge to take place through the air between them. If the distance is not too great, a stream of sparks will appear to pass without interruption. The energy transformed in each spark is greatly increased by the presence of the two Leyden jars shown in Fig. 290.

EXERCISES

1. Why is the metal disk of the electrophorus not charged negatively by contact with the negatively charged plate?

SUGGESTION. — Consider the nature of the plate and the fact that it touches the disk in only a very few places.

2. Explain why an influence machine turns with greater difficulty when it is developing a high potential between the balls O and O'.

SUGGESTION. — Consider the nature of the electrical forces existing between the stationary and moving charges.

3. Small Leyden jars (see Fig. 290) used in connection with the balls of the influence machine cause the production of much brighter sparks than would be produced without them. Explain.

4. With the Leyden jars removed, would the frequency with which sparks pass between O and O' be increased or decreased?

SUMMARY

1. Many substances, as glass, ebonite, sealing wax, etc., when rubbed with silk, flannel, fur, etc., possess the property of attracting light objects, and are therefore said to be *electrified* (§ 343).

2. Electrical charges are of two kinds, called *positive* and *negative* charges (§ 344).

3. Similar charges repel each other and dissimilar charges attract (§ 345).

4. The *electroscope* is used to determine the presence of a charge and its kind (§ 346).

5. Equal amounts of positive and negative electricity are always developed simultaneously (§ 348).

6. Charges of electricity are transferred by *conductors*, as metals, carbon, etc. A substance that will not act as a conductor is called an *insulator;* such are dry air, glass, mica, rubber, shellac, etc. (§§ 349 and 350).

7. An *electric field* is the space around a charged body within which neighboring bodies are affected by the charge (§ 351).

8. The electrical effect of an electrified body upon a neighboring conductor insulated from it is the result of *electrostatic induction* (§ 352).

9. The effect of induction by a given charge is to cause a dissimilar kind of electricity to appear on the nearer side of an insulated conductor and the similar kind on the remote side (§§ 353 to 355).

10. Charges of electricity distribute themselves over the exterior surfaces of conductors. The relative quantities per square centimeter depend on the curvature of the surface, being greatest at places of the greatest curvature, as at points (§ 356 and 357).

11. Charges are rapidly conveyed away from sharp points by air particles, which become charged by contact and are then repelled (§ 358).

12. Charges flow along conductors from places of higher to places of lower *potential.* The potential of the earth is regarded as zero (§ 360 and 361).

13. The union of positive and negative charges of the same size produces a cancellation of both (§ 362).

14. Unit charges are such equal quantities of electricity as exert upon each other a force of one dyne when the distance between them in air is one centimeter (§ 363).

15. Electrostatic *capacity* is measured by the number of units of electricity required to produce a given potential. The capacities of two insulated conductors are proportional to the number of units required to bring them to the same potential (§ 364).

16. The capacity of a *condenser* is made very large by bringing the conductor to be charged near another which is connected with the earth, but thoroughly insulated from the former. The most common form of condenser is the Leyden jar (§§ 365 to 368).

17. The process of induction is employed in the generation of large electrostatic charges. See the electrophorus (§ 369) and the Toeppler-Holtz machine (§ 371).

CHAPTER XVII

MAGNETISM

1. MAGNETS AND THEIR MUTUAL ACTION

372. Production of a Magnet. — Magnets present at least two properties that are familiar to every one: (1) the ends of a magnet will pick up small pieces of iron, as iron filings, etc., and (2) a magnet will take a north-and-south position when properly suspended. The following experiment will serve to show that an intimate relation exists between electricity and magnetism:

Break off several pieces of watch spring, and balance each of them on the head of a pin, as shown in Fig. 292. Observe that they will remain indefinitely *in any position* and will not pick up iron filings. Now wrap the pieces of spring in a small sheet of paper and place them within a *helix*, or spiral, made by winding ten or twelve turns of insulated copper wire upon a lead pencil. Discharge a Leyden jar through the helix of wire and then remove the pieces of spring. They will be found able to pick up iron filings and small sewing needles; and, when balanced on the head of a pin, each piece will become stationary only when *in a north-and-south position.*

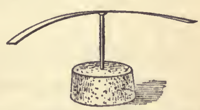


FIG. 292. — Piece of Watch Spring Balanced on Pin.

The experiment shows (1) that a moving charge of electricity which flows in a helix around a piece of steel magnetizes it, and (2) that a magnetized bar of steel retains at least a portion of the magnetism produced by the electric flow.

373. Natural Magnets or Lodestones. — In ancient times iron was mined on some of the islands of the Mediterranean and along the coasts of the Ægean sea. It was early observed that an occasional piece of the ore, magnetite (chemical symbol, Fe_3O_4), possessed the power of attracting small pieces of iron and also imparted this property by contact to pieces of iron and steel. According to some writers, the word "magnet" is derived from Magnesia in Asia Minor, a province in which magnetic iron ore is especially abundant. Specimens of ore that possess the properties of magnets are called *lodestones*, or *natural* magnets. See Fig. 293.

FIG. 293. — A Natural Magnet.

374. Poles of a Magnet. — If a magnet be placed on a sheet of paper and covered with iron filings, it will be found on lifting the magnet that the filings cling to the ends in great tufts but leave it bare in the middle, as shown in Fig. 294. The centers of attraction near the ends are called the poles of the magnet. The pole that points toward the north when the bar is suspended is the north-seeking, or N-pole, and the other the south-seeking, or S-pole.

FIG. 294. — Showing the Polarity of a Magnet.

375. Law of Magnetic Poles. — Balance upon a pinhead each of the magnetized pieces of steel used in § 372 and mark the N-poles with small labels. Now bring the N-pole of one piece near the N-pole of a suspended one and observe the effect. Present the N-pole of the former to the S-pole of the latter. In every case that can be tested it will be found that an N-pole repels another N-pole and attracts an S-pole. Likewise S-poles repel each other and attract N-poles.

The general law of pole action is made clear by this experiment, viz. *like poles repel each other and unlike poles attract.*

The force of attraction or repulsion between two poles is inversely proportional to the square of the distance between them, i.e. doubling the distance between two poles divides the force by four, tripling the distance divides the force by nine, etc. Compare with gravitation, § 67.

376. Artificial Magnets. — The fact that artificial magnets may be made of any desired form is of great practical value. The commonest forms are the straight *bar magnet* and the *horseshoe magnet* shown in Fig. 295. These can be produced in any size from the small toy magnet up to large ones capable of lifting an iron weight of several pounds.

Fig. 295. — A Horseshoe Magnet.

Draw the N-pole of a magnet along a steel nail, from the head toward the point. Present the head of the nail to the N-pole of a suspended magnet. The observed repulsion shows the head to be an N-pole. Also test the polarity of the point of the nail. Repeat the processes, using the S-pole of the first magnet, and ascertain the poles produced in the nail.

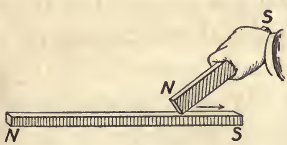


Fig. 296. — Magnetizing a Bar of Steel.

In every case it will be found that when a bar is magnetized by contact with one of the poles of a magnet, a pole of the opposite name is formed at the point last touched by the magnet, as illustrated in Fig. 296.

377. Magnetic Substances. — Practically only iron and steel are affected by a magnet, although the substances nickel and cobalt are slightly *attracted.* Bismuth, antimony, and some other substances are appreciably *repelled* when placed near a strong magnetic pole. Bodies belonging to the former class are called **paramagnetic** or simply **magnetic**, substances; and those of the latter, **diamagnetic** substances.

378. Magnetic Induction. — Let a strong magnet support an iron nail by its head. Test the polarity of the point of the nail. Let the point of the first nail support a second one by its head and test the polarity of this one also. If the nails are not too large, a chain of them may be formed as in Fig. 297. Now carefully remove the magnet from the first nail, and all will fall apart. Repeat the experiment after placing a piece of paper between the magnet and the first nail. Ascertain whether absolute contact between the magnet and the nail is necessary in order to enable the first nail to support the second. The action of the magnet on the nail is simply weakened by the intervention of the paper.

Fig. 297. — Each Nail Becomes a Magnet.

A piece of iron or steel becomes a **magnet by induction** *when brought in contact with, or close to, a magnetic pole.* The effect takes place through all substances except large masses of iron or steel. When the polarity of the nails is tested, it is found that the N-pole of the magnet, for example, produces an S-pole on the near end of the nails and an N-pole at the remote end. When the magnet is removed, it will be found that a portion of the induced magnetism is retained by the nails.

379. Retentivity of Magnetism. — It has been evident in many of the preceding experiments (§§ 372, 376, 378) that a magnetized piece of steel loses only a part of its magnetism when it is removed from the magnetizing influence. It is almost impossible to find a piece of iron that will not retain a little magnetism after being brought in contact with a magnet, although the amount retained is often very slight. *The property of retaining magnetism is called* **retentivity.** Hardened steel possesses the property of retentivity in a high degree, but in soft iron the retentivity is very small.

380. Magnetic Fields. — A magnetic field is a region in which magnetic substances experience magnetic forces. A magnetic field may be represented in the same manner as an electric field (§ 351). Each magnetic line of force shows at every point in it the direction of the force at that point. Magnetic fields are easily mapped by the help of iron filings, as in the following experiment:

Let a bar magnet be covered with a sheet of paper and fine iron filings strewn over its surface. If the paper is slightly jarred by tapping the table on which the magnet rests, the filings will arrange themselves in chains stretching from pole to pole. These chains show the direction of the magnetic forces at the points through which they pass.

The direction of the lines of force in the field around a short bar magnet is shown in Fig. 298. Each particle becomes a magnet by induction (§ 378) and turns until it lies lengthwise in a line of force. The poles of one particle attract the opposite poles of the neighboring particles, and thus the filings unite and form chains which extend from pole to pole. *The direction of a line of force is assumed to be from an N-pole to an S-pole through the air.* The strong parts of the field are the regions near the poles, as shown by the heavy, distinct lines of filings.

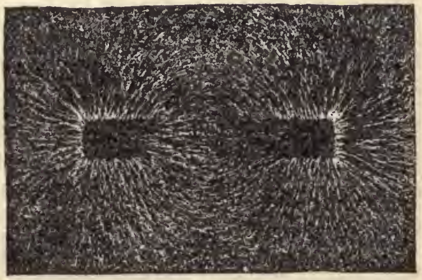


FIG. 298. — The Magnetic Field around a Bar Magnet.

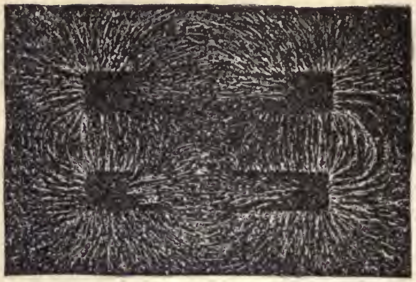


FIG. 299. — Showing the Magnetic Field between Unlike Poles.

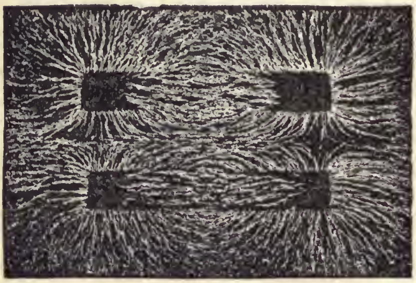


FIG. 300. — Showing the Magnetic Field between Like Poles.

Figure 299 was made by placing two bar magnets side by side with their *unlike* poles pointing in the same direction. It is at once evident that an intense field of force is produced between unlike poles when placed near each other. This fact is utilized in some of the practical applications of magnetism.

Figure 300 shows the result obtained by placing two short bar magnets parallel, but with their *like* poles turned in the same direction. A comparison of this with the figure just preceding shows at once the presence of a weak field of force between similar poles. In fact, the repellent action between the lines of force is obvious. No lines of force from one pole enter another pole of the same name. Since the poles are alike, it is plain that this field represents the case of repulsion, while Fig. 299 shows the condition for attraction.

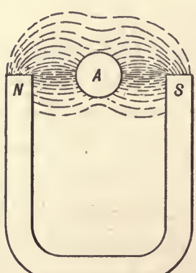


FIG. 301. — A Magnetic Field Distorted by a Piece of Iron *A*.

381. Magnetic Permeability. — Magnetic lines of force find an easier path through iron or steel than through air.

When a piece of iron, *A*, Fig. 301, is placed in a magnetic field, lines of force bend from their original course in order to pass through the iron. Thus lines of force are sent through the metal. *The relative ease with which magnetic lines of force pass through a substance is called its* **magnetic permeability.** The permeability of air is regarded as unity.

EXERCISES

1. What would be a suitable substance for permanent artificial magnets?
2. It is desirable in a certain instrument to use a substance that is easily magnetized, but which will lose its magnetism when the magnetizing influence is removed. What would be the proper substance?
3. Each of two nails hangs from the N-pole of a permanent magnet. Will the ends of the nails remote from the magnet attract or repel each other?
4. Four nails are placed lengthwise in a row without touching each other. Draw a figure showing the magnetic fields produced when the S-pole of a magnet is placed near one end of the row.
5. The N-pole of a magnet is held near a point on the circumference of an iron ring. Show by a diagram the position of the induced magnetic poles.

2. MAGNETISM A MOLECULAR PHENOMENON

382. Demagnetization by Heating. — Magnetize a piece of watch spring and note the quantity of iron filings that it will support. Heat the spring red-hot and test its magnetism again. It will be found to have lost its power of picking up filings as well as repelling either pole of a suspended magnet.

We have already learned in § 213 that heating a body simply increases its molecular motion. This experiment shows, therefore, that when the molecular motion reaches a certain degree, practically all magnetism is destroyed.

383. Demagnetization by Molecular Rearrangement. — Bend a piece of iron wire in the form shown in Fig. 302. Magnetize it by stroking it several times with a strong magnet. Test its power to pick up filings and to repel the pole of a suspended magnet. Now grasp the ends with pliers and give the wire a vigorous twist. If the wire is now tested as before, it will be found to have lost its magnetism.

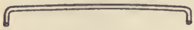


FIG. 302. — Wire may be Demagnetized by Twisting.

It is obvious that the effect of twisting is to give all parts of the wire a new molecular arrangement. Accompanying this disarrangement is the disappearance of the magnetism, as shown by the experiment.

384. Magnetism Not Simply at the Poles. — Magnetize a piece of watch spring about 5 in. long and test the location of its poles by presenting it to a suspended magnet. Break it at the center and test the pieces. It will be found that each piece is a perfect magnet, for two new poles will have developed at the point which was at first neutral. Break one of the pieces at its center and again each piece will be a perfect magnet.

Let the original magnet be represented by (1), Fig. 303. The polarity manifests itself only at the ends. When the magnet is broken, as at P, the condition shown in (2) is the result. Again, on breaking the two parts, the result shown in (3) is obtained. It may be imagined that this process be carried on even to the separation of ultimate particles, *i.e.* the molecules. There can be no doubt that each molecule would prove to be a magnet having two poles.

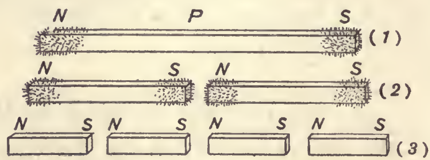


FIG. 303. — Effect of Breaking a Magnet.

385. Theory of Magnetization. — The results obtained in the experiments of the preceding sections lead to the theory of magnetization which assumes that *every molecule of iron or steel is a magnet even when the bar of which it is a part is not magnetized.* Magnetization consists in causing the molecules to arrange themselves in a certain order. In an unmagnetized bar of iron the condition is represented as in Fig. 304. Here each of the little rectangles represents a pivoted magnet, the N-pole being shaded. It will be seen that the small magnets arrange themselves in small groups so that unlike poles neutralize each other throughout the bar. The bar, therefore, manifests no polarity.

Fig. 304. — Illustrating the Condition in an Unmagnetized Bar of Iron or Steel.

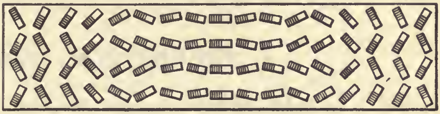


Fig. 305. — Illustrating the Condition in a Magnetized Bar.

Fig. 306. — Illustrating the Condition in a Saturated Magnet.

When a magnetizing influence is brought to bear upon the bar of iron, the molecules swing round until a condition approximating that shown in Fig. 305 results. Since the general direction of the N-poles is toward the left and the S-poles toward the right, the ends of the bar will show polarity. At a short distance from the ends and throughout the middle of the bar the proximity of unlike poles brings about a neutralization of polarity in these places. A jar serves to break up this artificial arrangement, which thereby destroys the magnetism, and the condition shown in Fig. 304 is resumed.

386. A Saturated Magnet. — If the theory of magnetization just described is correct, it will be found impossible to magnetize a piece of iron beyond the limit reached when all the molecules have been turned as represented in Fig. 306. This has been found experimentally to be the case. In this condition a magnet is said to be **saturated**. Hence, *a saturated magnet is one upon which an increase of the magnetizing influence has no effect.*

EXERCISES

1. How would you determine the poles of a magnet? Give two methods.

2. Why is a permanent magnet injured when dropped?

3. If iron be heated and then cooled in a magnetic field, it will be found to be magnetized. Explain.

4. Explain how jarring a bar of steel will aid in magnetizing it, but jarring a magnetized piece of steel will weaken its magnetism.

3. TERRESTRIAL MAGNETISM

387. The Compass. — One of the earliest properties discovered regarding a magnet is its tendency to take a definite position when placed on a pivot or suspended. *A magnet that is so pivoted as to turn freely in a horizontal plane is called a* **compass**. The invention of the compass is attributed to the Chinese.

The first satisfactory explanation of the action of the compass was given by Gilbert[1] about 1600 A.D. The

[1] WILLIAM GILBERT (1540–1603). The renown of Gilbert rests largely upon the fact that he was one of the first to recognize the value of experimentation and also upon his great work entitled *De Magnete*, which was published in London in 1600. His most celebrated experiments were made with magnets and magnetic bodies, and his results and conclusions are contained in the book just named. Gilbert made the discovery that the earth is a great magnet and demonstrated this by constructing a small sphere of lodestone. With this "terrella," or "little earth," he was not only able to show why magnets point to the north, but he also explained the declination and inclination of magnetic needles.

Gilbert also made important discoveries in the subject of Electricity. He was probably the first to clearly recognize a distinct difference between magnetized and electrified bodies. He showed that many substances besides amber could be electrified by rubbing; *e.g.* glass, resins, sealing wax, assumption is made that the earth is a great magnet, but the reason for its being one still remains a mystery. However, we are sure that the earth is surrounded by a magnetic field, and, in proof of this fact, the following experiment can easily be made.

Hold a bar of iron or a slender gas pipe about 2 feet long in a north-and-south plane, tilting the north end down 60° or more. Now give the bar several vigorous taps with a stone and then test the ends for polarity by presenting them to the poles of a suspended magnet. The end of the bar toward the north will be an N-pole, and the other an S-pole. Reverse the bar and repeat the processes. The polarity of the bar will be found reversed.

This experiment makes use of the earth's magnetic field in the magnetization of the iron bar. The jarring is effective only in lending assistance to the rearrangement of the molecules under the inductive influence of the earth's field (§§ 380, 385). The compass needle simply behaves as any small magnet would in a magnetic field, pointing in a general north-and-south direction because of the influence of the earth's lines of force.

388. Declination of the Needle. — Suspend a magnetized knitting needle on a fiber taken from the cocoon of a silkworm, or on an untwisted filament of silk floss. Stretch a cord below the needle precisely in a north-and-south line (given by the shadow of a plumb line or a vertical window frame at noon, sun time). The experiment should not be made within several feet of an iron pipe or beam, nor within a foot of steel nails. It will be observed that the needle does not take a position parallel to the north-and-south line except in certain localities.

etc. These he named "electrics," from the Greek word *electron*, meaning amber. He was also the first to use the word "electricity."

Gilbert was educated in medicine at Cambridge, England, and was appointed court physician by Queen Elizabeth. At the death of the queen in 1603, he was reappointed by her successor, James I, but his death occurred in November of that year.

The earth's magnetic lines do not coincide with the geographical meridians; consequently the magnetic needle, directed by the earth's field, will not point geographically north. This fact was known as early as the eleventh century; but it was first discovered by Columbus, on his memorable voyage of 1492, that the direction indicated by the compass changes as one passes from place to place over the earth's surface. *The angle between the direction of the needle and the geographical meridian is the* **declination** *of the needle.*

Lines that are so drawn upon a map as to pass through places at which the declination is the same are called

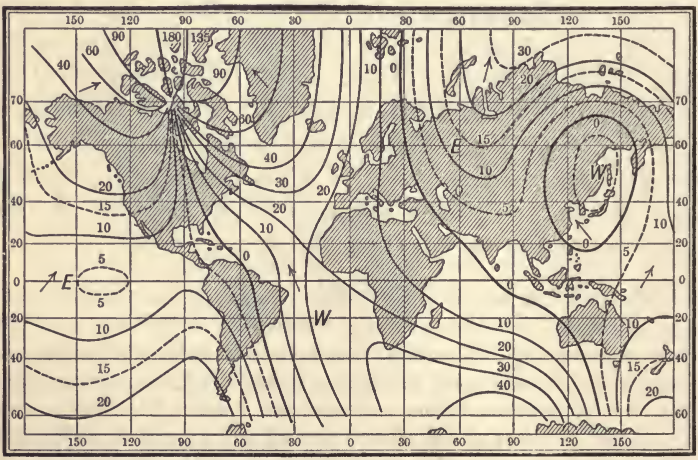


FIG. 307.—Map Showing Magnetic Declinations. Lines of no Declination are Marked "0." The Numbers Show the Declination in Degrees and the Letters Show Whether it is East or West.

isogonic lines. Figure 307 shows approximately the magnetic declination at all places on the surface of the earth. The heavy line, called the **agonic** line, shows the regions where the needle points due north. At all points in the United States and Canada lying east of the agonic line, the declination is towards the west; for all points west of this line, the declination is east. At the present time (1910) the agonic line passes a little west of Lansing, Mich., and slightly east of Cincinnati, Ohio, and Charleston, S.C. It is moving very slowly westward.

389. Inclination of the Magnetic Needle. — Thrust an unmagnetized knitting needle through a cork, and close to and at right angles with it pass a straight, slender sewing needle. Cut away a portion of the cork until the system will balance in any position on the edges of two tumblers when the longer needle is placed east-and-west. Magnetize the knitting needle by stroking one end with the N-pole of a magnet and the opposite end with the S-pole. Now place the system in a north-and-south position on the tumblers. The N-pole of the needle will appear heavier than the S-pole and will dip until the angle between the needle and the horizontal plane is about 70°. (See Fig. 308.)

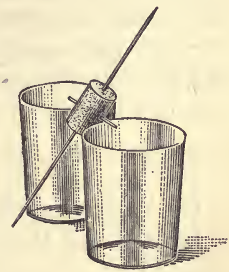


FIG. 308. — The N-pole of a Balanced Needle Tends to Dip Down.

The balanced, or dipping, needle simply places itself parallel with the lines of force of the earth. In the Northern States these lines are inclined about 70° from the horizontal. *The angle between the earth's magnetic lines of force and a horizontal plane is called the* **inclination,** *or* **dip,** *of the needle.* The angle of inclination increases as one approaches the magnetic poles of the earth, where it is 90°. Near the geographical equator the inclination is about 0°, and in the southern hemisphere the needle inclines with its S-pole below the horizontal plane passing through its axis.

EXERCISES

1. Account for the fact that the iron beams of a building, gas and water pipes, and other bars of iron are usually magnetized.

2. How would a compass behave while being carried entirely around the earth's magnetic pole along the Arctic Circle?

3. How would a dipping needle be of assistance in locating the magnetic poles of the earth?

4. Account for the fact that a surveyor's compass needle is often provided with a small adjustable weight for balancing.

5. Place a ruler upon the table, giving it approximately the declination of the needle at New York; at Los Angeles; at Iceland.

6. Ascertain by experiment whether a floating magnet tends to drift toward the north. Account for the result.

SUGGESTION. — Consider whether the attraction of one of the earth's poles for a pole of a magnet is greater than its repulsion for the opposite pole of the same magnet.

7. The upper end of a pipe driven into the ground was found to be an S-pole. Explain. Suggest an experiment by which your explanation can be tested.

SUMMARY

1. A bar of steel may be made a *magnet* by discharging a Leyden jar through a helix of wire wound around it (§ 372).

2. Magnets also appear in nature in an ore of iron called magnetite. Specimens of this ore that are magnets are called *natural magnets* (§ 373).

3. On account of the tendency of a magnet to place itself in a north-and-south position, one end is called the north-seeking, or N-pole; the other, the south-seeking, or S-pole (§ 374).

4. Poles of the same name repel each other, and those of unlike name attract (§ 375).

5. Artificial magnets may be made by stroking bars of steel with a magnetized body (§ 376).

6. Substances that are attracted by a magnet are called *paramagnetic* (or simply *magnetic*) substances; those that are repelled, *diamagnetic*. Iron is the most magnetic of all substances (§ 377).

7. A piece of iron or steel becomes a magnet by induction when brought near the pole of a magnet. An N-pole always induces an S-pole on the nearer end of a neighboring bar of iron or steel, and an S-pole induces an N-pole (§ 378).

8. The region around a magnet in which magnetic substances experience magnetic forces is called a *magnetic field* (§ 380).

9. Magnetic lines of force pass more readily through iron than air. Different grades of iron and steel differ also in the ease with which lines of force pass through them. They are therefore said to differ in *permeability* (§ 381).

10. The magnetism which would ordinarily be retained by iron or steel may be reduced or entirely destroyed by heating, jarring, twisting, etc. Hence magnetism is considered to be a molecular phenomenon. The molecules of iron and steel are supposed to be small magnets having two poles. Magnetization consists in giving the molecules a new arrangement in which the N-poles point in general in one direction and the S-poles in the other (§§ 382 to 386).

11. A magnet so pivoted as to turn freely in a horizontal plane is called a *compass*. It takes a general north-and-south position on account of the earth's magnetism (§ 387).

12. The angle that measures the deviation of the compass from the geographical meridian is called the *declination* of the needle. This angle varies greatly with different localities. The *agonic line* is the line passing through points where the declination is zero. It is now moving slowly westward (§ 388).

13. The angle between the earth's magnetic lines and a horizontal plane is called the *inclination*, or *dip*, of the needle. The dip varies from zero at the magnetic equator to 90° at the earth's magnetic poles (§ 389).

CHAPTER XVIII

VOLTAIC ELECTRICITY

1. PRODUCTION OF A CURRENT — VOLTAIC CELLS

390. Maintaining a Continuous Discharge of Electricity. — It was shown in § 372 that the moving charge of electricity obtained when a Leyden jar is discharged through a helix of wire is able to magnetize pieces of steel. But electricity would be of very little service if we were obliged to depend upon the *momentary* flow occasioned by such a discharge. The great practical value of electricity as a working agent lies in the fact that it is possible by different means, now to be studied, to maintain a *continuous* flow of electricity, or in other words, a *steady current.* The following experiments afford an opportunity to compare these two kinds of discharge, — viz. the *momentary* and the *continuous.*

1. Place one end of a coil of insulated wire within which are several pieces of soft annealed iron wire near one of the poles of a light pivoted magnetic needle as shown in Fig. 309. Discharge a highly charged Leyden jar through the wire of the coil and observe the effect upon the needle. The pole will be either attracted or repelled. Except for the effect of the magnetism remaining in the iron, *the action is only temporary.* Recharge the jar and discharge it through the coil

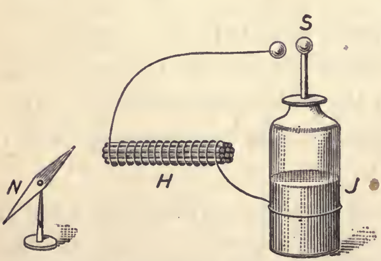


FIG. 309. — Magnetizing Iron by a Momentary Discharge of Electricity.

in the opposite direction. If the first discharge produced repulsion, this one will set up an attraction.

2. Attach one end of the coil shown in Fig. 310 to a strip of copper about 10 cm. long and the other to a strip of zinc. Now dip both plates in a very dilute solution (about 1 : 40, by volume) of sulphuric acid, keeping the plates from touching each other. One of the magnetic poles of the pivoted needle will swing round toward the coil and will not resume its original position so long as the plates remain in the liquid. Reverse the connections at the ends of the coil, and the opposite end of the needle will be attracted.

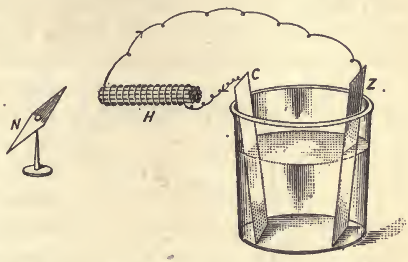


FIG. 310.—Magnetizing Iron by a Continuous Current.

These experiments show a marked similarity between the effect produced by the discharge of a Leyden jar and that produced by the plates of zinc and copper suspended in a solution of sulphuric acid. The iron is magnetized in both instances, and the magnetism is even reversed by reversing the connections. But since the effect in Experiment 2 lasts as long as both plates remain in the liquid, it is clear that a continuous discharge can be maintained by the means employed. *A continuous discharge, or movement, of electricity is called an electric current.*

391. The Voltaic Cell. — The combination of zinc, copper, and dilute sulphuric acid used in Experiment 2 of the preceding section constitutes a *voltaic cell.* This method of producing a current was first used by Volta[1] in the year 1800; hence the name. There are many kinds of voltaic cells, differing from one another in respect to the materials used in their construction. Some of the most important ones are treated later on.

[1] See portrait facing page 352.

392. Electrical Charges produced by Voltaic Cells. — Provide two perfectly flat metal disks about 3 in. in diameter. Attach one disk to the top of a sensitive gold-leaf electroscope and give it a thin coating of shellac. To the second disk attach an insulated handle and place it upon the first. The two plates thus form a condenser of large capacity. Why? See § 365. Form a series[1] of voltaic cells (discarded dry cells will usually do) and touch the wire leading from the zinc plate to the upper disk A, Fig. 311, and the wire leading from the carbon (or copper) to disk B. Remove the wires and lift the upper disk. The leaves of the electroscope will diverge. On bringing an electrified stick of sealing wax near the instrument, the divergence will be decreased, thus showing that the carbon (or copper) plate conveyed to the lower disk a positive charge. By repeating the experiment with the wires reversed, it will be found that a negative charge can be taken from the zinc of the cell.

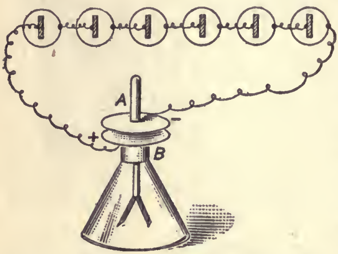


Fig. 311. — Charging an Electroscope from Voltaic Cells.

It becomes clear from this experiment that *the terminals of a voltaic cell and the wires leading from them are electrically charged, — a positive charge being borne by the carbon (or copper) and a negative charge by the zinc.* Hence the copper of a cell is called the *positive pole*, and the zinc, the *negative pole.* The voltaic cell, therefore, affords a case of two oppositely electrified bodies, which on being joined by a conductor set up an electrical discharge. Furthermore, this discharge, or current, is continuous; for, as fast as the charges become neutralized, they are renewed by the chemical changes taking place within the cell.

393. Electromotive Force. — As shown in the preceding section, a voltaic cell develops a charge of positive electrification on the copper plate and a charge of negative on

[1] If the disks are perfectly flat and the insulating material between them is thin enough, the experiment can be made by using a single cell.

the zinc. *The difference of potential that the cell maintains between these two charges when the plates are not connected by a conductor is the* **electromotive force** (abbreviated E. M. F.) *of the cell.* E. M. F. is sometimes called *electric pressure* and is the cause of an electric flow in a circuit. It is not a *force* in the sense in which that term is used in mechanics, since its tendency is to move electricity and not matter. The unit of E. M. F. is the *volt*, which is approximately the E. M. F. afforded by a cell containing zinc, copper, and dilute sulphuric acid.

394. An Electrical Circuit. — *The entire conducting path along which a current of electricity flows is called an* **electrical circuit.** The circuit comprises not only the wire with which the plates are connected, but also the plates and the liquid of the cell. When an instrument, for example, is to be *introduced into the circuit*, it is so connected with the plates of the cell as to be traversed by the current and is thus made a part of the circuit. Separating the circuit at any point is called *opening*, or *breaking*, the circuit; and joining the separated ends is called *closing*, or *making*, the circuit. When a circuit is broken, no current can flow, and the circuit is therefore said to be *interrupted.*

395. Action of a Voltaic Cell. — **1.** Place a strip of commercial zinc in very dilute sulphuric acid (about 1 : 40, by volume) and observe the effect. Very small bubbles will be seen to rise from the zinc and pass off at the surface of the liquid. These are bubbles of hydrogen gas which is liberated from the acid by the chemical action. If a small piece of zinc be left in the acid, it will soon dissolve, leaving behind only a few small flakes of insoluble impurities. Repeat the experiment with the strip of copper. No action will be observed.

2. Touch the zinc plate just used to a small quantity of mercury. Some of the mercury will be found to cling to the plate. With a cloth or sponge spread the mercury over the wet surface of the zinc. Now on placing the zinc in the acid, no bubbles will be seen. Place the copper plate in the acid with the zinc, but not in contact with it, and connect the two metals by means of a wire. Bubbles will now be observed rising from the copper plate. It is under these conditions, as we have seen in § 390, that a magnetic effect is derived from the wire which connects the two plates. If the circuit be now broken at any point, the bubbles cease to form. The mercury has thus prevented the formation of hydrogen bubbles at the zinc plate, which, nevertheless, continues to dissolve and decrease in size as long as the electrical connection is maintained between it and the copper.

It is clear from Experiment 1 that the acid used acts very unequally upon the two plates of a voltaic cell. It is this difference in the chemical action that gives rise to the difference of potential between the positive and negative charges found in § 392. *The greater the disparity in the chemical actions at the two plates, the greater the difference of potential maintained by the cell.* Furthermore, the experiment shows that the negatively charged plate (*i.e.* the zinc) is dissolved by the acid, while the positive copper plate from which hydrogen rises during the operation of the cell remains apparently unchanged.

396. Theory of a Voltaic Cell. — The theory of the simple voltaic cell described in § 390 rests upon the hypothesis of Clausius, (1822–1888), a German physicist. This hypothesis, which is based upon a large amount of experimental evidence, states that many of the molecules in a *dilute* solution of a substance "split up" into two parts called *ions*. Hence, when sulphuric acid (H_2SO_4)[1] is diluted, two kinds of ions are formed, those of hydrogen (H) and those of SO_4, called the *sulphions*. See Fig. 312. Furthermore, *the hydrogen ions bear positive charges of electricity, while the sulphions carry negative charges.* As shown in § 395, zinc has a strong tendency to dissolve in dilute sulphuric acid, while copper has not. In the process

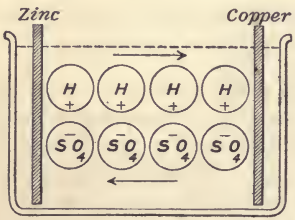


Fig. 312. — Diagram Showing Ions in a Voltaic Cell.

[1] This chemical formula expresses the fact that each molecule of sulphuric acid is composed of two atoms of hydrogen, one of sulphur, and four of oxygen.

the sulphions in the liquid attack the zinc plate, from which they abstract some of the metal to form zinc sulphate ($ZnSO_4$), a white substance that dissolves at once in the liquid. In this process the negative charge carried by each sulphion concerned in the action is given up to the zinc plate, which thus becomes charged with negative electricity. Again, for each negatively charged ion that engages with zinc, a positive hydrogen ion from the liquid gives up its charge to the copper, thus charging it with positive electricity. After the hydrogen ions have discharged their electricity to the copper plate, they become free hydrogen, which collects in small bubbles at this plate and rises to the surface.

397. Local Action and its Prevention. — The results of Experiment 2 (§ 395) show that a coating of mercury upon the zinc plate prevents the formation of hydrogen at its surface when it comes in contact with the acid. The reason is because pure zinc will not dissolve in pure sulphuric acid; and the mercury dissolves from the plate only the pure zinc which is then coated over the surface, thus overlaying the impurities with an amalgam of zinc. After a time these impurities, which are mainly carbon and iron, become exposed to the acid, and local currents are set up between them and the neighboring portions of the plate, as indicated by the arrows in Fig. 313. *The generation of electric currents between the zinc of a cell and its impurities is called* **local action.** Local action is a wasteful process, but can obviously be prevented by employing pure zinc or by coating an impure zinc plate with mercury. This treatment is known as the *amalgamation* of the zinc plate.

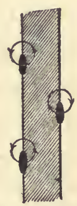


Fig. 313. — Wasting of the Zinc by Local Action.

398. A Mechanical Analogy. — The mechanical device shown in Fig. 314 is of assistance in making clear the action of a voltaic cell. Imagine a rotary pump *P* to be placed in a U-tube and arranged to be turned by the weight W. When the wheel of the pump is turned, water is forced to a greater height in one arm than in the other. The wheel, however, will come to rest when the back pressure due to the difference of level on the two sides of it just equals the pressure exerted by the wheel. If there is no friction, the device will maintain the difference in level h as long as no water escapes.

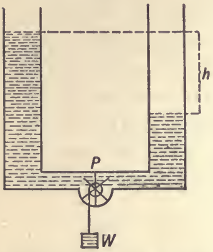


FIG. 314. — A Mechanical Analogy of Cell Action.

In this system the difference of level h maintained by the pump is analogous to the difference of potential (E. M. F.) maintained between the plates of a voltaic cell. Just as the pump ceases to turn when a certain difference of level is reached, so the chemical action of a perfect cell stops when the difference of potential between the positive and negative charges on the plates has attained a certain value, which will depend on the nature of the materials used in its construction.

399. A Cell in Action. — The case of a voltaic cell is modified as soon as the circuit is completed and a current allowed to flow; so also is that of the pump and water. Imagine a pipe T, Fig. 315, to connect the two arms of the U-tube. On account of the difference of water level, the liquid will flow through T, and the wheel will continue to turn, since now the back pressure will have been reduced. The difference of level will decrease to h', whose value will depend on the friction offered to the current flow in T.

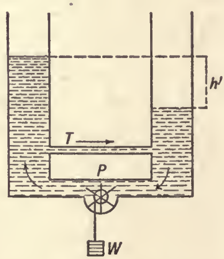


FIG. 315. — When Water is Allowed to Flow through T, the Wheel at P will Continue to Turn.

Similarly, when the plates of a voltaic cell are connected by a conductor, the charge on the positive plate (copper) moves toward the negative plate (zinc), and the potential difference is diminished. The chemical action now goes on vigorously in its attempt to restore the charge that has passed through the conductor. Hence the difference of potential between the two poles of a cell when a current is flowing will be less than that when the circuit is open. This value is no longer called the E. M. F. of the cell, but is termed the **fall of potential**, or **difference of potential** between the plates of the cell.

400. Deflection of a Magnet by a Current. — Hold the wire joining the plates of a simple voltaic cell over and parallel to a pivoted magnetic needle, Fig. 316, and then close the circuit by placing the plates in the liquid, or by means of a key inserted anywhere in the circuit. The needle will be turned on its pivot and finally come to rest at an angle with the conductor carrying the current. If the current be passed in the opposite direction above the needle, the deflection is opposite to the first. The wire may now be placed below the needle, and the direction of the deflections obtained. If the conductor be placed at the side, or near the end, of a suspended magnet, deflections are also obtained.

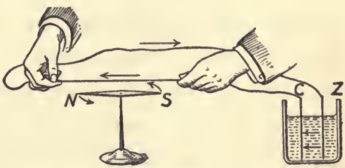


FIG. 316. — A Magnetic Needle is Deflected by an Electric Current.

This experiment confirms the result previously found in § 390, viz. that the region around a wire through which an electric charge is moving has magnetic properties. This experiment was first performed by Oersted,[1] a Danish physicist, in 1819. Oersted's discovery is of great historical interest, since it was the first evidence obtained in regard to the magnetic effect of a current of electricity and has led to results of the greatest practical importance.

[1] See portrait facing page 382.

By observing the direction in which the N-pole of the magnetic needle is moved in relation to the direction of the electric current, the following rule will be found to apply: *Let the fingers of the outstretched right hand point in the direction of the current flow in the wire and the palm be turned toward the needle; the extended thumb will then show the direction of the deflection of the N-pole of the needle.* The rule is of convenience in determining the direction of a current when its effect upon a magnetic needle is known. The manner of applying this rule is made clear by Fig. 317.

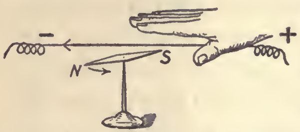


FIG. 317. — The Thumb Shows the Direction in Which the N-Pole Moves.

401. The Galvanometer. — The effect discovered by Oersted is of great service in the **galvanometer**, an instrument used for detecting and measuring electric currents. The following experiment will show how the effect of a current on a magnetic needle can be so increased as to make it possible to discover even very feeble currents.

Place a compass needle, below which is a graduated circle, within a single turn of wire as in (1), Fig. 318, and read the deflection on the scale. Now wind the same wire three or four times around the compass, always keeping the wire parallel to the original position of the needle, and again read the deflection. The second reading will be much greater than the first. A current that will scarcely move the needle when a single turn of wire is used will be found to produce a marked effect when tested with the coil of several turns.

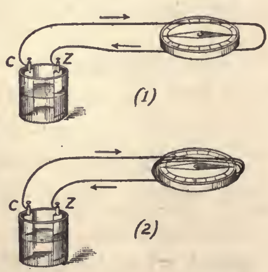


FIG. 318. — Oersted's Effect Applied in the Galvanometer.

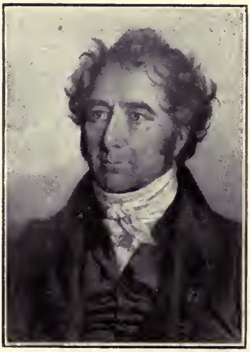
The magnetic forces due to the electric current in all parts of the coil tend to turn the needle in one

HANS CHRISTIAN OERSTED (1777–1851)

Oersted's famous experiment of 1819 on the deflection of a magnet by an electric current was the beginning of the science of electro-magnetism. This experiment demonstrated the long-sought connection between electricity and magnetism and served to point out the line of experimentation that has brought the science up to its present-day development.

Oersted was born in Langeland, a portion of Denmark, studied at Copenhagen, and afterwards became a professor at the university and polytechnic schools of that city.

DOMINIQUE FRANÇOIS JEAN ARAGO (1786–1853)



The year following Oersted's discovery, Arago, a noted Parisian astronomer and physicist, observed that iron filings cling to a conductor carrying an electric current. A little later it was shown by Sir Humphry Davy of England that the filings arrange themselves in magnetized chains around the conductor.

Arago was one of the first advocates of the wave theory of light. The beautiful tints produced when polarized light passes through certain crystals were discovered by him in 1811. Arago planned a method for measuring directly the velocity of light in air and water, but failing eyesight prevented carrying out his experiments. He lived, however, to see the work done by Fizeau and Foucault.

particular direction, a fact that becomes clear when the rule given in § 400 is applied. Hence by introducing a sufficient number of turns of wire and making the needle extremely light, a very small current will suffice to produce a deflection.

402. Polarization of a Voltaic Cell. — 1. Connect a simple zinc and copper cell with a voltmeter[1] (§ 430) or a high-resistance galvanometer and read the deflection produced. Short-circuit the cell for a short time by means of a wire, thus allowing hydrogen to form in large quantities at the copper plate, remove the wire, and again read the deflection. It will be less than at first. If the liquid is now stirred, the deflection will be increased. Keep the bubbles brushed from the copper plate and see if the current can be kept the same as at first.

2. Substitute a carbon plate for the copper and repeat Experiment 1. (A good carbon plate may be taken from a discarded dry cell.) A similar decrease in the current will be observed. While the cell is connected with the instrument, pour into the sulphuric acid a small quantity of sodium (or potassium) dichromate or chromic acid solution. The index of the galvanometer promptly indicates a strong increase of electromotive force. The cell may now be short-circuited as before, but the E. M. F. quickly resumes its original value after the short circuit is removed.

These experiments show clearly that the collection of hydrogen on the copper (or carbon) plate of a cell reduces the E. M. F. and, consequently, the current that it sends through the conductor. This effect arises from the fact that a hydrogen-coated plate now takes the place of the copper plate. *The diminution of the E. M. F. of a cell by the presence of hydrogen on the copper (or carbon) plate*

[1] For classroom demonstration purposes it is desirable that a voltmeter be provided having upwards of 50 ohms resistance and reading to about 5 volts. An ammeter having less than an ohm resistance and reading to 3 or 4 amperes will also be found of great service. In case such instruments are not available, a galvanometer of large resistance (50 ohms or more) may be substituted for a voltmeter, and one of small resistance (less than 1 ohm), for an ammeter.

is called the **polarization** *of the cell.* If the bubbles be removed by stirring, the current will remain near its original value. Experiment 2 serves to demonstrate the fact that the effect of the hydrogen can be largely overcome by chemical means, a method of which advantage is taken in many kinds of voltaic cells. In the prevention of polarization by chemical action the dichromate acts as a depolarizer for removing the hydrogen from the carbon plate. This it does by supplying an abundance of oxygen, with which the hydrogen unites chemically to form water.

403. The Dichromate, or Grenet, Cell. — In this cell a zinc plate *Z*, Fig. 319, is usually placed between two plates of carbon which are joined together by metal at the top. The liquid is dilute sulphuric acid (H_2SO_4) to which is added dichromate of sodium (or potassium) or chromic acid as a depolarizer. See § 402, Exp. 2.

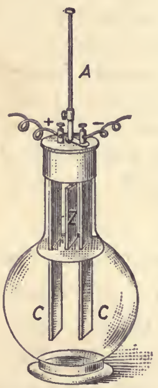


Fig. 319. — The Grenet, or Dichromate Cell.

The dichromate cell is capable of giving a strong current for a short time and for this reason has been largely used in experimental work. It has, however, been largely replaced by the "dry" cell (§ 407) and storage battery on account of their greater convenience. One disadvantage of the dichromate cell is the necessity of withdrawing the zinc from the acid by the rod *A* when the cell is not in use.

404. The Daniell Cell. — The Daniell cell, Fig. 320, consists of a glass jar containing a saturated solution of copper sulphate (blue vitriol, $CuSO_4$) in which stands a large sheet copper plate *C*. The copper plate encircles a porous cup of unglazed earthenware which contains a heavy bar of zinc *Z* immersed in a dilute solution of zinc sulphate ($ZnSO_4$). The porous cup does not check the flow of electricity, but does prevent the rapid mixing of the two solutions. Dilute sulphuric acid may be used in place of zinc sulphate.

In this cell the zinc is continually being dissolved and in the course of time must be renewed. On the other hand, copper (Cu) from the solution of copper sulphate ($CuSO_4$) is deposited slowly upon the copper plate, which in time grows into a massive sheet. In order to maintain a constant supply of copper ions in the solution, crystals of copper sulphate are added from time to time. Since copper, instead of hydrogen, is deposited on the copper plate, the nature of the plate is not changed, and, consequently, no polarization takes place. *On account of the complete non-polarization of the Daniell cell, it is often used when currents of great constancy are required.*

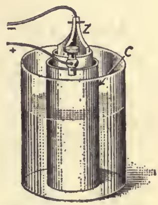


FIG. 320. — The Daniell Cell.

405. The Gravity Cell. — A dilute solution of zinc sulphate has less density than a saturated solution of copper sulphate; hence, the two will be kept separate by gravity and the former will float upon the latter. This fact is employed in the so-called *gravity* cell, Fig. 321, in which a copper plate lies upon the bottom of a jar surrounded by crystals of copper sulphate and a saturated solution of the same substance. Above this solution is one of dilute zinc sulphate which surrounds a massive zinc plate.

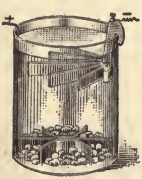


FIG. 321. — The Gravity Cell.

If a gravity cell be allowed to stand on an open circuit, the liquids slowly mix. In order to prevent this and thus keep the copper sulphate from reaching the zinc plate, the cell must be kept in more or less active operation. While producing a current the copper ions are continually moving away from the zinc, in which respect the action is the same as that of the Daniell cell, of which this cell is a modification. Gravity cells are extensively used in telegraphy and in circuits where constant currents are desired.

406. The Leclanché Cell. — The positively charged plate of the Leclanché (pronounced *Le clàn'shā'*) cell, Fig. 322, is a bar of carbon C which is packed in a porous cup together with small pieces of carbon and manganese dioxide. The porous cup is placed in a solution of ammonium chloride (salammoniac) in which stands a bar of zinc Z to serve as the negative plate of the cell.

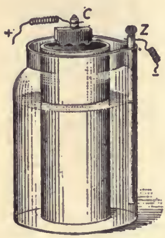


Fig. 322.—The Leclanché Cell.

When the circuit containing a Leclanché cell is closed, hydrogen is liberated at the carbon; but, on account of the presence of the manganese dioxide, the hydrogen is slowly oxidized, forming water. In this manner polarization is largely prevented. As a rule, however, the hydrogen is liberated so rapidly that the cell slowly polarizes, but regains its normal condition when allowed to stand for a time on an open circuit.

The Leclanché cell has had a very extensive use on account of the fact that it produces currents that are suitable for ringing bells, operating signals, regulating dampers, etc. The cell will remain in good condition for years with very little attention. At the present time it is being rapidly replaced by the more convenient and inexpensive "dry" cell, which is a modified form of the Leclanché.

407. The "Dry" Cell. — The Leclanché cell is made in the form of the so-called "dry" cell by embedding the carbon plate *C*, Fig. 323, in a paste *A* made by mixing zinc oxide, ammonium chloride, plaster of Paris, zinc chloride, and water. The whole mass is contained in a zinc cup *Z*, which serves as the negative plate of the cell. Evaporation is prevented by hermetically sealing the cup with melted bitumen or asphalt. Many different forms of dry cells are now on the market and are in great demand for operating the sparking devices of gas and gasoline engines, ringing bells, etc. A "dry" cell deteriorates rapidly on a closed circuit, and hence should always be connected with a spring key that automatically opens the circuit when the cell is not in use.

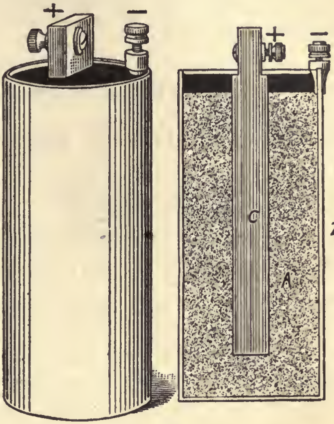


Fig. 323. — The "Dry" Cell.

EXERCISES

1. Explain how the direction of the current in a telegraph wire could be determined by means of a small compass.

2. Is the difference of potential between the plates of a voltaic cell large enough to cause a spark when wires from them are brought near together? Is it great enough to produce a shock when the plates are simultaneously touched?

3. Would you expect to get an E. M. F. by forming a cell of two copper plates or two zinc plates in dilute sulphuric acid? Make the experiment, using a sensitive galvanometer. Try the experiment with a polarized copper plate and one that is not polarized and account for the results.

4. Would you expect to derive a current from a zinc and copper cell containing a solution of common salt? Perform the experiment.

5. Why is the combination of zinc, copper, and dilute sulphuric acid a suitable one for experiments like those described?

6. Study the description of the Daniell cell and state which solution grows weaker and must ultimately be renewed. What materials, therefore, must be kept on hand for replenishing a system of Daniell or of gravity cells?

2. EFFECTS OF ELECTRIC CURRENTS

408. The Magnetic Effect. — It has already been shown that a current of electricity is capable of magnetizing bars of iron and steel (§ 390), and also that a magnetic needle placed near a current is deflected from its normal position. These are only special cases of the more general one illustrated by the following experiments:

1. Join several new dry cells in series and connect them through a key to a vertical wire passing through a horizontal sheet of card-

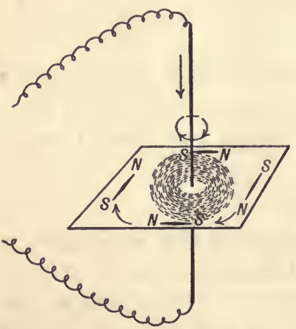


FIG. 324. — Magnetic Field around a Conductor Carrying a Current.

FIG. 325. — Iron Filings Arranged around a Conductor.

board as in Fig. 324. A current of at least 2 amperes is desirable. Sprinkle iron filings on the cardboard and close the key for a short time, meanwhile tapping lightly to jar the filings. The filings will arrange themselves in circular lines (see Fig. 325) around the wire as the center, thus showing the shape of the magnetic field about the conductor. Small pivoted magnets placed near the wire will turn until they are tangent to the circles with their N-poles as shown in Fig. 324.

2. Make a helix, or spiral, of insulated copper wire by winding about 30 turns upon a lead pencil. Connect the ends of the wire with a cell and present one end of the helix to the N-pole of a suspended magnet or compass needle. One end will be found to attract the N-pole while the opposite end repels it, and the end that attracts the N-pole will repel the S-pole. Reverse the connections of the cell and repeat the experiment. The end of the helix which formerly attracted a magnetic pole will now repel it.

3. Thread a spiral of copper wire through holes in a flat piece of cardboard or wood as shown in Fig. 326. Strew iron filings evenly over the surface and send the current (about 3 amperes are required) from several new dry cells through the wire. If the apparatus be tapped lightly, the filings will arrange themselves as shown in the figure.

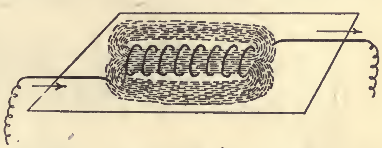


FIG. 326.—Magnetic Field Produced by a Current in a Helix, or Solenoid.

It is clear from these experiments that a conductor carrying an electric current is surrounded by a magnetic field. If the conductor is a straight wire, Experiment 1 shows that the lines of force are concentric circles around the conductor as the center. The direction of the lines is given by the direction in which an N-pole is urged when placed in the field. It is clear, therefore, from Fig. 324, that the direction of the lines is that indicated by the arrows. A convenient rule may be stated as follows:

Grasp the conductor with the right hand with the outstretched thumb in the direction the current is flowing. The fingers encircle the wire in the direction of the lines of force.

The shape of the field depends on the form of the conductor; for, when the conductor is a helix, the magnetic field resembles that about a straight bar magnet. (See Fig. 327.) In fact, it is shown in Experiment 2 that the helix has the properties of a magnet while a current is flowing through the wire. If the coil could be properly suspended and a current sent through it, the axis would assume the direction taken by a compass needle. Such a helix is also called a *solenoid.*

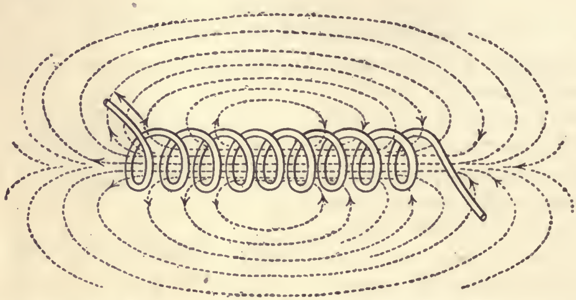


FIG. 327. — Showing the Relation between the Direction of the Current and the Magnetic Lines.

409. Poles of a Helix. — Let Experiment 2 of § 408 be repeated and the N-pole and the S-pole ascertained. Tracing the current from the positive pole of the cell, it will be found to flow around the N-pole in a direction contrary to the motion of the hands of a clock as one faces the pole and in the reverse direction about the S-pole as shown in Fig. 327.

The experimental result leads to the following convenient rule: *If the helix be grasped with the right hand so that the fingers point in the direction the current is flowing, the extended thumb will point in the direction of the N-pole of the helix.* (See Fig. 328.)

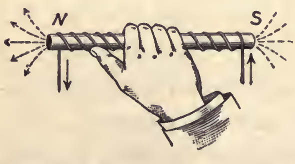


FIG. 328. — Rule for Determining the Poles of a Helix Carrying a Current.

Another convenient rule is applied by facing the end of the helix; if the current is flowing clockwise, the end of the helix is an S-pole, and, conversely, if counter-clockwise, an N-pole.

410. The Electro-magnet. — Probably none of the effects that can be produced by electric currents are of greater practical value or employed more extensively than the magnetic effect. The scale on which this effect can be produced is limited only by the dimensions of the apparatus used and the strength of the current.

Wind a helix consisting of about 75 turns of No. 22 insulated copper wire and provide an iron core that can be inserted into, or removed from, the helix as desired. Test the magnetic action of the helix without the core by presenting it to a magnetic needle while a current is flowing. Now insert the core and note the change. Its effect is more marked than before. Next send the current from a new dry cell through the helix and dip one end of the core into a box of tacks, Fig. 329. A large quantity of tacks will cling to the core and remain there until the current is interrupted.

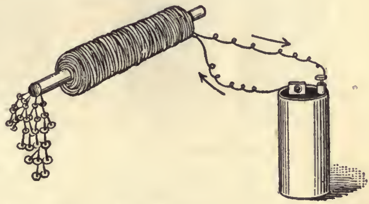


FIG. 329. — Magnetization of Iron by Means of an Electric Current.

Although a coreless helix of wire has magnetic properties when a current is flowing through it, its magnetic field is insignificant when compared to that which the same current will produce when an iron core is present, on account of the large permeability of iron (§ 381). The introduction of the core adds the lines of force of the magnetized iron to those produced by the current in the helix alone. *Any mass of iron around which is a helix for conducting an electric current is called an* **electro-magnet.** The small amount of magnetism retained by the core after the circuit is broken is termed *residual* magnetism.

Electro-magnets are made in many forms and often of extremely large dimensions for holding heavy masses of iron. The horseshoe form shown in Figs. 330 and 331 is most frequently used in electrical devices. The wire is so wound as to produce an N-pole at *N* and an S-pole at *S*. The bar of iron *A* which is held by the magnetism of the poles is called the *armature*. When the armature is against the poles, it will be observed that the lines of force find a

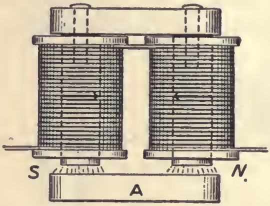


FIG. 330.—An Electro-magnet Showing Poles and Armature.

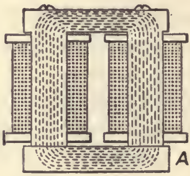


FIG. 331. — Showing the Path of the Lines of Force in an Electro-magnet.

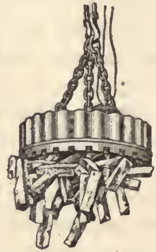


FIG. 332.—A Large Electro-magnet Used for Handling Masses of Iron in Factories.

complete magnetic circuit through iron, as shown by the dotted lines in Fig. 331. Figure 332 shows a large form of the electro-magnet that is widely used in manufacturing plants for the purpose of handling heavy masses of iron, as castings, plates, pig iron, etc. The lifting power is controlled mainly by the current used.

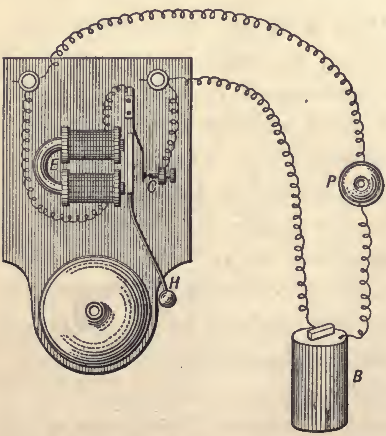


FIG. 333.—The Electric Bell.

411. The Electric Bell. — An important application of the magnetic effect of an electric current is found in the electric bell. The instrument consists of an electro-magnet *E*, Fig. 333, near the poles of which is an armature of iron attached to a spring. Extending from the armature is a slender rod bearing at its extremity the bell hammer H. The armature carries a spring that touches lightly against the screw point at C. The connections are made as shown in the figure. When the push button P is pressed, the circuit is completed by a metallic contact within the button, and the current from the cell B flows through the electro-magnet coils. This causes the magnet to attract the armature, and the hammer strikes the bell. The movement of the armature, however, breaks the circuit at C and thus interrupts the current. Since the cores of the magnet now lose their magnetism, the armature is thrown back by the spring; the contact at C is restored, and all the operations are repeated. Hence a steady pressure on the push button causes the hammer to execute a number of rapid strokes against the bell. The direction of the current is immaterial to the operation of the bell.

412. Mutual Action of Two Parallel Currents. — Suspend two light wires about 30 cm. long from two other wires a and b, Fig. 334, which are bent as shown. Let the lower ends of the vertical wires just dip into a mercury cup below. Now send a strong current through the apparatus (an alternating current of from 4 to 8 amperes suffices), which will flow down one wire and up the other. The two conductors will repel each other. Again, hang both wires on a and make the connections as shown in Fig. 335. Both currents will now flow in the same direction and an attraction will be observed.

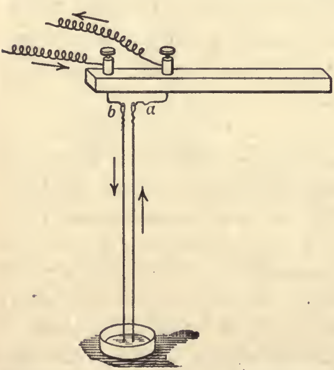


FIG. 334. — Illustrating the Mutual Action between Parallel Currents.

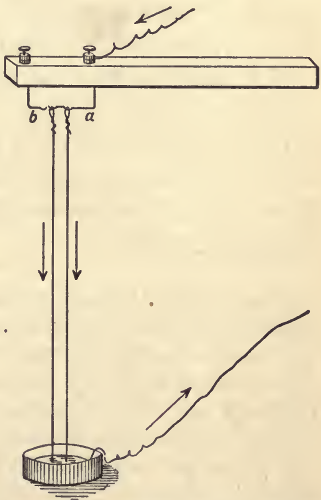


FIG. 335. — Parallel Currents in the Same Direction.

The results of the experiments may be stated as follows: *Currents flowing in the same direction attract each other, and those flowing in opposite directions repel each other.* The

Fig. 336. — Magnetic Fields around Parallel Conductors: (1) When Currents are Flowing in the Opposite Directions; (2) When Flowing in the Same Direction.

magnetic field around the two wires in each of the two cases is shown in Fig. 336.

413. Heating Effect of Electric Currents. — It is a familiar fact that electric currents produce heat. This is at once evident when the hand is placed in contact with the warm bulb of an incandescent lamp. The following experiment shows in another way the transformation of electrical energy into heat.

If an electric lighting current is available, construct a device for controlling it by mounting several lamp sockets on a board and joining them in parallel (§ 444). Connect each side to a binding post, screw a lamp into each socket, and a safe adjustable resistance is ready for use. Put the device just described in circuit with a piece of fine iron wire joined to one of copper wire of the same diameter. The pieces may be each several inches in length. Screw one lamp at a time into its socket until sufficient current flows to heat the wire. The iron wire will at length glow with heat while the copper wire of the same dimension is still comparatively cool. If a lighting current cannot be used, the experiment may be made by using shorter pieces of wire with several good dry cells joined in series. In this case no controlling resistance is necessary.

An experiment that succeeds well with a small current is made by connecting a cell with a fine insulated copper or iron wire that has been wound in a coil about the bulb of a thermometer. The rise of mercury indicates the heating effect of the current.

The heating effect of a current of electricity is employed in electric cooking devices, in various methods of heating, and in electric lighting. An electrically heated flatiron is shown in Fig. 337.

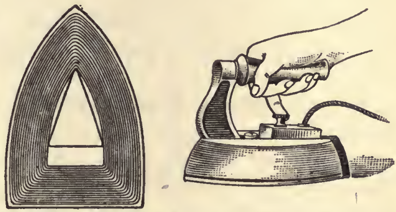


FIG. 337.—An Electrically Heated Flatiron.

414. Chemical Effects of Electric Currents.— Seal two platinum wires to which are attached platinum strips about 1 cm. wide and 3 cm. long in small bent glass tubes *e* and *f*, Fig. 338. Over each of these place an inverted test tube filled with water to which has been added a number of drops of sulphuric acid. Place a small quantity of mercury in each of the small tubes and connect the strips of platinum in circuit with three or four dry cells joined in series by inserting the connecting wires into the mercury. Small bubbles of gas will be seen rising from the platinum in each tube and collecting at the top. If *e* is joined to the positive pole of the battery, the gas above that strip of platinum will collect only one half as fast as that over the other. When the action has progressed until one of the tubes is filled with gas, remove it carefully and apply a lighted match. A blue flame will be seen caused by the burning of this gas, which is *hydrogen*. Now remove the other tube and insert a glowing pine stick, which will be observed to burst into a flame because of the *oxygen* contained in the tube.

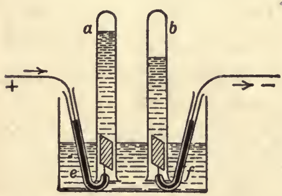


FIG. 338.—Electrolysis of Water. Oxygen is Collected at *a*, and Hydrogen at *b*.

The platinum strips employed in making electrical contact with the liquid in the tubes are called **electrodes**. While conducting the electric current from one electrode to the other, the water is decomposed into its constituent elements, hydrogen and oxygen. This decomposition is characteristic of all liquids, except liquid metals. *The process of decomposing a compound substance by means of an electric current is called* **electrolysis.** The substance decomposed is called an **electrolyte.** The electrode at which the current enters the electrolyte is the **anode;** the one at which it leaves, the **cathode.**

415. Theory of Electrolysis. — In the light of the theory of solutions stated in § 396, the process of electrolysis is easily explained. When a small quantity of sulphuric acid (H_2SO_4), for example, is introduced into water, the molecules split up into hydrogen (H) ions and sulphions (SO_4), the former bearing positive charges of electricity, the latter, negative charges. Now when two platinum electrodes that are connected to the poles of a battery are placed in the liquid, the cathode, which is charged negatively, attracts the positively charged H ions, while the anode, which is positively charged, attracts the negatively charged SO_4 ions. The H ions discharge their positive electricity to the cathode and are then free to collect and rise in bubbles to the surface. The SO_4 ions, on the other hand, discharge their negative electricity to the anode, where they react chemically upon the water (H_2O), setting free the oxygen (O). Thus each sulphion (SO_4) together with the hydrogen (H_2) taken from the water forms new molecules of sulphuric acid (H_2SO_4). In this manner the amount of acid present in the solution remains constant, while the quantity of water diminishes as the process of electrolysis advances.

416. Electrolysis of Copper Sulphate. — Introduce two platinum electrodes into a solution of copper sulphate ($CuSO_4$) and place them in circuit with three or four dry cells. After a few seconds the cathode, or negative electrode, will be found to be coated with metallic copper, while the anode remains unchanged. If the direction of the current be now reversed, copper will be deposited on the clean plate (now the cathode), while the copper coating on the anode gradually disappears.

In the electrolysis of copper sulphate, the ions present are copper (Cu) ions and sulphions (SO_4), the former bearing positive charges of electricity and the latter, negative. When platinum electrodes are introduced into the solution, the copper ions are attracted to the cathode, where they discharge their electricity and become free. Thus we find copper deposited upon this electrode. At the anode the sulphions react with water as in the case just described (§ 415). However, if the anode is a copper plate, the sulphions (SO_4) abstract copper from the plate at the instant they discharge their electricity and form copper sulphate ($CuSO_4$), which dissolves in the liquid.

The experiment illustrates the process of electroplating, a method by which one metal is given a coating of another. Thus by electrolytic action corrosive iron may be plated with non-corrosive nickel, or tarnishing brass with pure gold.

In the experiment, copper (Cu), which is a constituent of the copper sulphate ($CuSO_4$) in the solution, is always deposited on the cathode, or negative electrode. If the anode is made of platinum, oxygen will be liberated at its surface. If, however, the anode is made of copper, the case is modified; for, instead of setting oxygen free, copper is removed from the plate and carried into the solution.

417. Electroplating. — When we consider the vast number of plated articles in everyday use, we can scarcely overestimate the great commercial value of the electrolytic action of an electric current. When, for example, silver is to be plated upon the surface of a spoon, an anode plate of silver is suspended in a solution of silver cyanide, while the spoon is made the cathode by being connected with the zinc of a battery and completely submerged in the solution. See Fig. 339. The current is allowed to flow until the coating is of the desired thickness. In the nickel-plating pro-

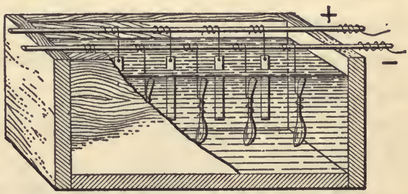


Fig. 339. — Illustrating the Process of Electroplating.

cess an anode of nickel is used in a solution of nickel nitrate and ammonium nitrate. *The article to be plated is always the cathode.* When the coating reaches the proper thickness, the final process of polishing gives the surface the desired appearance.

418. Electrotyping. — As a rule, books of which a large edition is to be printed are first *electrotyped.* In this process an impression is made in wax after the type has been set up, so that each letter leaves its imprint in the mold. A thin layer of finely powdered plumbago, or graphite, is brushed over the surface of the wax in order to render it a conductor of electricity. When thus prepared, the mold is placed in an electrolytic bath of copper sulphate and joined to the negative pole of a battery or other source of electricity. The anode is simply a copper plate. The current is allowed to flow until the coating of copper upon the wax is somewhat thicker than a sheet of paper. While the copper is being deposited upon the conducting graphite, it penetrates into even the smallest depressions of the mold and thus reproduces in copper the exact form of the type. The coating of copper is then removed from the wax, trimmed, and filled in at the back with molten type metal. The advantage gained by electrotyping is convenience, durability, and permanence, and the type from which the impression is taken on the wax may be distributed and used again without delay.

419. Refining Copper. — Copper as it comes from ordinary smelting works contains many impurities. Such copper is refined electrolytically by casting the crude metal in huge plates which are afterwards used as anodes in large depositing vats. The solution used is copper sulphate, and the cathode is a thin plate of pure copper. When a current of electricity is sent through the solution, copper is deposited on the cathode until it grows into a heavy plate. The copper anode is carried into the solution, while its impurities collect at the bottom of the vat. Copper thus refined is called *electrolytic copper*, and is much used in the manufacture of wire and in the construction of dynamos, motors, etc.

420. The Storage Battery. — The principle of the storage cell may be illustrated by making a small cell of two plates of lead about 2×6 inches and a solution of sulphuric acid consisting of one part of acid and about eight parts of water. Attach the plates to a piece of wood and hang them in the solution. Connect the lead plates to two good dry cells joined in series and allow the current to flow for a minute or more. Disconnect the dry cells and run wires from the lead plates to an electric bell. The bell will ring vigorously for a short time and then gradually cease. The power of the cell can be restored by again connecting it with the dry cells. If a galvanometer be introduced in the circuit, it will be found that the discharging current flows in opposition to the current used in charging. If the E. M. F. of the charged plates is measured by a voltmeter, it will be found to be about two volts.

The charging current decomposes the water as in § 414. Hydrogen is liberated at the cathode plate; but the oxygen produced at the anode changes the surface of the plate *from lead* (Pb) *into lead peroxide* (PbO_2), which may be recognized by its brownish color. When the plates are connected with the electric bell, a current flows from the peroxide plate through the bell to the other, which is simply lead. The current continues until the thin coating is used up.

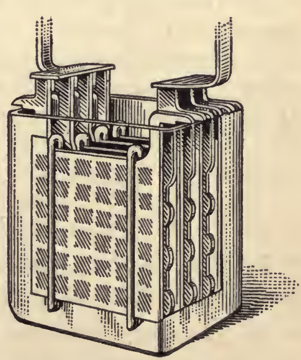


Fig. 340. — A Storage Cell, or Lead Accumulator.

It should be observed that the storage cell stores energy but not electricity. The work done by the charging current results in the production of the energy of chemical separation in the cell. When the circuit is closed, this amount of potential energy is transferred to the bell, where it appears as mechanical energy which is finally dissipated as sound and heat. A complete storage cell is shown in Fig. 340.

EXERCISES

1. What would be the effect produced upon the strength of the poles of a bar magnet if it were placed in a helix in which the direction of the current around the N-pole of the magnet was counterclockwise? In which it was clockwise?

2. A helix is suspended so as to turn freely in a horizontal plane and is placed above a strong bar magnet. If a current be sent through the helix, what will be its direction around that end which stops over the S-pole of the magnet?

3. Place a compass box upon one of the rails of an electric railway running north and south and see if you can detect the presence and direction of a current.

4. Would you expect a compass needle to point north and south in a moving trolley car? Why?

5. Which is most readily magnetized when placed in a helix, iron or steel? Which will retain the greater amount of magnetism? How could you produce a permanent magnet by the help of a dry cell and a helix?

6. How could the direction of an electric current be determined by means of an electrolytic cell through which it could be caused to flow?

7. What change would finally occur in the copper sulphate solution in an electrolytic cell having platinum electrodes if the current were allowed to flow?

8. Would the result in Exer. 7 be at all modified if the anode were copper? Explain.

9. Electric circuits in buildings are protected against too strong currents by lead wire fuses of the proper size placed in the circuit. If by accident the current becomes strong enough to be unsafe, the wire melts. Explain in full. If possible, inspect the wiring of some building and report on the form in which the fuses are made.

SUMMARY

1. An *electric current* is a continuous discharge, or movement, of electricity (§ 390).

2. Electric currents may be produced by chemical action, as in voltaic cells. It may be shown that one of the terminals of such a cell is charged with positive electricity, the other with negative (§§ 391 and 392).

3. The E. M. F. of a cell is the difference of potential between these charges when no current is flowing (§ 393).

4. Much of the potential energy of the zinc of a cell is wasted by "local action." This may be largely prevented by *amalgamation* (§ 397).

5. A current flowing in a conductor near and parallel to a magnetic needle tends to deflect it from its position. This principle is used in many electrical measuring instruments, as galvanometers, etc. (§ 400).

6. The E. M. F. is diminished by the accumulation of hydrogen on the copper (or carbon) plate. This effect is known as *polarization.* (§ 402).

7. An electric current is surrounded by a magnetic field. When the conductor carrying the current encircles a bar of iron or steel, the bar becomes a magnet (§ 408).

8. The *electro-magnet* consists of a helix of insulated wire wound upon a core of iron. The principle of the electro-magnet is employed in the electric bell and many other important devices (§§ 410 and 411).

9. Parallel currents flowing in the same direction attract each other, and those flowing in opposite directions repel (§ 412).

10. When an electric current flows through a conductor, the conductor is heated. This effect is applied in electric lighting, in heating and cooking devices, etc. (§ 413).

11. An electric current decomposes water into hydrogen and oxygen, hydrogen being liberated at the cathode. When a current flows through a solution of a metallic salt, the compound is decomposed and the metal liberated at (or plated upon) the cathode. This effect of an electric current is known as *electrolysis* and is used in electroplating, etc. (§§ 414 to 420).

CHAPTER XIX

ELECTRICAL MEASUREMENTS

1. ELECTRICAL QUANTITIES AND UNITS

421. Fundamental Electrical Magnitudes. — In every electric circuit there are three fundamental quantities which admit of measurement; viz. *current strength*, *electrical resistance*, and *difference of potential*. The first of these, current strength, may be likened to the rate at which water is delivered through a pipe; electrical resistance, to the friction encountered by a liquid current; and difference of potential, to a difference of level (or pressure) for any two chosen points between which the current is flowing.

422. Current Strength — the Ampere. — In many of the preceding experiments it has been obvious that the magnitude of many of the effects produced depended on a quantity that has been frequently referred to as the "strength of the current." For some effects the current must be strong, for others, weak. The expression refers to the rate at which positive electricity is being discharged from the positive to the negative pole through the circuit.

The unit of current strength is the *ampere*, so called in honor of Ampère,[1] a French physicist. *The* **ampere** *is that current which will deposit in an electrolytic cell* 0.001118 *grams of silver or* 0.0003287 *grams of copper per second.* Of the currents used in the experiments described in the preceding sections, the largest were required in § 413 and

[1] See portrait facing page 406.

amounted to 6 or 7 amperes. As a rule, the current used in most classroom demonstrations is less than 1 ampere in value. In the comparison and measurement of electric currents, different kinds of **galvanometers** are used.

423. The Tangent and Astatic Galvanometers. — The tangent galvanometer is a common form found in most laboratories. It consists of a circular coil of wire wound on a frame about 30 cm. in diameter. See (1), Fig. 341. The ends of the coil lead to binding posts on the base. At the center of the coil is placed a compass needle below which is a graduated circle. When in use the coil of the instrument is placed north and south and the current sent through the coil. The instrument derives its name from the fact that *the current is proportional to the tangent of the angle of deflection.* If the current necessary to deflect the needle 45° is known, other currents are easily measured.

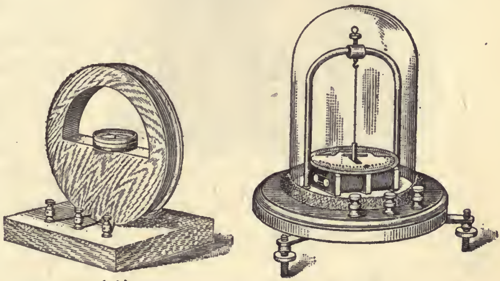


FIG. 341. — (1) The Tangent Galvanometer; (2) The Astatic Galvanometer.

For the measurement and detection of small currents the *astatic galvanometer* (2), Fig. 341, is sometimes used. Two similar magnets are attached to a vertical rod so that their N-poles point in opposite directions. This system is then suspended so that the lower needle swings within a coil of wire, as shown in Fig. 342. When a current is sent through the coil, all parts of it tend to throw the needles out of the north-and-south positions. This form of galvanometer may be made extremely sensitive to small currents.

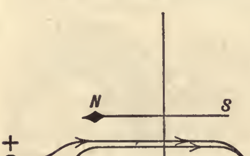


FIG. 342. — The Arrangement of the Magnets in an Astatic Galvanometer.

424. The d'Arsonval Galvanometer. — Galvanometers of the d'Arsonval type differ from those described in § 423 in that the magnet is stationary and the coil of wire movable. As shown in Fig. 343, the instrument consists of a light coil of many turns of fine wire, one end of which is the supension wire AC, while the other end extends below the coil and connects at B. When the instrument is joined in an electric circuit, the current is conducted through the wire of the coil. When no current is flowing, the plane of the coil is held by the suspending wire parallel to a line joining the two poles of a permanent magnet N and S. Since a current through the coil develops a magnetic field of its own at right angles to that of the magnet, the coil will turn until the magnetic forces are in equilibrium with the torsional resistance of the suspension wire. The deflections are read by the movement of a beam of light reflected from a small mirror M attached to the coil. For small angles of deflection the current is practically proportional to the angle; hence the instrument may be used in current measurements. The instrument also affords a very sensitive detector of currents and is, on this account, indispensable in a large class of experiments.

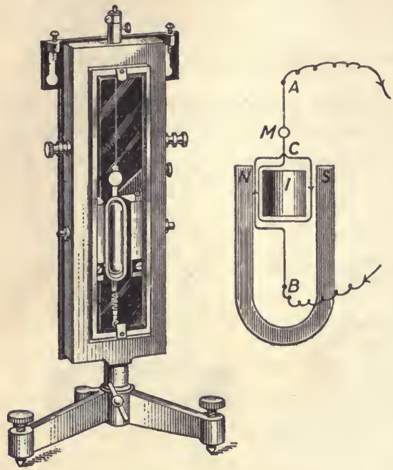


FIG. 343. — The d'Arsonval Galvanometer.

The principle involved in this galvanometer is employed in many electrical measuring instruments.

425. The Ammeter. — An instrument for measuring the strength of an electric current is called an *ammeter*, or *amperemeter*. In most ammeters the magnetic effect of a current is employed. The instrument may consist simply of a magnetic needle, which shows by the amount of its deflection the number of amperes of current. The best instruments, however, consist of a delicately pivoted coil of wire A, Fig. 344, turning between the poles

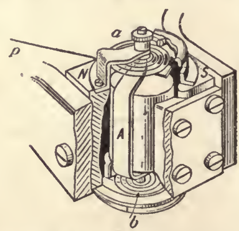


FIG. 344. — Section of an Ammeter.

of a strong permanent magnet *N* and *S* and held in position by two hair springs *a* and *b*. The principle involved in this class of instruments will be recognized at once as that of the d'Arsonval galvanometer. When a current is sent through the coil, the magnetic field developed by the current causes the coil to turn, and the pointer *p* moves over a scale which is graduated to read in amperes. When the current is interrupted, the coil is restored to its initial position by the springs which also serve to conduct the current into and out of the coil. The complete instrument is shown in Fig. 345.

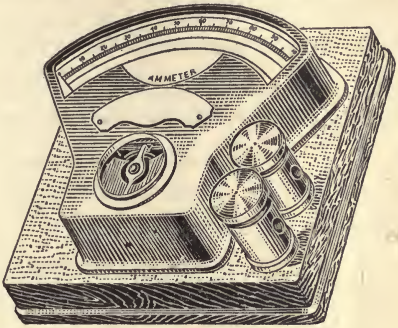


FIG. 345. — An Ammeter.

426. Electrical Resistance. — It was observed in § 350 that substances differ in respect to the readiness with which they transmit an electrical charge; thus bodies are classed as *good or poor conductors. The opposition that a conductor offers tending to retard the transmission of electricity is called* **electrical resistance.** Hence, with a given source of electricity, as a Daniell cell, the current strength will diminish as the resistance of the circuit is increased, and will rise in value as the resistance is decreased.

427. Laws of Resistance. — Construct a frame about 1 meter long and upon it stretch 4 wires terminating in binding posts. Let No. 1 consist of 1 meter of No. 30 (diameter 0.010 inch) German silver wire; No. 2, of 2 meters of No. 30 German silver wire; No. 3, of 2 meters of No. 28 (0.013 inch); and No. 4, about 20 meters of No. 30 copper wire. Connect wire No. 1 in series with one or two Daniell cells and a low resistance galvanometer, and read the deflection of the needle. Replace No. 1 by No. 2, and the deflection will be found to be less than before, thus indicating a greater resistance for the greater length. When the current is sent through No. 3, which is a larger wire, an increased deflection shows a decreased resistance. Finally, when the current is sent through wire No. 4, the deflection will be even larger than for No. 2, which is of the same size and only one tenth as long. Thus copper is shown to be more than 10 times as good a conductor as German silver. The experiment may be extended to other wires of various substances.

Accurate measurements verify the following laws:

1. *The resistance of a conductor of uniform size and composition is directly proportional to its length.*

2. *The resistance of a conductor is inversely proportional to its cross-sectional area; or, if circular in form, to the square of its diameter.*

3. *The resistance of a conductor depends upon the nature of the substance of which it is composed.*

For example, if the resistance of a copper wire of a certain diameter and length is 1 unit, the resistance of a wire of the same kind and size and twice the length is 2 units. If, now, the diameter be doubled, the resistance is divided by 2^2, *i.e.* reduced to 2 units $\div 4$, or $\frac{1}{2}$ a unit.

428. The Unit of Resistance. — The unit of resistance is called the **ohm** in honor of Dr. G. S. Ohm,[1] a German physicist. *The ohm is the amount of electrical resistance offered by a column of pure mercury* 106.3 *cm. in height, of uniform cross section, and having a mass of* 14.4521 *grams, the temperature being* 0° *C.* The cross-sectional area of such a column is almost exactly one square millimeter. Since such a column of mercury would be inconvenient to handle, coils of wire whose resistances have been carefully measured and recorded are used in practical measurements. A piece of No. 22 (0.025 inch) copper wire 60 feet in length has approximately one ohm of resistance. One meter of No. 30 German silver wire has a resistance of about 6.35 ohms. A convenient unit for rough work can easily be made by winding 9 feet and 5 inches of

[1] See portrait facing page 406.

ANDRÉ MARIE AMPÈRE (1775–1836)

The fame of Ampère rests mainly on the services he rendered to science in establishing the relation between electricity and magnetism and in developing the science of electro-dynamics. His chief experiments deal with the magnetic action between conductors in which electric currents are flowing. Ampère was also the first to magnetize needles by inserting them in a helix in which a current was flowing.

Ampère was born at Lyons, France. During the French Revolution his father was beheaded, an event which for years clouded the spirit of the young scientist. In 1805 he became professor of mathematics in the Polytechnic School in Paris, and later professor of physics in the College of France. In 1823 he published his mathematical classic on the theory of magnetism. The practical unit of current strength is named in his honor.

GEORGE SIMON OHM (1787–1854)

Following closely upon the experiments of Ampère on the magnetic action between currents came the researches of Ohm, a German, whose discoveries concerned the strength of an electric current.

Ohm's first experiments dealt with the conductivity of wires of different metals. He observed the deflections of a magnetic needle by currents from a given source, but flowing through different conductors. By changes in the E. M. F. in the circuits used, he obtained results which led him to the well-known expression $C = \frac{E}{R + r}$. He then investigated cells joined in parallel and series, and published his results in 1826. During the following year he published the theoretic deduction of the law bearing his name. This law is one of the most fruitful of the early electrical discoveries.

Ohm was born in Erlangen, where he was educated. His ambition to become a university professor was not realized until 1849, when he was elected to a professorship at Munich. The practical unit of resistance is called the *ohm* in his honor.

No. 30 copper wire on a spool and connecting its ends to binding posts.

429. Potential Difference — the Volt. — It was shown in § 393 that the difference of potential between the poles of a voltaic cell when no current is flowing is its *electromotive force.* This quantity is conceived to be the cause of current flow, and is analogous to the difference in level between two bodies of water, to the difference in pressure of gases compressed in two reservoirs, or to a difference of temperature in the case of heat. The unit of potential difference and, consequently, of E. M. F., is the **volt**. *The volt is that difference of potential between the ends of a wire having a resistance of one ohm which will produce a current of one ampere.* The E. M. F. of some of the voltaic cells in common use is given in the following table:

E. M. F. of Cells

Daniell, 1.1 volts.	Dry cells, 1.5 volts.
Leclanché, 1.5 volts.	Dichromate, 2.0 volts.

A Daniell cell would then cause a current of 1.1 amperes through a wire having a resistance of 1 ohm, provided there were no other resistance in the circuit. It is evident, however, that the resistance of the cell itself must always be considered in any given circuit.

430. The Voltmeter. — In order that an instrument may measure the E. M. F. of a cell, it is necessary that no appreciable current be permitted to flow. See the definition of E. M. F., § 393. In order, therefore, that a galvanometer may be adapted to this use, it must contain a large amount of resistance. Instruments of several hundred ohms are made whose deflections are practically proportional to the E. M. F. of cells to which they may be attached. When such instruments are graduated to read volts they are called *voltmeters.* Figure 346 shows the manner of connecting a voltmeter for determining the E. M. F. of the cell *B*, and Fig. 347 shows its connection

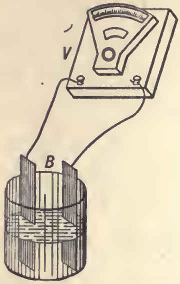


FIG. 346.—Illustrating a Use of the Voltmeter.

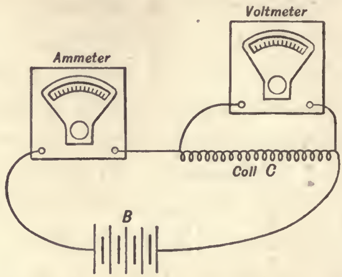


FIG. 347.—The Voltmeter Shows the Fall of Potential through Coil *C* and the Ammeter Measures the Current Strength.

for giving the potential difference between the terminals of a coil of wire placed in the circuit. One well-known form of a voltmeter is shown in Fig. 348.

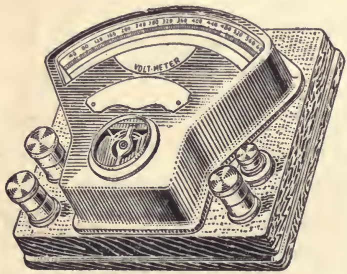


FIG. 348.—The Voltmeter.

431. Ohm's Law.—Connect a Daniell cell in circuit with a galvanometer or ammeter and a resistance box. Make the resistance of the circuit a suitable amount (say 200 ohms, including the galvanometer resistance) and measure the current strength. Replace the Daniell cell by a new dry cell and again ascertain the current. The two currents will be found proportional to the E. M. F. of the cells as stated in the table in § 429. Again introduce the Daniell cell and make the resistance of the entire circuit double the first amount. The current strength will be only one half as great. Make the resistance one half as great as at first, and the current will be doubled.

The experiment illustrates the law first published by Dr. G. S. Ohm in 1827. This law states that *the strength of an electric current is directly proportional to the E. M. F.*

furnished by the voltaic cell or combination of cells, and inversely proportional to the total resistance of the circuit. The ampere, volt, and ohm are so chosen that Ohm's law may be written

$$C = \frac{E}{R};$$

$$\textit{i.e.}\ \textbf{current (amperes)} = \frac{\textbf{E. M. F. (volts)}}{\textbf{resistance (ohms)}}. \qquad (1)$$

The law may be applied to any portion of an electric circuit, in which case it becomes

$$\textbf{current (amperes)} = \frac{\textbf{potential difference (volts)}}{\textbf{resistance (ohms)}}. \qquad (2)$$

For example, the current produced by a Daniell cell (1.10 volts) in a circuit in which the total resistance (including that of the cell) is 44 ohms is $1.1 \div 44$, or 0.025 ampere. It is clear from equation (1) that if any two of the quantities are given, the third can easily be computed.

432. Internal Resistance. — The current that any cell can produce is limited by the resistance which the current encounters in passing through the liquid of the cell. This is called the *internal resistance* of the circuit. In a fresh dry cell the resistance is a fraction of an ohm, but with age and use it increases to several ohms. That of a Daniell cell varies from about one to three or four ohms. *The resistance of cells may be decreased by using larger plates and also by reducing the distance between the plates.* It is furthermore dependent on the conductivity of the liquid. When more than one cell is connected in a circuit, the entire internal resistance depends on the manner in which they are joined together.

433. Cells Connected in Series. — Join each of three Daniell cells successively to a voltmeter, or a galvanometer of high resistance (not less than 50 ohms), and record the deflections produced. They should be about equal. Now connect two of the cells in series, as shown in Fig. 349, and lead wires to the instrument. The deflection will be twice as great as for one cell, and the addition of the third cell will make the current three times as strong. The deflections under these conditions are proportional to the number of cells, because their electromotive forces are added together when they are joined in this manner.

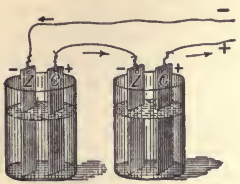


FIG. 349. — Two Cells Joined in Series.

Cells are in **series** *when the positive pole of the first is joined to the negative pole of the second, the positive pole of the second to the negative pole of the third, and so on.* A study of the figure will aid greatly in making the method clear. See also Fig. 311.

Although the electromotive forces are added by connecting cells in series, the resistances of the separate cells also add together and thus tend to reduce the current strength. For example, four similar cells in series will have four times the internal resistance of one cell.

Since the strength of the current is obtained by dividing the E. M. F. by the total resistance of a circuit,

$$C = \frac{E_1 + E_2 + E_3 + E_4 + \text{etc.}}{R + r_1 + r_2 + r_3 + r_4 + \text{etc.}}, \qquad (3)$$

where E_1, E_2, E_3, E_4, etc. are the electromotive forces of the individual cells, r_1, r_2, r_3, r_4, etc. are the corresponding internal resistances and R is the external resistance of the circuit. If we have, for example, five similar cells, the equation becomes

$$C = \frac{5E}{R + 5r}, \text{ and for n cells, } C = \frac{nE}{R + nr}, \qquad (4)$$

where E is the E. M. F. and r the resistance of a single cell.

434. Cells Connected in Parallel. — **1.** With the apparatus used in the experiment of the preceding section, record the deflection produced by each cell alone and then connect the positive poles of two of them to one terminal of the instrument and the two negative poles to the other terminal. The deflection will be the same as that obtained when only one cell is used. If all the cells are joined as shown in Fig. 350, the deflection of the instrument will not be greatly increased, if at all.

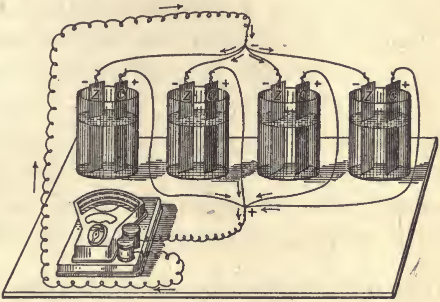


FIG. 350. — Four Cells Joined in Parallel.

2. Join the cells while connected in parallel to an ammeter or galvanometer of very small resistance and record the reading. Do the same with the cells connected in series and also with only one cell. It will be found that one cell alone will produce about as much current as all the cells when joined in series, but a much stronger current will be derived from them when they are in parallel.

The experiments show that the parallel arrangement of cells has an advantage over the series connection *when the external resistance is small.* In fact, when the external resistance is very small, as it is in some cases, the series arrangement of cells produces practically no more current than a single cell.[1] The following example will make the matter clear.

The current obtained from a single cell whose E. M. F. is 1.5 volts when the internal resistance is 3 ohms and the external resistance 0.2 ohm is, by equation (**1**), $C = \frac{1.5}{0.2 + 3}$, or 0.469 ampere.

[1] When cells of extremely small internal resistance are used, as new dry cells, the series arrangement is the better under nearly all circumstances. A poor cell, however, having a large resistance may prove to be more of a hindrance than a help.

Now if ten such cells are connected in series, the current is, by equation (4), $C = \frac{10 \times 1.5}{0.2 + 30}$, or 0.496 ampere, which is only slightly larger than the current obtained from one cell.

Experiment 1 shows that the E. M. F. remains unchanged when cells are connected in parallel; hence, for the ten cells just considered, *the E. M. F. is just the same as that of one cell*, viz. 1.5 volts. Again, since like plates are joined together, the entire combination of cells is like one cell having plates of ten times the area of those of one cell. Therefore, by § 427, the internal resistance is only one tenth as much, *i.e.* 3 ÷ 10, or 0.3 ohm. The equation for the parallel arrangement becomes $C = \frac{1.5}{0.2 + .3}$, or 3 amperes, which is about six times as much as the series arrangement would produce. Hence for n similar cells joined in parallel, Ohm's law is

$$\mathbf{C} = \frac{\text{E. M. F. of one cell}}{\mathbf{R} + \dfrac{\text{resistance of one cell}}{\text{number of cells}}}, \text{ or } \frac{\mathbf{E}}{\mathbf{R} + \dfrac{\mathbf{r}}{\mathbf{n}}}. \qquad (5)$$

EXERCISES

The pupil should represent each of the following cases diagrammatically.

1. In the discussion of the two methods of combining cells, for what kind of circuits is the series arrangement of cells shown to be suitable?

2. How much current will a dry cell of 2 ohms resistance and 1.43 volts send through a wire of 25 ohms resistance?

3. How much current would three cells similar to the one in Exer. 1 send through the same wire (1) when joined in series and (2) in parallel? *Ans.* (1) 0.13 ampere; (2) 0.0557 ampere.

4. What current would 8 Daniell cells of which the E. M. F. is 1.08 volts and the resistance 3 ohms each send through an ammeter of 0.4 ohm, when joined in series? What would be the current from one cell alone?

5. What would be the current produced from the same 8 cells connected in parallel and using the same ammeter?

6. Which is the better arrangement of 4 Daniell cells (E. M. F. = 1.1 volts and $r = 2.5$ ohms each) when the external resistance is 6 ohms?

7. Show that a small Daniell cell will give practically as much current as a large one through 1000 ohms of resistance, the resistance of the small one being 30 ohms while that of the large one is 4 ohms.

8. If a galvanometer gives the same deflection when connected with a very small cell as it does when connected with one several times as large but having the same E. M. F., is the resistance of the instrument large or small?

9. A dry cell whose E. M. F. is 1.5 volts produces a current of 0.2 ampere through an instrument whose resistance is 7 ohms. Find the resistance in the cell.

10. Which will produce the greater effect in an external circuit of 5 ohms, a dry cell whose resistance is 3 ohms or a copper-oxide cell (E. M. F. = 0.8 volt) having a resistance of 0.2 ohm?

11. What current will be derived from a Daniell cell whose resistance is 3 ohms and a dry cell with a resistance of 2 ohms when they are joined in series and the external resistance is 2.74 ohms?

12. What would be the value of the current in the preceding exercise if the poles of the Daniell cell were set in opposition to those of the dry cell? Diagram the connections.

13. A circuit contains 4 dry cells (E. M. F. of each = 1.5 volts) joined in series. Three of the cells are known to have a resistance of 0.5 ohm each, and the external resistance is 10 ohms. If the current is 0.15 ampere, what is the resistance offered by the fourth cell? Would the current be increased or decreased by removing this cell from the circuit?

2. ELECTRICAL ENERGY AND POWER

435. Energy of an Electric Current. — We have learned under the study of the effects of electricity (§§ 411, 413, 414) that the energy of a current may be expended in three ways: (1) it may produce mechanical motion, as in the electric bell; (2) it may produce chemical separation; and (3) it may be converted into heat in a conductor.

Let us imagine an electric circuit as shown in Fig. 351. Between the points A and B is a resistance of 4 ohms. If the battery produces a current of 5 amperes, for example, the potential difference between A and B will be, by Ohm's law (§ 431), 4×5, or 20 volts.

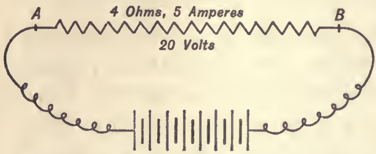


Fig. 351. — Electrical Energy is Transformed into Heat between A and B.

Now the work done, or energy expended, by the current between A and B depends on three factors, — (1) the potential difference, (2) the current strength, and (3) the time, — and it is measured by their product. Therefore we have the relation

energy = potential difference × current strength × time.

If the time is in seconds, the potential difference in volts, and the current strength in amperes, this product gives the energy in terms of a unit called the *joule*, which is equal to 10,000,000 ergs. Hence

$$\textbf{volts} \times \textbf{amperes} \times \textbf{seconds} = \textbf{joules}. \qquad (6)$$

In the electric circuit shown in Fig. 351 the amount of energy expended in 5 minutes (300 seconds) between the points A and B is, then, by equation (6), $20 \times 5 \times 300$, or 30,000 joules.

436. Power of an Electric Current. — Since *power* refers to the rate at which work is done or energy expended (§ 57), it may be found by simply dividing the total energy expended by the time. In an electric circuit, therefore, the power is measured by the product of the potential difference and the current strength; or,

$$\textbf{power} = \textbf{volts} \times \textbf{amperes}. \qquad (7)$$

It is plain that power is the number of joules per second expended by the current. A power of one joule per second is called a *watt*, in honor of James Watt (1736–1819) of Scotland. One horse power is equal to 746 watts.

In the example chosen in § 435, the power of the current in the circuit between A and B is 20×5, or 100 watts; *i.e.* if the energy of the current which is expended between the points A and B could be converted into mechanical energy, there would be developed $\frac{100}{746}$ horse power.

437. Quantity of Heat Developed by a Current. — When no mechanical or chemical work is done by a current of electricity, the energy is used simply in overcoming the resistance of the conductor and is converted into heat. Now one heat unit (a calorie) has been found by experiment to be equivalent to 4.2 joules; or, one joule equals $\frac{1}{4.2}$, or 0.24 calorie.

If we express the potential difference between two points of a circuit by E and the current strength by C, the number of joules expended in t seconds is, by equation (6), ECt joules. But by Ohm's law (§ 431), $C = \frac{E}{R}$; whence $E = CR$. Now if we substitute this value of E, we obtain for the energy expended C^2Rt, which represents the amount of electrical energy that is converted into heat in a resistance of R ohms when the current is C amperes and the time t seconds. Reducing joules to calories gives

$$\text{Heat} = 0.24\ C^2Rt \text{ calories.} \qquad (8)$$

This equation represents Joule's law, which may be stated as follows:

The heat developed in a conductor by an electric current is proportional to the square of the current, to the resistance of the conductor, and to the time the current is flowing.

438. Loss of Energy in Transmitting Electricity. — The conversion of electrical energy into heat in a conductor through which it flows has an important commercial bearing on the transmission of electric power over long lines. For example, if the resistance of the wires leading from a distant source of electrical power to a group of lamps is only 2 ohms, the waste of power in transmitting a current of 10 amperes is $10^2 \times 2$, or 200 watts. Now 10 amperes at 110 volts would operate 20 lamps, each of which consumes 55 watts. The total power required would be, therefore, $20 \times 55 + 200$, or 1300 watts. The loss in transmission is therefore $\frac{2}{13}$ of the total power produced. Now if the number of lamps is increased to 100, the lamp consumption is 5500 watts, while the line loss (since the current is now 50 amperes) is $50^2 \times 2$, or 5000 watts. Hence, of the 10,500 watts which must be produced at the power station, 5000 watts, or 47.6 per cent, are lost by being converted into heat in the line.

The loss of energy in long lines of wire can be reduced by constructing the line of larger wire and thus decreasing the resistance factor, but in many cases this method is impracticable from a commercial standpoint. However, by using modern devices involving principles that we shall study later, economical transmission of power over long distances is rendered possible.

EXERCISES

1. Compute the number of joules transmitted by a current of 10 amperes maintained for 20 minutes at a potential difference of 110 volts.

2. Compute the heat loss per hour in an electric line of 3 ohms resistance when the current is 5 amperes. What is the result if the current is 10 amperes?

3. If the power required for an incandescent lamp is 60 watts, what is the consumption of 50 lamps measured in terms of the horse power?

4. A piece of platinum wire is heated by a current of 2 amperes, and the potential difference between its ends is 8 volts. Compute the heat developed per minute.

3. COMPUTATION AND MEASUREMENT OF RESISTANCES

439. Resistances Computed. — It is customary to compute the resistance of a wire of given material, length, and size from the known resistance of a wire of that kind which is 1 foot in length and 0.001 inch (called 1 mil) in diameter. The following table gives this value in ohms for some of the common metals.

Ohms of Resistance in Wires 1 Foot long and 0.001 Inch in Diameter

Silver	9.5	Platinum	80
Copper	10.2	German silver	180
Iron	61.5	Mercury	570

According to the laws of resistance stated in § 427, *the resistance of a wire can be calculated by simply multiplying the number given in the table by the length of the wire in feet and dividing by the square of the diameter, which must first be reduced to thousandths of an inch.* For example, the resistance of a mile of iron wire 0.08 inch in diameter is 61.5 ohms $\times$ 5280 $\div$ 80^2, or 50.7 ohms.

EXERCISES

1. What is the resistance of each of the wires given in the following table?

Kind of Wire	Number	Diameter	Length
Copper	8	0.128 inch	1 mile
Copper	22	0.025 inch	500 feet
Copper	36	0.005 inch	40 feet
Copper	40	0.003 inch	25 feet
Iron	9	0.114 inch	1 mile
Iron	14	0.064 inch	25 feet
Iron	10	0.102 inch	5000 feet
German silver	20	0.032 inch	35 feet
German silver	30	0.010 inch	10 meters
Platinum	24	0.020 inch	2 feet

2. How long must a No. 36 copper wire be to offer a resistance of one ohm?

3. Find the diameter of a copper wire that has a resistance of 10 ohms per thousand feet.

4. A No. 22 wire 1000 ft. long offers a resistance of 285 ohms. Of what material mentioned in § 439 might the wire be made?

440. Resistance Boxes. — Coils of wire whose resistances have been carefully adjusted and marked are incased in boxes and sold by electrical supply houses. Such a box is shown in Fig. 352. The ends of each coil are soldered to brass blocks *A*, *B*, *C*, etc., Fig. 353, between which brass plugs may be inserted. The blocks are mounted on an ebonite or hardwood plate which forms the cover of the box. When the plugs are all in place, no resistance is encountered by an electric current in passing from one block to the next; but when a plug is withdrawn, the current must pass through the corresponding coil whose resistance is marked on the top of the box. Thus, by removing plugs, any resistance from the smallest up to the sum of all the resistances in the box can be obtained.

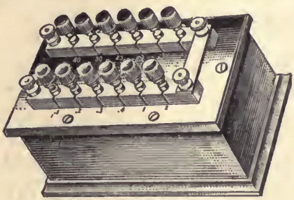


FIG. 352. — A Resistance Box.

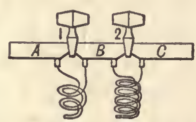


FIG. 353. — Showing Arrangement of Coils.

441. Resistance Measured by Substitution. — Place a coil of wire whose resistance is to be determined in circuit with a Daniell cell and a galvanometer and read the deflection produced by the current. Remove the coil from the circuit and insert a resistance box. Withdraw plugs from the box until the deflection of the galvanometer is the same as before. The sum of the resistances corresponding to the plugs that have been removed gives the resistance of the coil. Why?

442. Resistances Measured by Voltmeter and Ammeter. — The coil of wire *C*, Fig. 347, whose resistance is to be found, is placed in a circuit with the cell *B* and an ammeter. A voltmeter of high resistance is joined to the terminals of the coil C to indicate the potential difference at those points. Since by Ohm's law (§ 431) the current through C equals the potential difference in volts divided by the resistance in ohms (Eq. **2**, § 431), the value of C can be found by dividing the reading of the voltmeter by that of the ammeter. It is essential that the resistance of the voltmeter be so large that practically none of the current can pass through it.

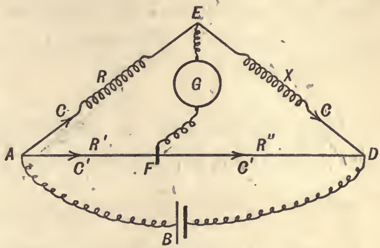


FIG. 354. — Diagram of Wheatstone's Bridge.

443. The Wheatstone Bridge. — When the current from a cell B, Fig. 354, passes through the point A, it divides into two parts, the portion C flowing through the resistance R *ohms*, and the portion C' through the resistance R' *ohms* in the straight uniform wire AD. Now a sensitive galvanometer G joining points E and F will be deflected *unless E and F do not differ in potential.* The point F which has the same potential as E is found by moving the contact along the wire AD until no deflection can be observed.

Under these conditions the difference of potential between A and E must equal that between A and F. By Ohm's law (§ 431),

$$\textit{fall of potential} \div \textit{resistance} = \textit{current}.$$

Hence $$\textit{fall of potential} = \textit{current} \times \textit{resistance};$$

whence $$\mathbf{CR = C'R'}. \qquad (1)$$

Again, the same current flows through X as through R, since the galvanometer is not deflected; and the same current flows through R'' as though R' for the same reason. Now, since the difference of potential between E and D is the same as that between F and D, we have

$$\mathbf{CX = C'R''}. \qquad (2)$$

Dividing (**2**) by (**1**) we have

$$\frac{X}{R} = \frac{R''}{R'}, \text{ or } X = R\frac{R''}{R'}.$$

Now the resistances R' and R'' are proportional to the lengths of the wire AF and FD (§ 427). Therefore,

$$\mathbf{X = R\frac{FD}{FA}}. \qquad (3)$$

Four resistances combined in this manner constitute *Wheatstone's Bridge*. It is obvious that if the resistance R and the lengths of the wire AF and FD are known, the value of the unknown resistance X can be calculated. The point F must be found experimentally as described above.

444. Conductors in Series and Parallel. — Conductors are said to be joined *in series* when they are connected in succession as shown in Fig. 355. In this condition the entire current passes through each conductor. The combined resistance between A and B is the sum of the resistances of the several parts.

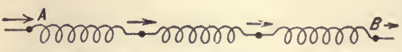


FIG. 355. — Conductors Joined in Series.

The case is very different when the conductors are joined as shown in Fig. 356. Resistances combined in this manner are said to be connected in *parallel*. The most important combination of this kind is that of two wires thus connected. The current from the cell C will obviously divide into two parts at point A which reunite at B. When the two resistances are equal, the two parts of the current are, of course, equal. But if the resistances are, for example, 3 and 7 ohms respectively, and the potential difference between A and B common to both branches is 1 volt, the current through the upper branch is $\frac{1}{3}$ ampere and that through the lower one $\frac{1}{7}$ ampere. Hence, while the resistances are as 3 is to 7, the currents are as 7 is to 3. The currents from the cell, therefore, in a two-branched circuit are *inversely proportional to the corresponding resistances.*

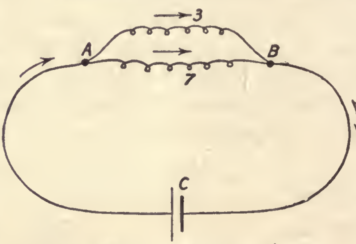


FIG. 356. — Conductors Joined in Parallel.

Again, the total current flowing between A and B is $\frac{1}{3}+\frac{1}{7}$ ampere. If, now, we let R be the combined resistance between A and B, the total current is $\frac{1}{R}$ ampere.

Therefore $$\frac{1}{R}=\frac{1}{3}+\frac{1}{7}, \qquad (4)$$

whence $R=\frac{3\times 7}{3+7}$, or 2.1 ohms. It is clear from the example chosen, that *the combined resistance offered by the two branches of a divided electric circuit is the product of the two resistances divided by their sum;* or, expressed as an equation,

$$R=\frac{r_1 \times r_2}{r_1+r_2}. \qquad (5)$$

445. Shunts. — A shunt is a conductor which is connected in an electric circuit parallel to another conductor. It may be likened to a side-track, or to a by-pass that is frequently used in the case of the conduction of water or gas through pipes. The term is applied to a resistance coil S, Fig. 357, which is joined in a circuit parallel to a galvanometer G or some other instrument. In this case it is used to reduce the flow of electricity through the instrument; for, as shown in § 444, the current divides into parts that are inversely as the two resistances. Suppose, for example, the resistance of G is 45 ohms and that of S is 5 ohms. Then $\frac{5}{50}$ of the entire current passes through the galvanometer and $\frac{45}{50}$ of it through the shunt. Thus it is clear that when a shunt contains $\frac{1}{9}$ as much resistance as a galvanometer, $\frac{1}{10}$ of the total current passes through the galvanometer, and $\frac{9}{10}$ through the

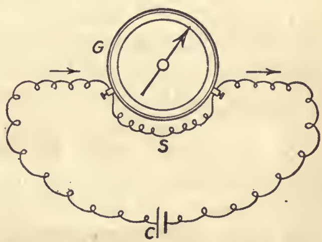


FIG. 357. — Illustrating the Use of a Shunt.

shunt. In the same manner the division of a current in any given case may be computed.

EXERCISES

The pupil should diagram the conditions expressed in each of the following exercises.

1. A current of 4 amperes divides and passes through two parallel coils of wire, one of 5 ohms and the other of 8 ohms. What is the current that goes through each branch?

Ans. 2.461 amperes and 1.539 amperes.

2. Find the combined resistance offered by the two parallel conductors in Exer. 1.

3. Two instruments of 3 and 4 ohms respectively are connected parallel between the poles of a dry cell (E.M.F. 1.5 volts) whose resistance is 1 ohm. Find (1) the external resistance of the circuit, (2) the current strength of the cell, and (3) the current flowing through each branch.

Ans. (1) 1.714 ohms; (2) 0.553 ampere; (3) 0.316 ampere, 0.237 ampere.

4. Two electro-magnets of 4 and 12 ohms respectively are connected in parallel to each other and then placed in a circuit containing a coil of 3 ohms and a battery of 10 ohms. Find (1) the total resistance of the circuit, and (2) the strength of the current in each part of the circuit, the E. M. F. being 4.8 volts.

Ans. (1) 16 ohms; (2) 0.225 ampere and 0.075 ampere.

5. A galvanometer of 30 ohms has a shunt of 30 ohms. When connected in an electric circuit, what part of the whole current will pass through the instrument? By what number must the current measured by the galvanometer be multiplied to give the entire current?

6. If the resistance of the shunt in Exer. 5 is reduced to 20 ohms, what part of the total current will the galvanometer carry, and what will be the multiplier? *Ans.* The multiplier will be 2.5.

7. If a galvanometer has 300 ohms of resistance, what must be the resistance of a shunt so that only one tenth of the entire current will pass through the wire of the instrument? *Ans.* $33\frac{1}{3}$ ohms.

SUMMARY

1. The unit of current strength is the *ampere.* It is the current that will deposit 0.001118 g. of silver per second. Current strength is measured by an *ammeter* (§ 422).

2. The electrical resistance of cylindrical wires of uniform size and composition is directly proportional to the length and inversely proportional to the square of the diameter. It also varies with the nature of the substance of which it is composed. The unit of resistance is the *ohm* (§§ 427 and 428).

3. The unit of potential difference is the *volt.* It is the potential difference required to force a current of an ampere through a resistance of one ohm. Potential differences are measured by the *voltmeter* (§§ 429 and 430).

4. Current, potential difference, and resistance are related mathematically as shown by the equation $C = \frac{E}{R}$, or *amperes* $=$ *volts* $\div$ *ohms.* This relation is known as Ohm's Law (§ 431).

5. The resistance of circuits includes *internal* and *external* resistances; the former is the resistance offered by the cells, the latter, that of the remaining portion of the circuit (§ 432).

6. For n similar cells joined *in series* the current is

$$C = \frac{nE}{R + nr} \text{ (§ 433).}$$

7. For n similar cells joined *in parallel* the current is

$$C = \frac{E}{R + \frac{r}{n}} \text{ (§ 434).}$$

8. The energy expended in any part of a circuit is the product of the potential difference, current strength, and time, or

$$\textit{energy} = ECt \textit{ joules} \text{ (§ 435).}$$

9. The power, or rate at which the current is working, is

$$power = EC \textit{ watts} \text{ (§ 436).}$$

10. The quantity of heat developed by a current of C amperes in a resistance R ohms in t seconds is

$$heat = 0.24 \ C^2Rt \textit{ calories} \text{ (§ 437).}$$

11. The resistance of a cylindrical wire may be computed by multiplying the resistance of one foot of it one mil in diameter by the whole length and dividing by the square of the diameter in mils (§ 439).

12. The combined resistance of conductors joined *in series* is their sum. The combined resistance of two conductors joined *in parallel* is

$$R = \frac{r_1 r_2}{r_1 + r_2} \text{ (§ 444).}$$

CHAPTER XX

ELECTRO-MAGNETIC INDUCTION

1. INDUCED CURRENTS OF ELECTRICITY

446. Currents Induced by Magnetism. — Oersted's discovery of the effect of a current of electricity upon a magnetic needle (§ 400) in 1819 led to the invention of the electro-magnet by Sturgeon in 1825, and induced many experimenters to seek for a method of producing an electric current by means of a magnet. Two physicists, Joseph Henry[1] in America and Michael Faraday[1] in England, independently discovered the process for doing this about 1831.

Connect the ends of a coil of insulated wire *C*, Fig. 358, consisting of a large number of turns, directly to the terminals of a sensitive d'Arsonval galvanometer. Arrange a large horseshoe magnet with its poles upward as shown. Now move the coil down quickly into the magnetic field. The galvanometer will reveal the presence of a current of electricity, *but the index will go back to zero as soon as the coil stops moving.* If the coil be now removed from the magnetic field, the galvanometer will show that a current is produced in the opposite direction. Repeat the experiment, but move the coil more slowly. Turn the coil over and repeat the experiment. Each deflection will be in a direction opposite to the corresponding one produced at first.

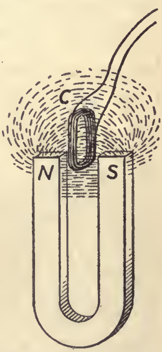


Fig. 358. — Inducing an Electric Current.

The experiment shows clearly the production of an electric current *without the aid of a voltaic cell.*

[1] See portraits facing page 426. See also Maxwell, facing page 432.

Such a current is called an *induced current.* The experiment shows also that the induced current flows only while the coil is moving in the magnetic field, *i.e.* only while the number of lines of force through the coil is changing. Since a current is always due to an E. M. F., it is obvious that the movement of a coil of wire in the magnetic field develops such an electromotive force in the wire. This is called an *induced electromotive force.* Furthermore, the slower the movements, the less rapid the change, and the smaller the induced E. M. F. The following general laws may be stated:

1. *A* **change** *in the number of magnetic lines of force threading through a coil (or a single loop) of wire induces an electromotive force in that coil.*

2. *The induced electromotive force is proportional to the* **rate** *at which the number of lines of force is changed.*

447. Special Cases of Current Induction. — According to the laws of induction given in the preceding section, an induced electromotive force may be brought about either by an increase or by a decrease in the number of magnetic lines through a coil of wire. This effect may be produced in several different ways, as the following experiments will show.

1. Connect the ends of a coil of wire with a sensitive galvanometer and thrust the N-pole of a magnet into the coil. A deflection will be produced. On pulling the same pole out of the coil, a deflection in the opposite direction results. Turn the coil over and repeat the operations. Every effect is just the reverse of the corresponding one produced at first. The experiment should be repeated, using the S-pole in the same manner.

2. Connect a coil of wire, Fig. 359, to the poles of a voltaic cell and thrust this coil into the coil used in Experiment 1. In every operation similar to those performed above, the effect is the same as that produced by using a magnet. Repeat with a soft iron core within the smaller coil.

MICHAEL FARADAY (1791–1867)

Faraday was one of the most distinguished chemists and physicists of the nineteenth century. He was the son of a blacksmith at Newington, near London, England, and became a bookbinder in 1804. Hearing by chance some lectures on chemistry by Sir Humphry Davy of the Royal Institution, he became interested in the subject, and in 1813 was appointed assistant in Davy's laboratory. In 1825 he became director of this laboratory, and in 1833 was made professor of chemistry in the Royal Institution for life. He died at Hampton Court in 1867.

Faraday's achievements in the domain of chemistry are largely overshadowed by his numerous and brilliant discoveries in electricity, of which the most far-reaching was that of the induction of electric currents by magnets, made in 1831. This subject has led to results of tremendous value in the commercial applications of electricity.

His further discoveries deal with the capacity of condensers, the laws of electrolysis, the rotation of the plane of polarized light by a magnetic field, and diamagnetism.

Faraday was a prolific writer and a popular lecturer on scientific subjects. He is best known by his *Experimental Researches in Chemistry and Physics.*

JOSEPH HENRY (1797–1878)

After the time of Franklin, Henry was the first in the United States to make original researches in the subject of electricity. After the invention of a practical electromagnet by Sturgeon in 1825, Henry developed a magnet, in the construction of which he employed many turns of copper wire insulated with silk. The magnet was capable of supporting over fifty times its own weight. It is believed on good evidence by many physicists that the discovery of electromagnetic induction was made by Henry at Albany, New York, in 1830, although he did not publish his results until 1832, a year later than the publication of Faraday's results.

3. Open the battery circuit used in Experiment 2, insert a key, and place one coil within the other. Complete the battery circuit by pressing the key. A deflection of the galvanometer will indicate the presence of an induced current which at once dies away. Open the key, and an induced current in the opposite direction will be obtained.

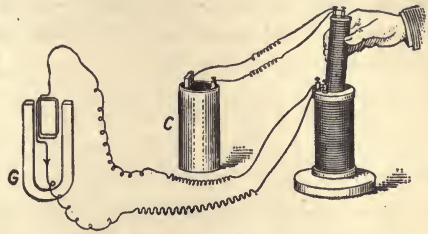


FIG. 359. — Inducing a Current by a Current.

4. If the galvanometer is sufficiently sensitive, simply turning the coil with which it is connected in the earth's magnetic field will suffice to produce an appreciable current.

It is to be observed that in each of the cases chosen in the experiments, a *change* in the number of magnetic lines through the coil is brought about in the process. In Experiment 1, the magnet carries its lines of force with it when it is thrust into the coil; in Experiment 2, the magnetic lines of force set up by the battery current in one coil are carried through the second coil when the former is thrust into the latter; in Experiment 3, the change in the number of lines is produced by alternately making and breaking the circuit which contains the battery. In every case the induced current is set up by magnetic action, *i.e.* either by an *increase* or a *decrease* in the number of magnetic lines through the coil connected with the galvanometer.

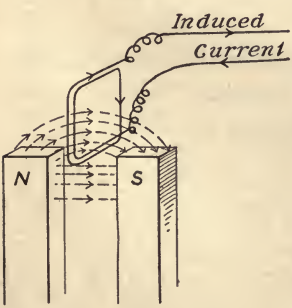


FIG. 360. — Showing the Direction of the Induced Current.

448. Direction of the Induced Current. — Lenz's Law. — Connect a small piece of zinc with one terminal of a sensitive galvanometer and a piece of copper with the other. Hold the two metals in the fingers and observe the deflection of the galvanometer. The moisture of the hand is sufficient to establish a current from the copper to the zinc through the instrument. Now repeat the experiment of § 446, and from the direction in which the galvanometer is deflected, determine the direction of the induced current in the wire.

While the coil is moving into the magnetic field placed as shown in Fig. 360, the current in the wire will be found to have the direction shown by the arrows. Thus the induced current tends to set up a magnetic field of its own whose lines of force are *opposite* in direction to those of the magnet. This fact is easily verified by applying the rule given in § 408. See Fig. 361. Hence, *increasing the magnetic lines through the coil produces a current which tends to oppose that increase.* On the other hand, moving the coil away from the magnet induces in the coil a current in the direction opposite to the former current. Hence, *a decrease in the number of lines through the coil sets up a current that tends to prevent that decrease in number.* In fact, every case of current induction may be shown to obey the following law:

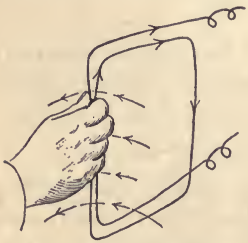


Fig. 361. — Applying Lenz's Law.

The direction of an induced current is such as to produce a magnetic field that will tend to prevent a change in the number of magnetic lines of force through the coil.

This is known as **Lenz's Law** and may be viewed as an application of the more general law of the Conservation of Energy (§ 64). From this law we know that neither electrical nor any other form of energy can be derived unless an equivalent amount of work be performed. In this case the work is done when the coil is forced into the magnetic field in opposition to the repulsion between the field of the magnet and that of the induced current.

449. The Induction Coil. — One of the most important applications of the principles of electro-magnetic induction is found in the *induction coil.* The instrument consists of a so-called *primary* coil of coarse wire *A*, Fig. 362, wound on a core of soft iron, a *secondary* coil of several thousand turns of very fine insulated wire *S*, and a current *interrupter* *I.* The terminals of the secondary coil are at *p* and *q*.

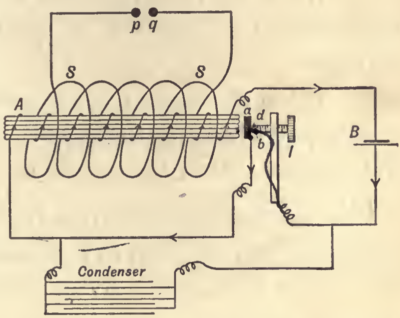


FIG. 362. — Diagram Showing the Parts of an Induction Coil.

When the current from a battery *B* flows through the primary coil, it magnetizes the iron core. The iron hammer *a* is then drawn toward the core and away from the screw *b*. This operation breaks the primary circuit at the screw point *d*. The interruption of the current at *d* causes the core to lose its magnetism and release the hammer *a*, which is restored to its original position by the spring to which it is attached. The contact at *d* is thus made again, and all the operations are repeated. It is clear that most of the lines of force set up by the battery current in the primary coil pass through each turn of the secondary. Hence, when the current is interrupted and the lines of force decrease, an E. M. F. is induced in the secondary coil. Even with small coils, the potential difference between *p* and *q* is often sufficient to cause a spark to pass across a short gap from one to the other. Again, when the primary circuit is made at *d*, the resulting increase in the number of lines of force induces a contrary E. M. F. in the secondary, but this is of a much smaller intensity than the former induced E. M. F. Lenz's law applied to the induction coil shows the following statements to hold true :

1. *When a current is started in the primary coil, a momentary current is induced in the secondary in the opposite direction.*

2. *When the current in the primary coil is interrupted, a momentary current is induced in the secondary in the same direction as that in the primary.*

The *condenser* is introduced as an accessory part of the induction coil in order to increase the rate of demagnetization of the iron core. By its aid a very quick interruption of the current in the primary coil is accomplished, and consequently the induced E. M. F. is proportionately increased. The use of an extremely large number of turns of wire in the secondary coil makes it possible to secure a sufficiently large potential difference between p and q to produce sparks many inches in length. The actual form of the instrument is shown in Fig. 363.

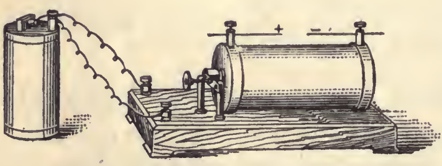


FIG. 363. — An Induction Coil.

If metallic handles that can be grasped with the hands are joined to the terminals of the secondary coil, it will be found that the shock experienced when the primary circuit is made is much lighter than at the break. When the primary circuit is made, the current requires a large fraction of a second to attain its maximum value ; consequently the rate of change of the lines of force is comparatively small. At the break of the primary, however, the current falls to zero very suddenly, thus producing a high rate of decrease in the lines of force. For this reason (§ 446) the E. M. F. is correspondingly greater at the interruption of the primary than at the making of the circuit.

The induction coil is widely used in many operations. It is used in gasoline engines to ignite the mixture of air and vapor (§ 271), for igniting explosives in blasting, and in chemical laboratories for many experimental purposes. Small induction coils are used in medicine for the remedial effect of the electric current and in the speaking part (transmitter circuit) of the modern telephone. Coils of large dimensions are employed in sending messages by wireless telegraphy and in the production of X-rays.

450. An Induced E.M.F. in a Moving Conductor. — By referring to Fig. 360 it becomes clear that in order to change the number of lines of force threading through the coil of wire, the conductor has to cut through, or across, some of these lines. The following experiment will show the effect of this operation in another way.

Hang a loose copper wire across the classroom and connect its ends with a sensitive galvanometer. If the wire be set swinging, the galvanometer will be deflected first to the right and then to the left *as the conductor cuts through the earth's lines of force.* Connect one of the galvanometer wires to the center of the swinging conductor and repeat the experiment. The induced E. M. F. will be about one half as great as before, as shown by the reduced deflection of the galvanometer.

The experiment shows that *when a conductor cuts across magnetic lines of force, an E. M. F. is induced within it.* If, for example, the conductor *ab*, Fig. 364, move from right to left, and the galvanometer be used to determine the direction of the E. M. F., it will be found that the E. M. F. acts through the conductor from *a* towards *b*; *i.e.* *b* will act temporarily like the

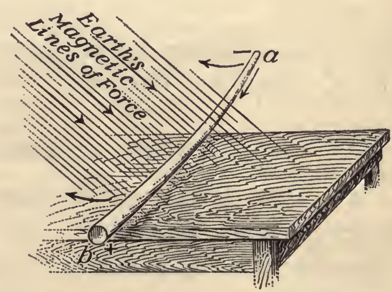


FIG. 364. — A Conductor *ab* Cutting Magnetic Lines of Force.

positive pole of a cell, and *a* like the negative pole. While the wire is swinging in the opposite direction, the lines of force are cut from the opposite side, and the induced E. M. F. is reversed. In the second part of the experiment the lines of force are cut only one half as fast, and the reduced deflection shows that the value of the E. M. F. varies with the rate at which the lines of force are cut by the moving conductor.

It is clear that the direction of the induced E. M. F. (or current) depends (1) on the direction of the lines of force and (2) on the direction of the motion of the conductor. The three quantities, viz. *Motion*, *Force*, and *Current*, have a definite relation which has given rise to the following rule:

Let the thumb and the first two fingers of the right hand be bent at right angles to each other, as in Fig. 365. *If, now, the thumb point in the direction of the* **Motion** *of the wire, and the* **First** *finger point in the direction of the lines of force in the* **Field**, *then the* **Center** *finger indicates the direction of the* **Current.**

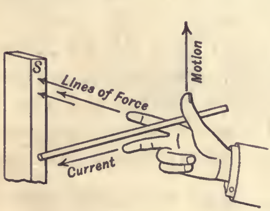


Fig. 365. — Finding the Direction of an Induced Current.

The key to the rule is found in the corresponding initial letters in the words *First* and *Field*, *Center* and *Current*.

2. DYNAMO-ELECTRIC MACHINERY

451. Principle of the Dynamo. — The laws of induced currents find their most important application in the *dynamo*, a machine which facilitates the conversion of mechanical energy into electrical. The familiar examples of electric street lighting and the operation of city and

JAMES CLERK-MAXWELL (1831–1879)

Maxwell ranks as one of the greatest of mathematical physicists on account of the important practical results to which his theories have led. He was born in Edinburgh, Scotland, where he received his early education. In 1856 he became professor of physics in Marischal College at Aberdeen, in 1860 professor of physics and astronomy in King's College of London, and in 1871 first professor of physics in Cambridge University.

Maxwell built upon the experimental discoveries of Faraday, so arranging and relating them as to make them yield to mathematical treatment. He advocated the view that electric and magnetic forces result from certain changes in the distribution of energy in the ether. He showed that electro-magnetic action must travel through space in the form of transverse waves and with the velocity of light. Later on this theory was corroborated by the experiments of Hertz, who was first to produce such waves and show that they could be reflected, refracted, and polarized like light. By these two physicists, the one attacking problems from the mathematical standpoint, the other building his experiments upon the theoretical results obtained, the intimate relation between light and electricity has been amply confirmed.

Among other subjects investigated by Maxwell was the kinetic theory of gases. The results were published with others in a memorial edition of two volumes by the Cambridge Press.

Maxwell's works also include his *Theory of Heat* (1871), *Electricity and Magnetism* (1873), and a clear and concise treatise of dynamics entitled *Matter and Motion* (1876).

interurban railroads have been brought about by the development of the dynamo.

Make a rectangular coil of 200 or 300 turns of very small copper wire of such dimensions as to rotate between the poles of a horseshoe magnet. Connect the ends with a d'Arsonval galvanometer and place the coil in the magnetic field as shown in Fig. 366. Starting with the plane of the coil at right angles to the lines of force, *i.e.* vertical, rotate it through 90.° A deflection will show the presence of an induced current. Continue the rotation through the next 90°. A deflection in the same direction will be observed. If, now, the rotation be continued, a deflection in the opposite direction will be produced until the coil has returned to its initial position.

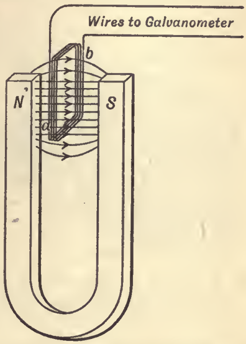


Fig. 366. — Illustrating the Dynamo Principle.

When the coil is revolved from the position shown in Fig. 367, the two portions *a* and *b* begin to cut the lines of force of the magnet and thus decrease the number of lines passing through the coil. A current is therefore set up by the induced E. M. F., which, according to the rule given in the preceding section, is upward in *a* and downward in *b*. For the first 180° of rotation, the portion *a* of the coil continues to cut the lines of force in the same direction; and the same is true of portion *b*; hence the induced current thus far is continuous and in one direction. When, however, the coil begins to revolve through the second 180°, each part of it begins to cut magnetic lines in the opposite direction; hence the induced current will accordingly flow through the wire in the opposite direction. It is now obvious that *a continuous rotation of the coil develops a current whose direction reverses*

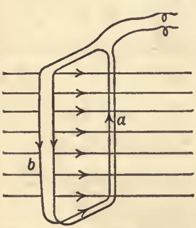


Fig. 367. — Loops of Wire Perpendicular to Lines of Force.

29

every time the plane of the coil is perpendicular to the lines of force. Such a current is called an **alternating current.**

452. The Alternating-current Dynamo. — Alternating-current dynamos, or *alternators*, are based upon the principles of induction as shown by the experiment in the preceding section. In its simplest form, the machine consists of a coil of several turns of wire arranged to rotate between the poles of a strong electro-magnet, as shown in Fig. 368. The wire is wound on a core of soft iron C and mounted on a shaft A. The revolving coil is called the **armature**, and the electro-magnet whose poles are N and S is called the **field magnet.** In order to provide for the flow of the induced current to and from the rotating coil, the ends of the wire of the armature are connected with the two insulated metal collecting rings a and a'. Stationary "brushes," b, b, rest against these rings as shown and conduct the induced current to the external circuit X. The efficiency of the machine is greatly increased by the presence of the iron core C, since it multiplies and concentrates the number of magnetic lines of force between the poles N and S.

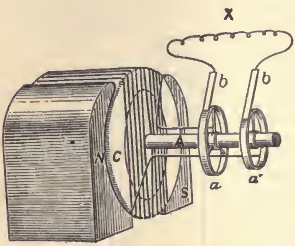


FIG. 368. — Diagram of a Simple Alternating-current Dynamo.

When the armature is rotated, an induced alternating current is set up in the coil, and through the rings and brushes is transmitted to the external circuit. The E. M. F. will be determined by the number of lines of force cut per second. This will of course be greatest when the coils are *moving at right angles to the magnetic lines, i.e.* when the plane of the coils is horizontal. The E. M. F. is zero at the instant of reversal, for at that time all portions of the coil are *moving parallel to the lines of force.*

Hence, the E. M. F. rises from zero to a maximum value and then decreases to zero again, at which time it reverses. Such a current is shown diagrammatically by the curve in Fig. 369.

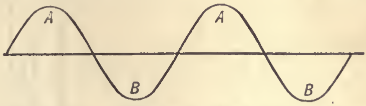


FIG. 369.—Diagram of an Alternating Current.

453. Multipolar Machines.—Alternators are of great commercial value in generating electricity for both lighting and power. They have largely replaced the so-called direct-current dynamos which have had a world-wide use. But for practical purposes it is desirable that the reversals of an alternating current attain, or even exceed, 120 per second. This, of course, cannot be accomplished with the two-pole machine described in the preceding section. In the multipolar machine there are several poles arranged around the armature as shown in Fig. 370. In this machine there are as many coils of wire in the armature as there are poles in the field magnets.

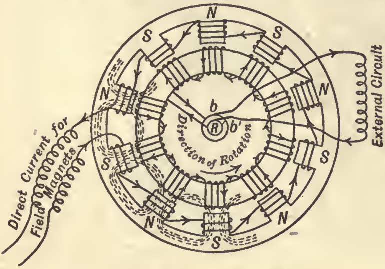


FIG. 370.—Diagram of a Multipolar Machine.

As the moving coils cut through the lines of force, an induced E. M. F. is set up in each. The several coils are so connected that the E. M. F. at the brushes is the sum of that produced in the separate coils. The field magnets are excited by means of a continuous direct current from a small dynamo (§ 454). Figure 371 shows a type of multipolar alternator which is in common use.

454. The Direct-current Dynamo.—The alternating-current dynamo described in § 452 may be employed to produce a unidirectional current by the introduction of a so-called **commutator.** The commutator in this case consists of a metal ring divided into two semicircular parts

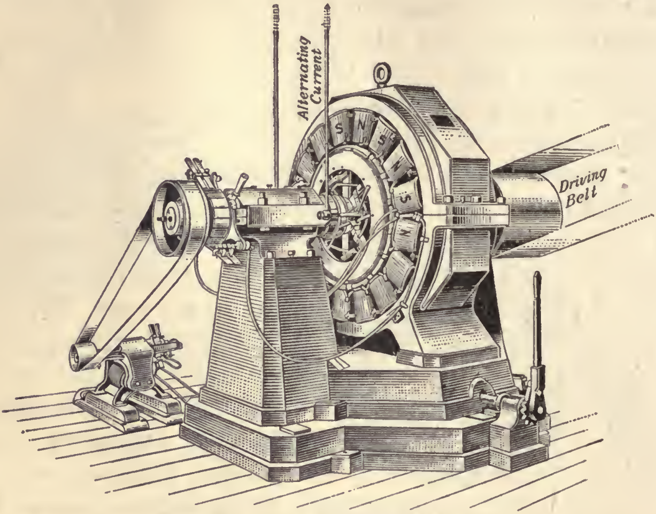


Fig. 371. — A Multipolar Alternator.

a and *a′*, Fig. 372, called *segments.* These are insulated from each other and mounted on the shaft which carries the armature. Each of the two ends of the armature coil connects with a segment of the commutator as shown in the figure. The brushes *b* and *b′* which conduct the induced current to and from the external circuit *X* are set on opposite sides of the commutator. Their position is very important. They must be so placed that *each brush changes its point of contact from one segment to the other at the instant the current reverses in the armature coil.* Thus *b*, for example, will continually be in contact

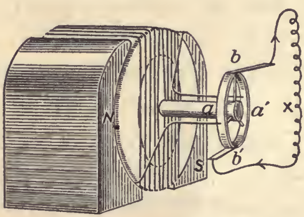


Fig. 372. — Diagram of a Direct-current Dynamo.

with a positively charged segment, and b' with a negatively charged one. Hence, in this case, the current will always flow into the external circuit through b and return to the armature through b'. Such a current is *pulsating* in nature, since the E. M. F. falls to zero twice during each revolution (§ 452). A comparison of the alternating current in the armature with the pulsating current taken off at the brushes is made in Fig. 373.

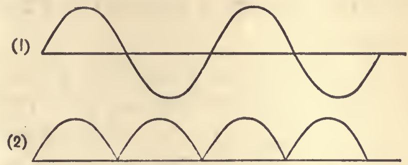


FIG. 373. — (*1*) Diagram of the Armature Current. (*2*) Diagram of the Current Taken from the Brushes.

455. Dynamos for Steady Currents. — The pulsating currents produced by dynamos of one coil (Fig. 373) are unsatisfactory for most purposes. The difficulty may be overcome and a continuous current developed by the use of several coils distributed uniformly over the armature. The first armature wound in this manner was the so-called **Gramme ring**, invented by a Frenchman in 1870. The core of the armature is an iron ring so mounted on an axle as to turn between the poles, N and S, Fig. 374, of a strong electro-magnet. The ring is wound with several coils of copper wire placed at equal distances. These are represented in the figure by the single turns numbered from *1* to *12*. At each junction of two adjacent coils a connection is made with a segment of the commutator C, which consists of as many insulated bars as there are coils in the armature. The lines of force

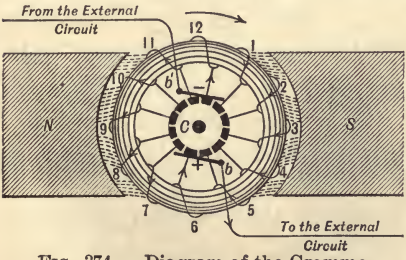


FIG. 374. — Diagram of the Gramme Ring Armature.

follow through the iron of the ring from *N* to *S*, as shown by the dotted lines; hence the outer portions of each coil are the only ones that cut the lines when the armature revolves.

If, now, the armature is revolved in the direction shown by the arrow at the top, the conductors from *1* to *5* cut through the lines in a downward direction; hence the E. M. F. throughout these coils is in the direction shown by the arrowheads on the wire (§ 450). At the same time the conductors from *7* to *11* are cutting the lines in an upward direction, which develops an E. M. F. within them as shown by the affixed arrows. An inspection of the figure will show that upon both the right and the left side of the armature the tendency is to raise the lowest segment of the commutator to a high potential and to reduce the topmost one to a low potential. Therefore, by placing the brushes *b* and *b′* in contact with these points, a direct current is led through the external circuit in the direction shown in the figure. The E. M. F. produced by such an armature is practically constant when the rate of rotation is uniform.

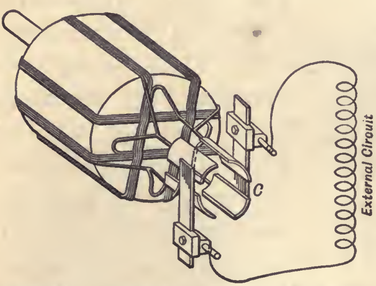


FIG. 375. — Diagram of the Drum Armature.

456. The Drum Armature. — The modern direct-current dynamo is provided with an armature of the "drum" type. This consists of a cylindrical core, or drum, of iron upon which are wound numerous coils of wire equally spaced. The construction is made clear by a study of Fig. 375. If the windings are traced, the coils will be found to be joined in series, and at four points connections are made with the segments of the commutator C. If the armature is now rotated between the poles of a magnet, the E. M. F. will at no time be zero at the brushes; for at every instant some of the conductors are cutting magnetic lines. By setting the brushes at the proper points, a direct and fairly steady current will be transmitted to the external circuit.

457. Field Magnets. — The magnetic poles between which the armature of a dynamo rotates receive their excitation from coils of wire carrying an electric current. In direct-current machines this current is produced by the dynamo itself. The initial current developed in the armature depends upon the residual magnetism (§ 379) of the pole pieces, which serves to induce a small current. This in turn increases the magnetism until the poles finally reach their full strength.

In the *series-wound* dynamo, (1), Fig. 376, the entire current is led from the brushes through the few turns of

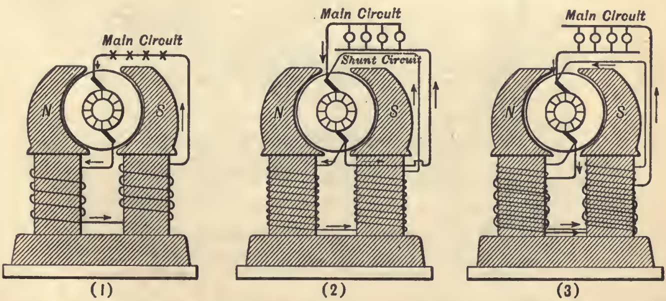


FIG. 376. — (*1*) Series-wound Dynamo. (*2*) Shunt-wound Dynamo. (*3*) Compound-wound Dynamo.

thick wire of the field magnets, which are joined *in series with the external circuit* as shown.

In the *shunt-wound* dynamo, (2), Fig. 376, only a portion of the entire current is led through the many turns of rather fine wire in the coils of the field magnet, while the main portion is conducted through the external circuit. The field magnet coils thus form a *shunt* (§ 445) to the external circuit.

In the *compound-wound* dynamo, (3), Fig. 376, the field magnets are wound with two coils, one being joined in series with the external circuit, and the other connected as a shunt between the brushes. A compound-wound machine adjusts itself to variations in the resistance of the external circuit in such a way as to maintain a constant difference of potential between the brushes.

458. The Acyclic, or Unipolar, Generator. — It is of interest to note that the alternations of the current induced in the armature of a dynamo can be prevented by employing a mechanism of the proper form. Figure 377 shows a sketch of a so-called *acyclic*, or *unipolar*, generator, which

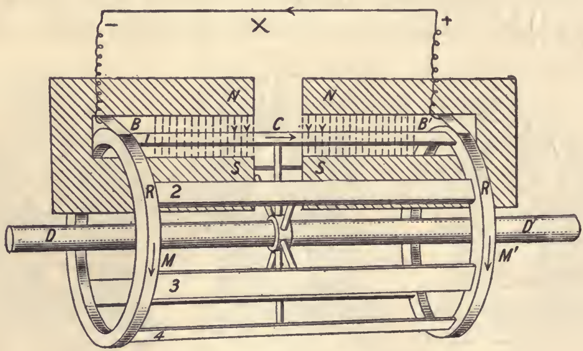


FIG. 377. — Diagram of an Acyclic Generator.

consists of two collecting rings R and R' joined together by conducting bars *1*, *2*, *3*, etc. These bars are attached at their centers to a revolving shaft D, but are insulated from it. Magnets are placed with their poles as shown at N and S. When the armature is revolved in the direction shown by the arrows *MM'*, conductor *1* cuts through the field of the magnets, and an E. M. F. is induced in it (§ 450) in the direction shown by the arrow at *C*. Hence a current may be taken from the brush at *B'*, which rests continually against the collecting ring, through the external circuit *X* back to the brush *B*. Since each conductor cuts the magnetic lines of force in the same direction, *B'* is always the positive brush and *B* the negative, and hence a direct current flows through both the internal and external parts of the circuit.

In the commercial form of this type, Fig. 378, the magnetic field extends completely around the armature and is

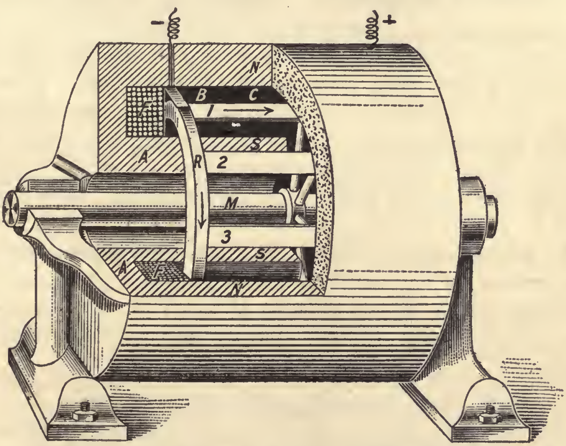


Fig. 378. — An Acyclic Generator.

excited by means of a current in the field coils *FF* which lie in concentric circles around the cylindrical steel core *AA*. The armature conductors *1*, *2*, *3*, etc., are attached to the periphery of a steel cylinder from which they are insulated.

Unipolar generators are designed to run at a high speed, and a desired E. M. F. is obtained by arranging several independent sets of conductors and their corresponding collecting rings in the armature of the machine and joining their brushes in series. In this manner the E. M. F.s of the separate sets are added together. The greatest advantages of this form of generator over that of the commutator type are the elimination of commutator difficulties and lower cost of construction.

EXERCISES

1. In order to show that a current of electricity is produced when a magnet is thrust into a coil of wire, why is it usually necessary to employ a coil of many turns?

2. Account for the enormous difference of potential induced between the terminals of the secondary of an induction coil?

3. Connect one terminal of a spark coil (induction coil) to the outer coating of a Leyden jar and bring the other close to the knob. Put the coil in operation and show that the jar becomes charged. Explain how the charge can "jump" into the jar, but cannot escape.

4. How many revolutions per minute would have to be made by a two-pole alternator to produce 120 alternations of the current per second?

5. An alternator has 16 poles. How many alternations per second will be produced when the speed is 375 revolutions per minute?

6. What will be the effect upon the E. M. F. of a shunt-wound dynamo of introducing resistance into the field magnet circuit?

Suggestion. — Consider the effect of the added resistance on the current in the field coils and also on the magnetic field.

7. Does this suggest a way in which the E. M. F. of a shunt-wound machine can be regulated?

3. TRANSFORMATION OF POWER AND ITS APPLICATIONS

459. The Electric Motor. — Electrical energy is transformed into mechanical by means of electric motors. The direct-current motor does not differ greatly in construction from the dynamo. The principle underlying its action is illustrated by the following experiment.

Suspend a wire on the apparatus described in § 412 so that its lower end just dips into mercury. Hold a horseshoe magnet as shown in Fig. 379 and send a current from a new dry cell through the wire. The wire will be found to move at right angles to the lines of force, as shown by the arrow. If the current is now reversed, the wire will move in the opposite direction.

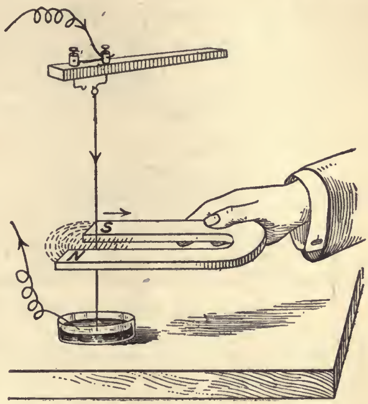


FIG. 379. — Movement of a Conductor in a Magnetic Field.

The experiment shows clearly that *a conductor in which a current is flowing tends to move in a direction at right angles to the lines of force of a magnetic field in which it is placed.* The motion may be determined by applying the rule of § 450, but by using the *left* hand instead of the right.

Let this principle be applied to Fig. 374. If a current from some source be sent through the armature from b' to b, the current through the coils will take the direction indicated by the arrows, dividing as it leaves b'. According to the principle shown in the experiment, all the conductors from *1* to *5* will be urged in an upward direction, and those from *7* to *11*, downward. Since the current employed by the motor flows through the field magnet coils, a powerful magnetic field is produced in which the armature will rotate with sufficient power to turn the wheels of factories and propel electric cars, launches, automobiles, etc.

460. The Electric Railway Car. — A familiar application of the electric motor for the generation of mechanical power is found in the electric railway, Fig. 380. A current from the generator at the power house is transmitted through the trolley wire, or in some cases through a third rail, and by means of a metallic arm A to the motors placed under the floor of the car at M. The axles of the car are so geared to those of the motors that *the car is propelled by the rotation of the motor armatures.*

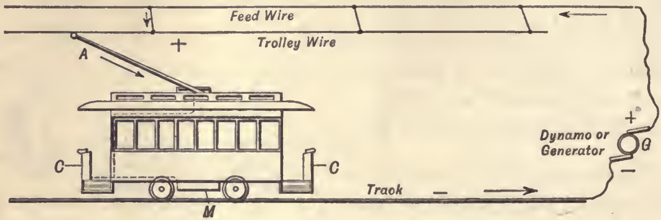


FIG. 380. — Diagram of an Electric Railway System.

From the motors the current returns to the power house through the track, the rails being carefully "bonded" together with copper conductors. In circuit with the motors is placed the *controller* C, operated by the motorman. By means of a series of resistance coils connected with the controller, the current can be increased or diminished and the speed of the car thus regulated. Another device enables the motorman to reverse the motors. A third accessory serves in applying the air brakes (§ 159) to check the speed after the current has been completely interrupted at the controller.

461. The Alternating-current Transformer. — Alternating currents owe their extensive application to the fact that the potential difference between two points can be easily reduced from a dangerous one of many thousand volts to one of a safe value for dwelling-house use and for many other purposes. This is accomplished by means of a *transformer*, which is simply a modified induction coil.

The principle involved in the transformer is easily understood from a study of Fig. 381. An iron core R is wound with two independent coils of wire P and S.

Let an alternating current be sent through the primary coil P, and let the secondary be connected with a group of lamps L. The current in P magnetizes the iron core in one direction, then demagnetizes and remagnetizes it again in the opposite direction, while the magnetic lines follow the iron core through the secondary coil S. As a result of these magnetic changes in the core an alternating current is induced in S and flows through the lamps.

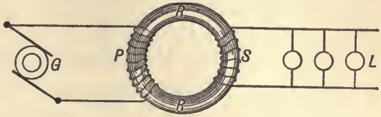


FIG. 381. — Diagram of a Transformer.

If there are more turns of wire on the secondary coil S than on the primary coil P, the potential difference at the terminals of S will be greater than at P, because of the larger number of loops of wire in which the magnetic changes occur. In this case the transformer is called a *step-up* transformer. If, however, the secondary contains the fewer turns, its potential difference is lower than that of the primary, and the transformer is a *step-down* transformer. *The ratio of the two potential differences is equal to the ratio of the number of turns of wire in the two coils.* For example, a transformer that is to be used to reduce a voltage of 2000 to a lower one of 100 volts would be constructed by winding the primary coil with 20 times as many turns as the secondary. The same transformer would reduce a potential difference of 2200 volts to one of 110 volts.

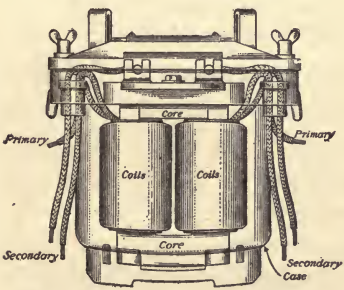


FIG. 382. — A Commercial Transformer.

462. Utility of the Transformer. — The value of the transforming process is made clear by the study of a specific case. For example, electric power is to be transferred from a power station to a large city over several miles of wire. It is desired that the number of amperes (*C* in Eq. **8**, § 437) be small in order to reduce to a minimum the loss of energy due to the heating of the conducting wires. The alternator used at the power house develops a potential difference of 2000 volts. This current is led through the primary coil of a "step-up"

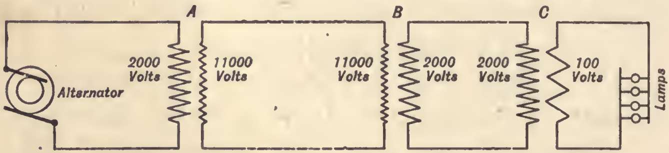


FIG. 383. — The Transformer System of a Long Distance Power Circuit.

transformer *A*, Fig. 383, where the potential difference is raised to 11,000 volts, while the number of amperes becomes proportionately reduced. At this voltage the current would be about 7 amperes per 100 horse power. Since wires of so great a potential difference are unsafe to lead into houses, the voltage is reduced from 11,000 to 2000 volts, by a "step-down" transformer *B* where the line enters the city, and again at the houses, from 2000 to 100 volts by the transformer *C*.

Since the power transmitted by an electric current is the product of the number of volts and amperes (Eq. **7**, § 436), a current of 10 amperes under a potential difference of 11,000 volts, for example, delivers a power of 110,000 watts, which is about 147.5 horse power. It is plain, therefore, that a large amount of power under a high voltage can be transferred by a small current. The power generated at Niagara Falls is distributed to Buffalo, Syracuse, and other places with potential differences of from 22,000 to 60,000 volts.

Again, if the transformer *C*, Fig. 383, deliver a current of 5 amperes under a potential difference of 100 volts for lighting a building, the power delivered is 5×100, or 500 watts. In the long distance circuit between A and B where the voltage is 11,000, the current would be $500 \div 11,000$, or 0.045 ampere. Hence, to take 5 amperes at 100 volts would increase the current in the long line only 0.045 ampere.

The heat losses in long distance power transmission render it impracticable to convey large currents, as shown in § 438. But the examples above show that by raising the potential difference to several thousand volts the current is proportionately reduced; consequently a given power can be transferred with far less heat loss, since the heat generated is proportional to the square of the current. It is in this manner that the transformer has solved the problem of long distance transmission of power from places where power is comparatively cheap to distant manufacturing centers where it would be expensive. The distribution of power over long electric railway lines is accomplished by means of so-called high tension, *i.e.* high potential, apparatus.

The alternating current transformer affords one of the most striking examples of the transformation and transference of energy to be found. The power in one circuit is transferred to another *entirely without any mechanical connection between the two.* It is necessary only that the magnetic lines set up by the current in the primary coil pass through the secondary. No mechanical motion is concerned in the process. The power loss in a good transformer is usually not more than 3 or 4 per cent.

463. Incandescent Lighting. — It is a familiar fact that the heating effect of an electric current is employed in the process of electric lighting. The simplest case to study is the incandescent lamp. See Fig. 384. In this lamp the current is sent through a carbon filament C, which is heated to incandescence. In order to prevent the carbon from burning, as well as to prevent loss of heat by convection, it is inclosed in a highly exhausted glass

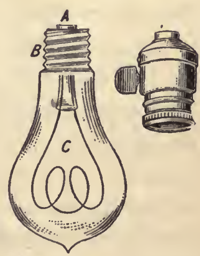


Fig. 384. — An Incandescent Lamp with Carbon Filament and Socket.

bulb. Connections are made with the ends of the filament by means of two short pieces of platinum wire sealed in the glass. One of these leads to the contact *A* in the center of the base, the other to the brass rim *B* which holds the lamp in its socket *S*. Through these the current is transmitted to and from the lamp.

On account of its large consumption of power (about 3.5 watts per candle power), many efforts have been made to produce lamps of higher efficiency. At the present time the carbon filament lamp is being rapidly replaced by those provided with metallic filaments of the rather uncommon metals tantalum or tungsten. These metals admit of being drawn out into very thin wires which can be heated white-hot without melting. These thin filaments are mounted in glass bulbs in much the same manner as the carbon filaments. Not only is the light which metallic filament lamps emit whiter than that given off by the carbon filaments, but the efficiency is far greater. In the tungsten lamp the consumption is about 1.25 watts per candle power.

Ordinarily the potential difference on a lamp circuit is maintained at 110 or 220 volts. Lamps are adapted to the voltage of the circuit on which they are to be used. A 16-candle-power lamp with a carbon filament requires a current of slightly more than 0.5 ampere when the voltage is 110 and about 0.25 ampere when the voltage is 220. The power necessary is, by equation (7), § 436, 110×0.5, or 55 watts.

The Nernst lamp employs a short rod, or "glower," Fig. 385, composed of oxides which are maintained at a high temperature by the passage of a current of electricity. Although the glower is a nonconductor at ordinary temperatures, it becomes a conductor when heated. The glower is mounted close to a heater coil of fine platinum wire through which a current passes when the lamp is turned on.

As soon as the glower becomes sufficiently hot, the heater coils are automatically thrown out of circuit, and the current then flows only through the glower. Since the glower is incombustible, it can be used without being inclosed. The efficiency of the Nernst lamp is considerably below 2 watts per candle power.

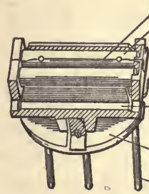


FIG. 385. — Showing Parts of a Nernst Lamp.

464. Incandescent Lamp Circuits. — Incandescent lamps are connected in parallel between the two main wires leading into a building. These wires are maintained by the dynamo D, Fig. 386, at a potential difference of 110 volts, so that any lamp may be turned on or off without interfering with the others. Each 16-candle-power lamp requires a current of half an ampere and, consequently, has a resistance when hot of 220 ohms. Lamps are often joined in groups, as shown at A. In this case the switch placed at S controls all the lamps of the group.

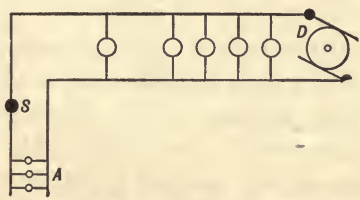


FIG. 386. — Showing the Connection of Incandescent Lamps in a Circuit.

465. Cost of Electric Power. — Electrical energy is sold at a certain rate per **watt-hour**. *A watt-hour is a volt-ampere-hour;* in other words, an ampere of current flowing under a potential difference of one volt for one hour delivers a watt-hour of energy. Hence a 110-volt lamp carrying 0.5 ampere requires $110 \times 0.5 \times 1$, or 55 watt-hours for every hour it is used. In commercial lighting a meter which is designed to register the consumption of energy in *kilowatt-hours* is placed in the circuit at the point where the wires enter each consumer's house. Thus readings of the meter show the amount of electrical energy for which the user is charged.

30

466. The Electric Arc. — The first electric arc was exhibited in 1809 by the great English scientist, Sir Humphry Davy. For this purpose Davy employed over 2000 voltaic cells joined to two pieces of charcoal which were touched and then slightly separated. The same experiment may be easily made on any commercial lighting circuit.

Wind bare copper wire, about No. 18, around pieces of electric light carbons. Join these in series with a resistance of about 10 ohms of iron wire to the terminals of a 110-volt lighting circuit. The resistance may be made by winding 150 to 200 feet of No. 18 or 19 iron wire on a suitable frame. Touch the tips of the carbons together and at once separate them about a quarter of an inch. An intensely bright light will be produced as the current continues to flow across the gap.

By the separation of the carbon rods, a high temperature is produced by the current, which vaporizes some of the carbon, forming a conducting layer from one to the other. The resistance of this mass of vapor may not be more than 3 or 4 ohms. If the current is a direct one, the temperature of the positive carbon rises above that of the negative, and from it comes the greater portion of the light. In this case the positive carbon is consumed about twice as fast as the negative and becomes hollowed out, as in Fig. 387, while the negative remains pointed. When an arc is produced by an alternating current, light is given out equally from the two points, and the two rods are consumed at the same rate.

FIG. 387. — The Electric Arc Produced by a Direct Current.

467. Arc Lamps. — With the development of the dynamo, the arc lamp as a means of illumination has come into extensive use. In the so-called hand-feed lamp, Fig. 388, the positions of the carbons are controlled by the screw heads *S*. They are at first permitted to touch and then are separated. As fast as the rods are consumed, they are slowly fed together by the operator. Such lamps are used mainly in projection lanterns (§ 324). In the arc lamp of automatic feed, Fig. 389, the mechanism has two duties to perform: (1) to separate the carbons when the current starts, and (2) to feed the carbons together as fast as they are consumed, always keeping the proper space between them.

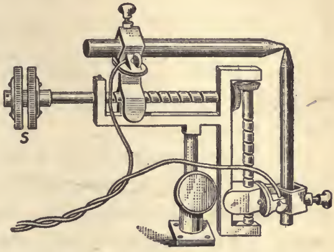


FIG. 388.—The Hand-feed Electric Lamp.

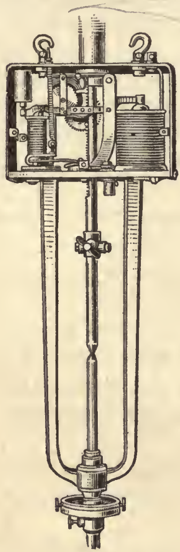


FIG. 389.—The Automatic Arc Lamp.

The consumption of the carbon rods in the arc lamp can be largely reduced by inclosing the arc in a globe that is nearly air-tight as shown in Fig. 390. In this form of lamp the carbon burns from 60 to 100 hours.

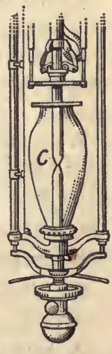


FIG. 390.—Inclosed Electric Arc Lamp.

In the lamps just described the light is emitted by the incandescent ends of the carbons. If the carbons, however, are cored with a mixture of carbon and metallic salts, a highly luminous vapor is maintained by the current between the two terminals. In *flaming arc lamps* the carbons are cored with calcium salts which serve to give the light a bright yellow color.

The open arc is operated by a current of from 5 to 10 amperes and 45 to 50 volts, and the candle power in the direction of greatest intensity is about 1000. The inclosed arc lamp requires a voltage of about 80 and a current of from 5 to 8 amperes.

EXERCISES

1. According to Lenz's law, in what direction in a circuit will a current be induced by the sudden interruption of a current in a neighboring conductor?

2. If the experiment of § 450 be repeated with a long loop of wire, it will be found that no deflection of the galvanometer will be produced by swinging both parts of the loops together across the earth's lines of force. Explain. By swinging the two parts of the loop in opposite directions, a large deflection is obtained. Why?

3. Would you class the induction coil as a "step-up" or a "step-down" transformer?

4. Explain why the interrupter is an accessory part of the induction coil but not of the transformer.

5. A transformer carries a current of 10 amperes in its primary coil, under a potential difference at its terminals of 2000 volts. If it delivers a current of 194 amperes with a potential difference of 100 volts, how much energy is transformed per hour, how much delivered, and how much wasted?

6. If a building contained 50 110-volt incandescent lamps, what voltage would have to be supplied to operate them if they were all joined in series? Show that this would be impracticable.

7. Show by a diagram the manner of connecting ten 16-candle-power 110-volt incandescent lamps in parallel. What would have to be the voltage and how much current would be required?

SUGGESTION.— Consider that in the case of parallel conductors the total current is the sum of that in the separate parts.

8. Why would the alternating-current transformer be entirely ineffective in transforming a continuous current?

9. Compute the monthly cost of operating five 16-candle-power incandescent lamps when current costs 10 ct. per kilowatt-hour, allowing an average use of 3 hr. per day and $3\frac{1}{2}$ watts per candle power.

10. Compute the monthly cost of current for an open arc lamp requiring 8 amperes at 50 volts, allowing 3 hr. per day and 10 ct. per kilowatt-hour.

11. If the potential difference between the trolley of an electric railway and the track is 550 volts, show how it would be possible to operate 110-volt incandescent lamps by properly connecting them together. Diagram the system.

12. Show how a workman standing on the top of an electric car can safely handle the trolley wire with bare hands, while one standing on the ground would be severely shocked by coming in contact with any wire forming a connection with the trolley wire.

13. 25 street lamps each operated by a potential difference at its terminals of 45 volts are joined in series. What potential difference must be maintained at the terminals of the dynamo? What would have to be the current strength? Disregard the line loss.

14. What has been the effect of economical long distance power transmission upon manufacturing industries, railway development, etc.?

4. THE TELEGRAPH AND THE TELEPHONE

468. The Morse Telegraph. — The most extensive use of the magnetic effect of electric currents is made in the telegraph systems in common use. In 1831 Joseph Henry produced audible signals at a distance, but the system generally employed in this country is that designed by Samuel F. B. Morse and first used in 1844. The instruments found at each station are the key, sounder, and relay.

The *key*, Fig. 391, is merely a convenient device for making and breaking an electric circuit at A by operating the lever L. It is also provided with a switch S, so that the circuit may be left closed when the key is not in use.

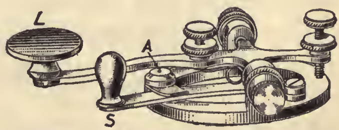


FIG. 391. — Telegraph Key.

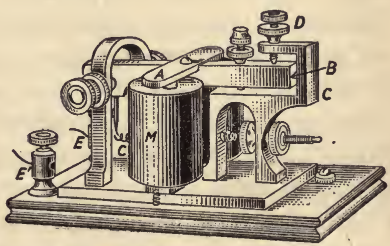


FIG. 392. — Telegraph Sounder.

The *sounder*, Fig. 392, consists of an electro-magnet M which attracts the iron armature A whenever a current is sent through it, thus causing the heavy brass bar B to be drawn down against C with a sharp click. When the current is interrupted, the armature is no longer attracted, and the bar is lifted against D by an adjustable spring. Transmitted messages are read by ear from the clicks of this instrument.

469. Plan of a Short Telegraph Line. — Short lines in which the resistance is small require at each station only the key and sounder, as in Fig. 393. The connection is

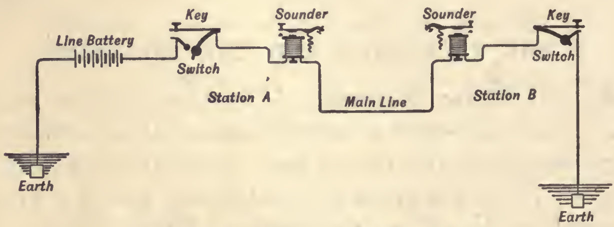


FIG. 393. — A Short Telegraph Line.

usually made by a single wire, the circuit being completed by joining a wire at each end to a metal pipe or to a metal plate buried in the ground. Thus the earth forms a part of the circuit. The battery may be placed anywhere in the line.

When the operator at *Station A* wishes to send a message to *Station B*, he opens the switch on his key, which breaks the circuit. The sounder cores are thus demagnetized, and the bars are thrown up by the springs. Now, by means of the key, the operator A can make and break the circuit which will cause both sounders to click off the "dots" and "dashes" composing the message. A *dot* is produced by a quick stroke of the key which closes the circuit for only an instant; a *dash* is a slower stroke which leaves the circuit closed for a slightly longer time.

The operator at *B*, skilled in the interpretation of the clicks of the sounder, reads and records the message. Since a telegraph circuit is left closed when not in use, a "closed circuit," cell like the Daniell or gravity must be used.

The Morse code given below is composed entirely of dots, dashes, and spaces. A small space is left between letters and a slightly longer one between words.

THE MORSE TELEGRAPH CODE

a - —	h - - - -	o - -	u - - —
b — - - -	i - -	p - - - - -	v - - - —
c - - -	j — - — -	q - - — -	w - — —
d — - -	k — - —	r - - -	x - — - -
e -	l ——	s - - -	y - - - -
f - — -	m — —	t —	z - - - -
g — — -	n — -		

470. The Relay and Its Use. — In long telegraph lines on which there are many instruments, the resistance is usually so great that the current in the main line is too feeble to operate the sounders with sufficient loudness. The difficulty is avoided by the use of the *relay*,

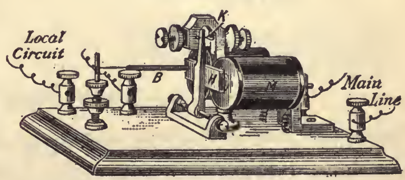


FIG. 394. — The Telegraph Relay.

which is a more sensitive instrument than the sounder. The *relay*, Fig. 394, consists of an electro-magnet *M* containing several thousand turns of wire (about 150 ohms) which is placed in the main line at each station together with the key. The armature and bar *H* of the relay are made very light, and all the adjustments of the instrument may be made with great precision. The function of the relay is to open and close at *K* a *local circuit* which contains merely the sounder and two or three cells. It will readily be seen that since there is little resistance to reduce the current, very distinct clicks will be produced on the sounder by the current from this local battery every time the relay automatically closes and opens the local circuit.

471. The Long Distance Telegraph System. — The usual arrangement of the parts of a telegraph system is shown diagrammatically in Fig. 395. In sending a message from

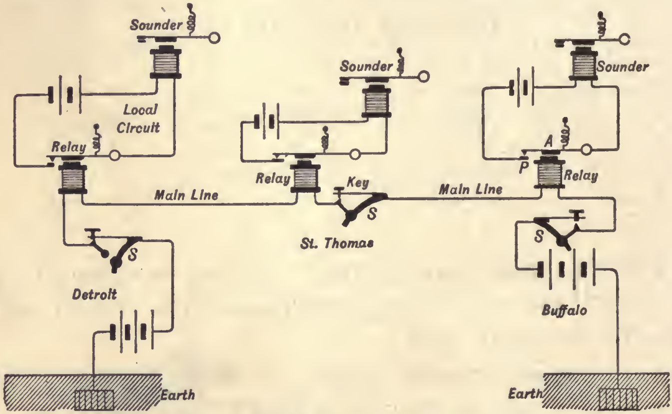


Fig. 395. — A Long Distance Telegraph System.

Detroit to Buffalo, for example, the Detroit operator uses precisely the method described in § 469. When the circuit is opened at Detroit, no current flows from the main line battery, and all the relays on the line release their armatures and thus open every local circuit. When the Detroit operator presses his key, the main line battery sends a current out over the line, and the electro-magnets of the relays draw the armatures down and thus close the local circuits, causing every sounder to produce a sharp click. Thus the dots and dashes comprising a message may be read by every operator along the line.

The relay may also be used to repeat a message to another line instead of transmitting it to a short local circuit. A message from Chicago to New York may be repeated to a Detroit-Buffalo line at Detroit, to a Buffalo-Syracuse line at Buffalo, and finally to a Syracuse-New York line at Syracuse. Since the repeating is performed automatically at each of these stations, no time is lost in the transfer from one line to another. The relay when used in this manner is called a *repeater*.

472. The Telephone. — The simplest manner in which speech produced at one station can be reproduced by electrical means at another is by means of two telephone

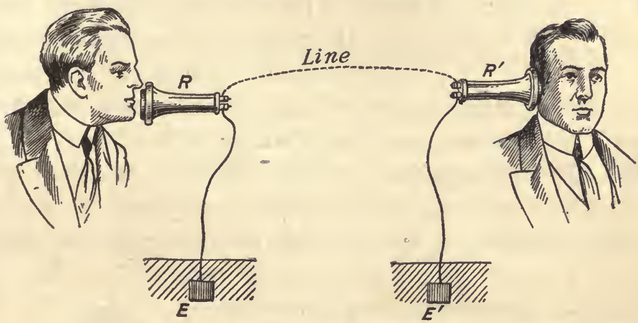


Fig. 396. — A Simple Telephone System.

"receivers" connected by two wires, or by one wire and the earth. See Fig. 396. The telephone receiver was invented in 1876 almost simultaneously by Alexander Graham Bell and Elisha Gray, both Americans. It consists simply of a permanent bar magnet *M*, Fig. 397, surrounded at the end by a coil of fine insulated copper wire *C*. An iron disk *D* is mounted so as to vibrate freely close to the end of the magnet.

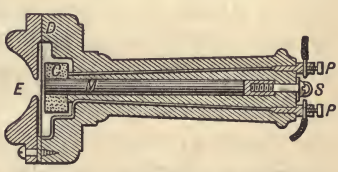


Fig. 397. — Section of a Telephone Receiver.

If a person speaks into the receiver, the sound waves set the disk in vibration. Each vibration of the disk changes the number of magnetic lines of force through the coils of wire and thus induces a current whose nature depends entirely upon the loudness, pitch, and quality of the sound. In this manner a pulsating current is sent over the line to a similar instrument at the distant station.

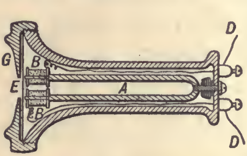


FIG. 398.—A Recent Type of Receiver.

When the current generated at the first station flows in such a direction as to strengthen the magnet at the second one, the disk is drawn in; when it flows in the opposite direction, the magnet is weakened and the disk released. Thus the vibrations at the first station are reproduced on the instrument at the second. A modern receiver is shown in Fig. 398.

473. The Transmitter.—The telephone receiver just described is not sufficiently powerful when used as a *transmitter*, or sender, of speech; but it is a *receiver*, or reproducer, of sound of extremely great sensibility. For this reason the transmitters in general use are based on an entirely different principle from that of the receiver. The modern form used in long-distance telephony consists of two carbon buttons c and c', Fig. 399, between which are carbon granules g. The metal disk, or diaphragm, D is attached to the button c. When the disk is set in vibration by the sound waves, the variation of pressure against the carbon granules causes large variations in the electrical resistance between the buttons. Hence, if such an instrument be placed in a battery circuit and so connected that the current passes through the

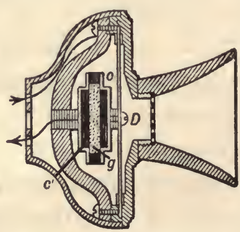


FIG. 399.—The Transmitter.

granules from c to c', *the strength of the current changes precisely in accordance with the vibrations of the diaphragm.*

474. A Long Distance Telephone System. — In all telephones operating with a local battery, the connections are made as shown in Fig. 400. The current from the local

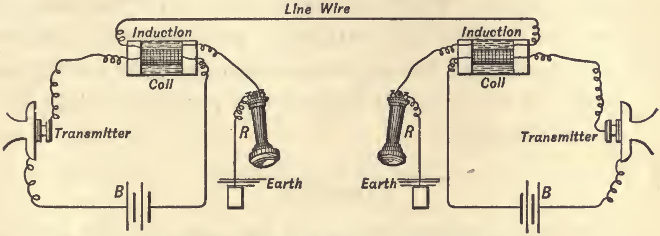


Fig. 400. — A Long Distance Telephone System.

battery B is led through the transmitter and the primary of a small induction coil back to the battery. The main line contains at each station the secondary of the induction coil and the receiver. Two wires are generally used to connect the two stations, although one may be replaced by the earth.

When a person speaks into the transmitter, the vibration of the diaphragm changes the pressure at the contact points of the carbon granules which conduct the current flowing through the primary coil. When the diaphragm is forced in, the resistance of the transmitter is lowered, and a comparatively large current flows; when it moves outward, the current is reduced. *These changes in the primary coil induce currents in the secondary which pass out over the line and set up vibrations in the receiver at the distant station.*

For the purpose of calling attention, an electric bell is placed at each station. When the receiver is lifted from the hook upon which it hangs, the bells are disconnected from the line, and the connections are made as shown in

Fig. 400. The downward motion of the hook restores the bell to the line when the receiver is hung up.

In cities and villages the telephones are all connected with a central exchange, where the operator upon request connects the line from any instrument with the line leading to any other. Such exchanges can now be found in even the small towns and will serve to show to the student of electricity one of the ways in which the study of physical principles has contributed to the prosperity and convenience of mankind.

EXERCISES

1. Compute the resistance of an iron telegraph line 150 mi. long, the size of the wire being 0.1 in., the line containing also 10 relays of 150 ohms each. Allow nothing for the earth connections.

2. A telegraph wire offers a resistance of 35 ohms per mile. If the line contains five 150-ohm instruments, what current will be produced by 30 Daniell cells of 2 ohms each in a line 80 mi. long? Do you think this current would operate a sounder?

3. Connect a telephone receiver with the terminals of a very sensitive galvanometer and press in on the diaphragm. A deflection will be produced. Why? Release the diaphragm. A contrary current will be obtained. Why?

4. If the telephone receiver can be used as a transmitter and requires no battery in its operation, why is it not so used?

SUMMARY

1. Currents of electricity may be produced by *induction* by increasing or decreasing the number of magnetic lines of force which thread through a coil of wire if it is in a "closed circuit." The induced E. M. F. is proportional to the rate of change in the number of lines (§§ 446 and 447).

2. The direction of an induced current is always such as to produce a field that tends to prevent a change in the number of lines of force through the coil (§ 448).

3. When an electrical conductor cuts magnetic lines of force, an E. M. F. is induced within it (§ 450).

4. Currents of electricity are produced on a large scale by making use of the *dynamo*, which depends for its action on the principles of electro-magnetic induction. Dynamos are *alternators* or *direct-current* dynamos according as they produce alternating or direct currents (§§ 451 to 458).

5. Dynamos are *series-wound*, *shunt-wound*, or *compound-wound*, depending on the manner in which the field cores are excited (§ 457).

6. The direct-current *motor* depends upon the tendency of a conductor carrying an electric current to move in a magnetic field. Motors are used wherever electric power is to be converted into mechanical power for moving machinery (§ 459).

7. Alternating-current *transformers* are used to convert alternating currents of low potential into alternating currents of high potential and *vice versa*. By their use unsafe currents of many thousand volts can be reduced to safe ones for domestic and commercial use (§§ 461 to 464).

8. Electric power is sold by the *kilowatt-hour*. The number of kilowatt-hours consumed is given by the equation

$$\textit{kilowatt-hours} = 0.001\ EC \times \textit{hours} \quad (\S\ 465).$$

9. The telegraph and telephone employ the magnetic effect of the electric current in transmitting messages (§§ 468 to 474).

CHAPTER XXI

RADIATIONS

1. ELECTRO-MAGNETIC WAVES

475. An Electrical Discharge is Oscillatory. — When an electric spark jumps across a short gap, it appears to be only a single flash. The eye is incapable of determining whether or not this is actually the case. The following experiment may be used to investigate the nature of such a discharge.

Bend 2 or 3 feet of wire into a hoop R, Fig. 401, leaving a gap of about 1 millimeter at S. Connect the hoop in series with a Leyden jar L and the spark gap of an induction coil I as shown. If the induction coil be now put into operation, sparks will be observed to pass across S every time they jump across the gap at I.

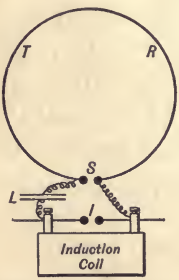


FIG. 401. — Illustrating an Effect of an Electric Discharge.

The spark at S indicates that the air gap offers a better path than the metal loop STR. But we know that the resistance of the wire STR is but a fraction of an ohm, while that of the air at S is perhaps millions of ohms. The discharge through the loop, therefore, must meet with some impedance other than that which would be encountered by a direct current. From this and other effects we are led to infer that *the discharge at I is a rapid surging of electricity back and forth*, but one which lasts only for a fraction of a second. At the first rush of current in the ring a magnetic field is suddenly set up within the loop. This sudden magnetic change induces, according to Lenz's law (§ 448), an opposing E. M. F. which effectually prevents the flow of a large portion of the current in the loop. Likewise the sudden reversal of the discharge reverses the magnetic lines and again induces an opposing E. M. F. in the loop. Hence the greater portion of the discharge finds a better outlet through the gap *S*. Stronger proof of the oscillating nature of a discharge has been obtained by photographing a spark by the help of a revolving mirror. The result is shown in Fig. 402. The period of oscillation has been shown by this method to be of the order of a millionth of a second. As a rule the oscillations subside very quickly, as shown by the curve in Fig. 403.

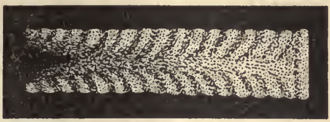


Fig. 402.— Result Obtained by Photographing an Electric Spark.

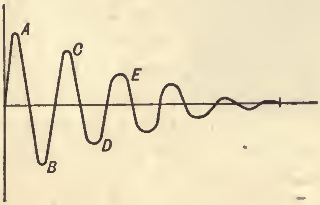


Fig. 403. — Diagram of a Spark Discharge.

476. Electro-magnetic Waves in the Ether. — In 1888 Hertz[1] of Germany showed that each electrical oscillation that occurs when a spark passes across an air gap produces a disturbance in the surrounding ether which is propagated outward in wave form in much the same manner as water waves move outward from a falling pebble, or sound waves from a vibrating bell. These ether waves are called *electro-magnetic waves.* Inasmuch as light itself is propagated in wave form in the ether, it might be inferred that the speed of the two should be the same. Such has been found to be the case.

[1] See portrait facing page 464. See also Kelvin, frontispiece.

477. Detection of Electro-magnetic Waves. — The process of transmitting messages by means of electro-magnetic ether waves is dependent on the detection of such waves at the receiving station. One method that can be employed for this purpose is illustrated by the following experiment.

Bend a glass tube about 5 centimeters long and 3 or 4 millimeters in diameter as shown in Fig. 404. Place a few coarse iron filings in the bend and introduce a globule of clean mercury into each end of the tube as shown at A and B. Insert small iron wires into the mercury and connect the device in series with a cell and an ammeter or galvanometer. No deflection should be produced. Now cause a spark to jump a short air gap several feet away by using an induction coil, an influence machine (§ 371), or a Leyden jar. A deflection will be observed at once. If the tube of filings be now tapped lightly, the circuit is again broken, and the experiment may be repeated.

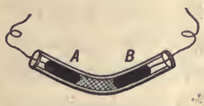
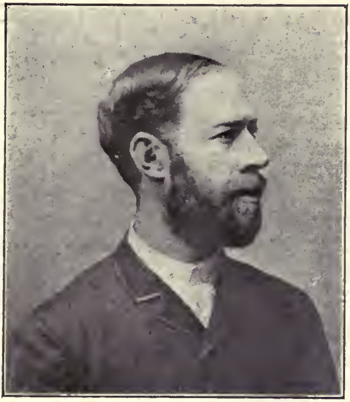


FIG. 404. — A Coherer.

The tube of metal filings used as a detector of electro-magnetic waves is called a **coherer**. Ordinarily the filings offer a large resistance to the flow of electricity through the many points of contact between the several pieces. The waves emitted by the oscillatory discharge at the spark gap cause the filings to cling together, and the resistance of the tube immediately falls to a few ohms. A tap or jar breaks the filings apart, and the resistance rises again to its former value.

478. Wireless Telegraphy. — The possibility of sending out electro-magnetic waves from an induction coil and of detecting them by a coherer has led to the transmission of messages by the so-called *wireless telegraph systems.* A plan of a simple sending and receiving equipment is shown in Figs. 405 and 406.

Connect the coherer described in the preceding section in series with the electro-magnet of a relay R and a cell C, Fig. 406. In the



HEINRICH HERTZ (1857–1894)

About the middle of the last century Maxwell advanced the idea that waves of light are electro-magnetic in character. If this were true of light, then it would also be true of radiant heat. In 1888 Hertz of Germany succeeded in demonstrating experimentally the truth of this assumption. During the discharge of electricity between two polished knobs, so-called electro-magnetic waves are radiated into space. Hertz was able to detect these waves and to reflect, refract, and polarize them and to cause interference to take place between them. He also measured the velocity with which they are propagated through space, and found it to be equal to the velocity of light. Thus the hypothesis of Maxwell was placed upon an experimental basis, and the way opened for long-distance communication between stations without the necessity of connecting wires. The practical value of Hertz's results can hardly be overestimated. Among others who have contributed greatly to the knowledge of electro-magnetic waves may be mentioned Sir Oliver Lodge of England, Righi of Bologna, and Branly of Paris.

In 1880 Hertz was made assistant to Helmholtz at Berlin. In 1885 he became professor of physics in the Technical High School at Karlsruhe. It was in the latter place that his epoch-making experiments were first performed. In 1889 he was elected professor of physics at Bonn, where he died at the age of thirty-seven years. Electro-magnetic waves are called Hertzian waves in his honor.

make-and-break circuit of the relay connect an electric bell B (§ 411) and a cell C'. The bell is so placed that its hammer strikes the coherer D whenever it is set in vibration. One end of the coherer should be connected to the earth and the other joined to a high aërial wire. For short distances the aërial wire may be very short or left off altogether. At the sending station (Fig. 405), simply connect one side of a spark gap of an induction coil with the earth and join the other side with an aërial conductor which is merely a rod or wire, one end of which is lifted some distance above the apparatus. A key K should be placed in the circuit of the primary coil. Closing the key and thus producing sparks at S causes the bell to ring at the receiving station until the circuit at the sending station is broken.

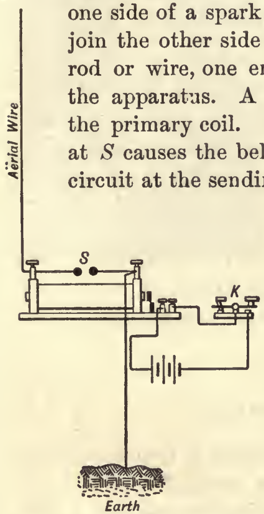


FIG. 405. — Apparatus for Sending Wireless Telegraph Signals.

When sparks pass at the so-called oscillator S, electro-magnetic waves are emitted which affect the coherer at the receiving station, causing the filings to conduct as shown in § 477. The current from the cell C then flows through the coherer and the electro-magnet of the relay R. The relay closes at A the circuit through the bell which is rung by the current from the cell C'. The filings continue to cohere until sparks cease to pass at S, when the taps of the bell hammer jar them apart. The circuit

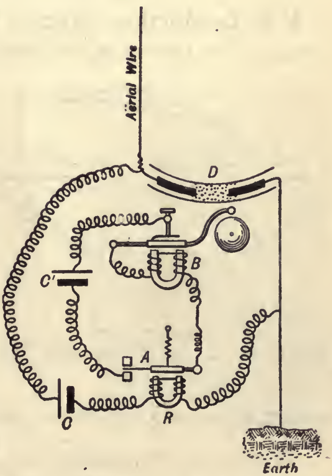


FIG. 406. — A Receiving Station for Wireless Messages.

through the relay is thus broken, which in turn opens the bell circuit at *A*. Another spark at *S* again causes *D* to conduct and the bell to ring. Many other forms of receiving devices are in common use.

Systems of wireless telegraphy have reached such a state of development that ocean-going vessels are, as a rule, at all times in communication with land stations or with one another. Passengers on sinking vessels have thus been rescued by timely assistance obtained through wireless messages which were received at stations many miles away. Naval fleets equipped with a good system may be kept continually informed by the controlling department of the government, and the department may be kept acquainted with every movement of the fleet. Even transatlantic messages are now transmitted without the use of wires or cables.

2. CONDUCTION OF GASES

479. Conduction through Vacuum Tubes. — Connect the terminals of an induction coil with electrodes *A* and *B*, Fig. 407, which are sealed in the ends of a glass tube 2 or 3 feet in length. Connect the tube with an air pump and put the coil in operation. Sparks will jump across the gap *S* because it is the better path. Put the air pump in action and exhaust the tube while the coil is running. When the pressure has been sufficiently reduced, the discharge will begin to take place through the long exhausted tube rather than over the shorter path through the air at *S*.

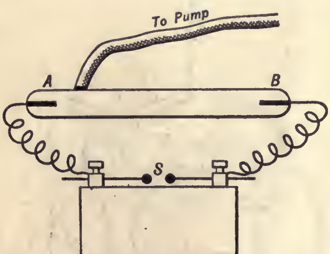


Fig. 407. — Discharge through a Partial Vacuum.

When the exhaustion of the gas has been carried much farther, there may be seen a radiation from the cathode proceeding in straight lines and traceable by a slight

SIR WILLIAM CROOKES (1832-)

The discharge of electricity through partially exhausted tubes has been a subject of much research since the middle of the last century. In 1853 Masson of Paris discharged electricity from a large induction coil through a Torricellian vacuum. Later Geissler, a German, constructed excellent tubes containing small amounts of different gases, which became famous on account of the great beauty of color manifest upon discharging electricity through them. Crookes began his experiments in 1873 with tubes in which the exhaustion was carried to a high degree. In these he showed that so-called "radiant matter" is thrown out from the electrodes in straight lines, casts shadows when intercepted by solids, and is capable of producing mechanical effects when brought into collision with light movable vanes. Furthermore, he showed that the stream of particles is deflected from a straight line by a magnet.

WILHELM KONRAD RÖNTGEN (1845-)

The most striking phenomena accompanying discharges in highly exhausted tubes were discovered by Professor Röntgen at Würzburg, Germany, in 1895. A Crookes tube was left in operation on a table. Beneath it was a book containing a key as a bookmark; below the book was a photographic plate in a plate holder. Later, on using the plate in a camera, and developing it in the usual manner, a well-defined shadow of the key became visible upon it. Further experimentation showed the discoverer that he had found a new kind of radiation which he named X-rays. The important ends attained by the use of X-rays in the practice of medicine and surgery will always serve to keep the name of Röntgen before the public.

luminescence occasioned in the gas remaining in the tube. Where these rays fall upon the glass, it is made warm and luminous. Ordinary glass glows with a soft greenish yellow light; and a solid, as marble, placed in the path of the radiations will glow with a characteristic color. These radiations are known as *cathode rays.* Such highly exhausted and hermetically sealed tubes are known as *Crookes'*[1] *tubes.*

480. Cathode Rays. — Cathode rays are characterized by three main properties, viz. (1) they are deflected from a straight line when caused to pass through an electric or a magnetic field, (2) they convey a negative charge of electricity to the object upon which they fall, and (3) they raise the temperature of any solid object which obstructs their path. The inference is, therefore, that *cathode rays are swiftly moving particles charged with negative electricity.* The velocity with which the particles move sometimes reaches the enormous value of over 50,000 miles per second, — nearly one third of the velocity of light. A continuous stream of such particles, all of which carry negative charges, is equivalent to a current of electricity; hence their deviation in passing through a magnetic field.

481. X-Rays. — The most important application of the action of cathode rays is employed in the production of the so-called X-rays, which were discovered in 1895 by Röntgen,[1] a German physicist. A special vacuum tube of the form shown in Fig. 408 is used for this purpose. This is connected with the secondary of an induction coil in such a manner that C, a concave electrode, is

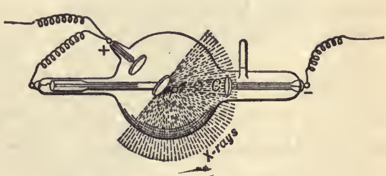


FIG. 408. — An X-Ray Tube.

[1] See portrait facing page 466.

the cathode, and P the anode. Since the cathode particles are sent off at right angles to the surface which they leave, they are focused against the solid piece of platinum P placed near the center of the tube. Accompanying the impact of the cathode rays against P appears a new radiation of an entirely different character. It is to these that the name of *X-rays*, or *Röntgen* rays, has been given.

482. Properties of X-Rays. — The properties which give to X-rays their great practical utility is their power (1) to penetrate bodies of matter that are opaque, and (2) to make a permanent impression upon photographic plates. Substances are not all equally transparent to the rays; *e.g.* they more readily penetrate a thick book than a thin coin; and while flesh is quite transparent, bones are more or less opaque. Hence, if the hand be held between an X-ray tube and a photographic plate, the bones will shield the plate more than the flesh and thus produce shadows. Fig. 409 shows an X-ray picture of a broken wrist, and the same wrist is shown in Fig. 410 after having healed.

3. RADIO-ACTIVITY

483. Radio-active Substances. — Place a gas mantle that has been pressed out flat upon a photographic plate and inclose it in a light-proof box. After three or four weeks develop the plate in the usual way. A distinct image of the fabric of the mantle will become visible. Such a plate is shown in Fig. 411.

Similar experiments may be made with compounds containing uranium, especially by using the mineral uraninite, or pitchblende. These experiments show that *certain substances spontaneously emit a kind of radiation capable of affecting a photographic plate.* Such substances are said to be *radio-active*, or to possess the property of *radio-activity.* The radio-active element contained in a gas mantle is thorium. The element *radium* is remarkable for its

FIG. 409.—X-Ray Picture Showing the Fractured Bones of a Wrist.

FIG. 410.—X-Ray Picture of the Wrist Shown in Fig. 409, after the Bones had Knitted.

FIG. 411.—The Radio-active Effect of a Portion of a Gas Mantle on a Photographic Plate.

intense radio-activity. The discovery of this element by Monsieur and Madame Curie[1] of Paris, in 1898, resulted from the fact that its presence in microscopic quantities in tons of the mineral from which it was taken was detected by its exceptionally large radio-active effects.

484. Radio-Activity. — The property of radio-activity was discovered by Henri Becquerel[2] of Paris in 1896. In honor of the discoverer the radiations emitted by radio-active substances are often called *Becquerel* rays. Experimental researches have shown that Becquerel rays are of a complex nature. They consist (1) of negatively charged particles called *beta* rays, (2) of positively charged particles called *alpha* rays, and (3) of radiations resembling X-rays in nature, which are called *gamma* rays.

Becquerel rays are detected not only by their power to affect a photographic plate as shown in § 483, but also by their power to discharge electrified bodies near which they are placed. This effect is due to the fact that the air surrounding a radio-active body is rendered a conductor of electricity by the radiations emitted. Furthermore, if minute crystals of zinc sulphide be placed in the immediate neighborhood of an extremely small quantity of radium, they show intermittent flashes of light as they are bombarded by the alpha particles expelled by the radium.

485. Electrons. — Beta rays, or the negatively charged particles emitted by a radio-active substance, are called *negative electrons*. Negative electrons are separated from alpha and gamma rays on passing through a strong electric or magnetic field on account of the difference in the kind of charge carried by them. The path of the negative electrons is curved in one direction by the field, while that of

[1] See portrait facing page 470.

the alpha rays is bent in the opposite direction. The gamma rays remain unchanged in direction.

Knowledge of electrons began with the discovery by Sir J. J. Thompson of England that the cathode rays produced in a vacuum tube (§ 480) consist of swiftly moving particles charged with electricity. These particles have since been found to be identical with the negative electrons emitted by a radio-active substance. The speed attained by the electrons in a cathode tube is about 62,000 miles per second, but the electrons emitted by radium move with speeds that reach as high as 165,000 miles per second, which is about $\frac{9}{10}$ the velocity of light.

The mass of an electron has been ascertained and found to be about $\frac{1}{1700}$ the mass of an atom of hydrogen. Differences in the electrical, magnetic, and other properties of matter are attributed to variations in the arrangements and movements of the electrons associated with the molecules. For example, in electrical conductors the electrons are less firmly attached to the molecules than they are in non-conductors, and are consequently moved readily by potential differences. This motion of electrons through a conductor constitutes an electric current.

486. Alpha Particles. — The alpha rays emitted by a radio-active substance are of atomic size and have been found to be charged with positive electricity. Experiments have also shown that they are atoms of the well-known gas *helium*, and that their velocity may reach as high as $\frac{1}{10}$ the speed of light. The energy that is developed by a radio-active body is due mainly to the alpha particles which it hurls outward with this enormous rate of motion. It is estimated that the total amount of energy that can be given off by a gram of radium is about equal to that developed by the combustion of a ton of coal.

ANTOINE HENRI BECQUEREL (1852–1908)

A new epoch in the theory of matter was inaugurated by the discovery of radio-activity in 1896 by Becquerel of Paris. It had long been known that compounds containing the element uranium would produce an effect upon photographic plates in the dark. This singular action had been attributed merely to their property of phosphorescence. Becquerel proved that all compounds containing uranium act similarly on plates, even those which are not phosphorescent. This phenomenon corresponds to a continuous emission of energy, for which the old view of matter did not account.

MADAME CURIE (1867–   )

Soon after the discovery of radio-activity, investigations were made by Madame Curie of Paris and others in order to ascertain if radio-activity were not a general property of matter. Various compounds were tested; but the strange property appeared to be confined to substances containing uranium and another element, thorium. However, it was observed by Madame Curie that certain pitchblendes (minerals containing oxide of uranium and other well-known elements) were many times more radio-active than uranium. It therefore seemed probable that an unknown element of great radio-activity was present in the mineral. From this grew up the celebrated experiments of Monsieur and Madame Curie which led to the discovery of the remarkable element radium. Although the separation of radium from minerals is attended with enormous difficulties on account of the small quantity which they contain, enough of it has been secured for experimental research in the laboratories of the world to lead to the generalizations in regard to the constitution of matter contained in the concluding sections of this book.

487. Disintegration of Matter. — The emission of electrons and of positively charged particles which are comparable in mass with atoms of hydrogen leads at once to the conclusion that a radio-active substance must be continually experiencing a molecular change. Although a gram of radium develops hourly an amount of heat equal to 100 calories, its transformation is so slow that nearly 13 centuries would elapse before one half its mass would suffer a change in its nature. Experiments have shown that it is along with this transformation that the element helium is produced. Thus the radio-active elements are not permanent, but by the emission of electrons and alpha particles they are being constantly transformed into elements of smaller atomic weights. *Uranium*, for example, is supposed to be breaking up and forming for one of its products the element *radium*; radium in turn forms so-called *radium emanation*, and so on through a series. It is while these transformations are going on that the alpha, beta, and gamma rays are emitted.

According to the disintegration theory of matter advanced by Rutherford and Soddy, the atoms of radio-active substances are unstable systems which break up spontaneously with explosive violence, expelling a small portion of the fractured atom with great velocity. The remaining portion of each atom forms a new system of a smaller atomic weight and possessing properties differing from those of the atoms of the parent substance. This new substance may be unstable also and undergo an atomic change similar to the preceding. The process may continue by stages until at last a stable form is finally attained.

488. The Domain and Future of Physics. — The study of Physics is primarily an investigation of environment to the end that the knowledge obtained shall be conducive to the comfort of mankind, and lead to an increase of man's power in his range of action. If this were all, the pursuit of physical science would be amply justified, for the field is a broad one, and the results to which the study has led are overwhelming. It has opened the entire world to the traveler, it has brought the West within speaking distance of the East, it has overthrown the dangers and solitude of the sea, it has brought distant worlds within our range of vision and exposed the sources of disease, it reveals the secret of aërial flight, and, now, at the present rate of advancement man may soon acquire that knowledge which the human mind has long sought to secure in answer to the all-important question, "What is matter?"

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(2)